ECON 203
Midterm on Consumer Theory

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 19:40 and end at 21:20

1. *(20 points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test.

   Name and Surname: _

   Student ID: _

   Signature:

   _

2. *(15 points)* What three axioms must a person satisfy in order to be considered normatively rational? For each of the three axioms write down a definition (words or mathematics will be fine) and give a counter-example showing that sometimes real people do not have these preferences.

   Solution 1

   (a) Reflexivity: $A \succsim A$ where $A \succsim B$ means that A is at least as good as B.
   Counter Example: The best counter-example (due to Amartya Sen) is the "second largest slice of cake preferences." Say that you are at your potential in-laws house, and his/her father brings out a cake after dinner which he baked himself. He cuts slices into various sizes and asks you which one you want. You don't want to choose the largest—because that's greedy, or the smallest—because that says you don't think he can cook. You choose the second largest slice, which can easily be shown to be not reflexive. For example out of $\left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$ you choose $\frac{1}{4}$, out of $\left\{\frac{1}{4}, \frac{1}{5}, \frac{1}{6}\right\}$ you choose $\frac{1}{5}$. The first choice indicates $\frac{1}{4} \succ \frac{1}{5}$, the second that $\frac{1}{5} \succ \frac{1}{4}$, and thus we conclude $\frac{1}{4} \succ \frac{1}{4}$. Crazy? Only because you assumed that preferences only depended on the choices, these depend on the options available as well. (I should mention that $A \succ B$ means that A is strictly better than B).

   (b) Transitivity: If $A \succsim B$ and $B \succsim C$ then $A \succsim C$
   Counter Example: This rules out decision cycles. We all go through these when making a major purchase. A is reasonably priced and good, but B is only a little bit more expensive but has many features, but then C is much cheaper than B even though it doesn't have as many features as either A or B. The last time I went through this was when I was buying a TV. Transitivity says that I can resolve such cycles, and I did because I bought a TV.

(c) Completeness: for any A and B either $A \succsim B$ or $B \succsim A$ or both.

Counter Example: This says you can compare all options, so take any two options that are so weird you can't do the comparison and there you go. For example a Chalet on the Moon versus an Apartment on Mars. Personally I have no idea which one I would want... but if I have complete preferences I should be able to say.

3. *(28 points total)* About Giffen goods:

(a) *(12 points)* Write down the Slutsky equation in elasticity form. Define each term.

Solution 2

$$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x$$

where $e_x(p_x)$ is the own price elasticity of Marshallian demand, $e_{h_x}(p_x)$ is the same for the Hicksian or Income compensated demand, $e_x(I)$ is the income elasticity of demand, and $s_x = \frac{p_x X}{I}$ is the share of my income spent on x.

(b) *(3 points)* Show that if $F = Y^\tau$ then $e_F(Y) = \tau$ (where $e_F(Y)$ is the elasticity of F with regards to Y.)

Solution 3

$$\begin{aligned}
\frac{dF}{dY} &= \tau Y^{\tau-1} \\
e_F(Y) &= \frac{dF}{dY}\frac{Y}{F} = \tau Y^{\tau-1}\frac{Y}{Y^\tau} = \tau
\end{aligned}$$

(c) *(13 points)* Consider the following three curves. For each first find the elasticities with regard to price (p_x) and income (I). (The impact of all other factors is held fixed. They're in the constant.) Second state whether each can be a demand curve. Finally if it can be a demand curve, find how large the share of income spent on that good must be.

β	α	μ	s_x	ρ	γ	τ	s_x	κ	ϕ	λ
10	1	2	$\frac{1}{3}$	15	$\frac{1}{3}$	$\frac{2}{3}$	4	20	$\frac{1}{2}$	$\frac{2}{5}$
8	1	3	$\frac{1}{4}$	8	$\frac{2}{3}$	$\frac{2}{3}$	5	3	$\frac{1}{4}$	$\frac{1}{10}$
6	2	3	$\frac{1}{2}$	12	$\frac{1}{4}$	$\frac{3}{4}$	5	9	$\frac{1}{5}$	$\frac{3}{11}$
15	1	4	$\frac{1}{5}$	4	$\frac{3}{4}$	$\frac{3}{4}$	7	15	$\frac{2}{5}$	$\frac{1}{5}$

i. $X = \beta p_x^\alpha I^{-1-\mu}$

Solution 4

$$\begin{aligned}
e_x(p_x) &= \alpha > 0 \\
e_x(I) &= -1 - \mu
\end{aligned}$$

this is a giffen good, it is inferior so that's OK, the final thing we can use to find out if it is legal or not is the slutsky equation.

$$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x$$

or

$$e_{h_x}(p_x) = e_x(p_x) + e_x(I)\, s_x$$

now the left hand side has to be negative, so we need:

$$\begin{aligned} e_x(p_x) + e_x(I)\, s_x &\leq 0 \\ \alpha - (1+\mu)\, s_x &\leq 0 \\ \frac{\alpha}{1+\mu} &\leq s_x \end{aligned}$$

and since $\alpha < 1 + \mu$ *this could be a demand curve.*

ii. $X = \rho p_x^{1+\gamma} I^{\tau-1}$.

Solution 5

$$\begin{aligned} e_x(p_x) &= 1+\gamma > 0 \\ e_x(I) &= \tau - 1 < 0 \end{aligned}$$

again this is a giffen good, or at least potentially. Checking the second condition we must have:

$$\begin{aligned} e_x(p_x) + e_x(I)\, s_x &\leq 0 \\ 1+\gamma - (1-\tau)\, s_x &\leq 0 \\ \frac{1+\gamma}{1-\tau} &\leq s_x \end{aligned}$$

and since $\frac{1+\gamma}{1-\tau} > 1$ *this is not possible. Thus this can not be a demand curve.*

iii. $X = \kappa p_x^{\phi} I^{\lambda}$

Solution 6

$$\begin{aligned} e_x(p_x) &= \phi > 0 \\ e_x(I) &= \lambda > 0 \end{aligned}$$

if this was a giffen good it would have to be inferior, but it is a normal good and thus this can not be a demand curve.

α	β	σ	BFB_c	BFB_f	$C(p_c, p_f, I)$	$F(p_c, p_f, I)$
1	16	3	$\frac{1}{C^4 p_c}$	$\frac{16}{F^4 p_f}$	$\frac{p_c^{\frac{3}{4}}}{p_c^{\frac{3}{4}}+2p_f^{\frac{3}{4}}} \frac{I}{p_c}$	$\frac{2p_f^{\frac{3}{4}}}{p_c^{\frac{3}{4}}+2p_f^{\frac{3}{4}}} \frac{I}{p_f}$
1	8	2	$\frac{1}{C^3 p_c}$	$\frac{8}{F^3 p_f}$	$\frac{p_c^{\frac{2}{3}}}{p_c^{\frac{2}{3}}+2p_f^{\frac{2}{3}}} \frac{I}{p_c}$	$\frac{2p_f^{\frac{2}{3}}}{p_c^{\frac{2}{3}}+2p_f^{\frac{2}{3}}} \frac{I}{p_f}$
8	1	2	$\frac{8}{C^3 p_c}$	$\frac{1}{F^3 p_f}$	$\frac{2p_c^{\frac{2}{3}}}{2p_c^{\frac{2}{3}}+p_f^{\frac{2}{3}}} \frac{I}{p_c}$	$\frac{p_f^{\frac{2}{3}}}{2p_c^{\frac{2}{3}}+p_f^{\frac{2}{3}}} \frac{I}{p_f}$
16	1	3	$\frac{16}{C^4 p_c}$	$\frac{1}{F^4 p_f}$	$\frac{2p_c^{\frac{3}{4}}}{2p_c^{\frac{3}{4}}+p_f^{\frac{3}{4}}} \frac{I}{p_c}$	$\frac{p_f^{\frac{3}{4}}}{2p_c^{\frac{3}{4}}+p_f^{\frac{3}{4}}} \frac{I}{p_f}$

4. *(31 points total)* For the utility function $U(C,F) = -\frac{\alpha}{\sigma C^{\sigma}} - \frac{\beta}{\sigma F^{\sigma}}$.

(a) *(6 points)* Find the marginal utility of food and clothing and the marginal rate of substitution.

Solution 7 $U = -\frac{\alpha}{\sigma}C^{-\sigma} - \frac{\beta}{\sigma}F^{-\sigma}$, $\frac{\partial U}{\partial C} = \left(-\frac{\alpha}{\sigma}\right)(-\sigma)C^{-\sigma-1} = \frac{1}{C^{\sigma+1}}\alpha$, $\frac{\partial U}{\partial F} = \left(-\frac{\beta}{\sigma}\right)(-\sigma)F^{-\sigma-1} = \frac{1}{F^{\sigma+1}}\beta$

$MRS = \frac{\partial U}{\partial C}/\frac{\partial U}{\partial F} = \frac{\frac{1}{C^{\sigma+1}}\alpha}{\frac{1}{F^{\sigma+1}}\beta} = \frac{\alpha}{\beta}\left(\frac{F}{C}\right)^{\sigma+1}$

(b) *(6 points)* Which common assumptions about utility functions does this satisfy? You only need to consider $C > 0$ and $F > 0$. (Note: Of course it has to satisfy the axioms required to have a utility function. I will only give credit for other assumptions.)

Solution 8 *It is* ***strictly monotonic*** *because for* $C > 0$ *and* $F > 0$ $MU_c > 0$ *and* $MU_f > 0$.

It is ***strictly convex*** *because* $\frac{\partial MRS}{\partial C} = \frac{\alpha}{\beta}(\sigma+1)\left(\frac{F}{C}\right)^{\sigma+1-1}(-1)\frac{1}{C^2} = -\frac{1}{C^2}\frac{\alpha}{\beta}(\sigma+1)\left(\frac{F}{C}\right)^{\sigma} < 0$

(c) *(2 points)* Set up the objective function for utility maximization. Let the price of clothing be $p_c > 0$, the price of food be $p_f > 0$ and the income be $I > 0$.

Solution 9

$$\max_{F,C}\min_{\lambda} -\frac{\alpha}{\sigma C^{\sigma}} - \frac{\beta}{\sigma F^{\sigma}} - \lambda(p_c C + p_f F - I)$$

(d) *(3 points)* Find the first order conditions.

Solution 10

$$\begin{aligned}\frac{1}{C^{\sigma+1}}\alpha - \lambda p_c &= 0\\ \frac{1}{F^{\sigma+1}}\beta - \lambda p_f &= 0\\ -(p_c C + p_f F - I) &= 0\end{aligned}$$

(e) *(6 points)* Find the bang for the buck of food and clothing, and a function for F in terms of C and the prices.

Solution 11

$$\begin{aligned}\frac{1}{C^{\sigma+1}}\alpha - \lambda p_c &= 0\\ \lambda &= \frac{1}{C^{\sigma+1}}\frac{\alpha}{p_c} = BFB_c\\ \frac{1}{F^{\sigma+1}}\beta - \lambda p_f &= 0\\ \lambda &= \frac{1}{F^{\sigma+1}}\frac{\beta}{p_f} = BFB_f\end{aligned}$$

$$
\begin{aligned}
\frac{1}{C^{\sigma+1}}\frac{\alpha}{p_c} &= \frac{1}{F^{\sigma+1}}\frac{\beta}{p_f} \\
\alpha p_f F^{\sigma+1} &= \beta p_c C^{\sigma+1} \\
F^{\sigma+1} &= \frac{\beta p_c}{\alpha p_f} C^{\sigma+1} \\
F &= \left(\frac{\beta p_c}{\alpha p_f}\right)^{\frac{1}{\sigma+1}} C
\end{aligned}
$$

(f) *(4 points)* Find the demand curve for C. Simplify if possible.

Solution 12

$$
\begin{aligned}
p_c C + p_f F - I &= 0 \\
p_c C + p_f \left(\frac{\beta p_c}{\alpha p_f}\right)^{\frac{1}{\sigma+1}} C - I &= 0
\end{aligned}
$$

$$
\begin{aligned}
C &= \frac{I}{p_c + p_f \left(\frac{1}{\alpha}\beta\frac{p_c}{p_f}\right)^{\frac{1}{\sigma+1}}} \\
&= \frac{p_c}{p_c} \frac{1}{p_c + p_f^{1-\frac{1}{\sigma+1}} p_c^{\frac{1}{\sigma+1}} \left(\frac{1}{\alpha}\beta\right)^{\frac{1}{\sigma+1}}} I \\
& \frac{1}{\frac{p_c}{p_c} + p_f^{1-\frac{1}{\sigma+1}} p_c^{\frac{1}{\sigma+1}-1} \left(\frac{1}{\alpha}\beta\right)^{\frac{1}{\sigma+1}}} \frac{I}{p_c} \\
& \frac{1}{1 + \left(\frac{1}{\alpha}\beta\right)^{\frac{1}{\sigma+1}} \left(\frac{p_f}{p_c}\right)^{\frac{\sigma}{\sigma+1}}} \frac{I}{p_c} \\
& \frac{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}}}{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}} + \beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}} \frac{I}{p_c}
\end{aligned}
$$

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(g) *(4 points)* Find the demand for F. Simplify if possible.

Solution 13

$$
\begin{aligned}
p_f F + p_c C &= I \\
p_f F + p_c \left(\frac{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}}}{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}} + \beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}} \frac{I}{p_c} \right) &= I \\
p_f F &= I - \frac{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}}}{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}} + \beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}} I \\
&= \left(1 - \frac{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}}}{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}} + \beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}} \right) I \\
&= \left(\frac{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}} + \beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}}{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}} + \beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}} - \frac{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}}}{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}} + \beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}} \right. \\
&= \frac{\beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}}{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}} + \beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}} I \\
F &= \frac{\beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}}{\alpha^{\frac{1}{\sigma+1}} p_c^{\frac{\sigma}{\sigma+1}} + \beta^{\frac{1}{\sigma+1}} p_f^{\frac{\sigma}{\sigma+1}}} \frac{I}{p_f}
\end{aligned}
$$

5. *(6 points)* Write down one of the four great insights of rationality (notice that two of these are called 3a and 3b in the handout). Define the insight and explain it by giving an example.

Solution 14 *The four insights are:*

(a) *It's the margin not the average.*

(b) *It's all relative.*

(c) *Sunk costs are Sunk Costs*

(d) *It's opportunity costs, not accounting costs.*
For examples I refer you to my introductory handout.