ECON 203
Final on Producer Theory and Equilibrium

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 9:40 and end at 11:20

1. *(10 points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test.

   Name and Surname: _

   Student ID: _

   Signature:

   _

2. *(14 points total)* About Pareto Efficiency.

   (a) *(3 points)* Define *Pareto Dominance* or (equivalently) *Pareto Improvement.*

   Solution 1 *An allocation A Pareto Dominates (or Pareto Improves) on an allocation B if everyone likes A more and some strictly prefer it.*

   (b) *(3 points)* Define *Pareto Efficiency.*

   Solution 2 *An allocation A is Pareto Efficient if there is no allocation that Pareto dominates it. Equivalently every allocation that make some (strictly) better off makes others strictly worse off.*

   (c) *(4 points)* Explain why suicide bombings must be Pareto Efficient (though morally abhorrent.)

   Solution 3 *Since someone is giving their life for the (opportunity to) kill people, this must be the best possible outcome according to his or her preferences. Thus any feasible allocation where they are stopped must be strictly worse for them, and thus does not Pareto dominate this allocation.*

   (d) *(4 points)* Monopolies are illegal. Thus when a court case proves a firm is a monopolist the result is either the monopoly is broken up into smaller companies or taken over by the government. Explain why this is not a Pareto Improvement and how it could be made to be. (Note: Monopolies cause dead weight loss.)

Solution 4 *The statement above does not mention compensating the Monopolist. In order to Pareto Improve on the current allocation (with a Monopolist) the monopolist must be made happier as well as the consumers. This is possible, the government could buy the Monopoly and then either break it up or take it over, but has never been implemented as far as I know.*

α	β	τ	X	ρ	χ	B_a^*	C_a^*	B_p^*	C_p^*	R^*	B_b^*	C_b^*
1	7	1	135	10	1	3	126	5	110	160	2	140
1	4	$\frac{1}{4}$	36	12	4	4	32	6	27	180	3	36
1	$\frac{5}{2}$	$\frac{1}{4}$	54	8	2	6	45	8	38	140	5	50
1	2	$\frac{1}{4}$	80	5	1	8	64	10	55	105	7	70

3. *(34 points total)* Robison Crusoe is a poor, impoverished Imperialist. Being stranded on an island, he's too afraid to scout around nearby islands for help. Instead he exists consuming Boa Constrictors (B) and Coconuts (C). His preferences are $U(B,C) = B^\alpha C^\beta$ and his production possibilities frontier is $C + \tau B^2 \leq X$.

   (a) *(1 point)* Set up the Lagrangian of his utility maximization problem.

$$L(\lambda, B, C) = B^\alpha C^\beta - \lambda\left(C + \tau B^{(2)} - X\right)$$

   (b) *(3 points)* Find the first order conditions.

$$\begin{aligned} \frac{\partial L}{\partial \lambda} &= -\left(C + \tau B^{(2)} - X\right) = 0 \\ \frac{\partial L}{\partial B} &= \alpha \frac{U}{B} - \lambda \tau (2) B^{(2)-1} = 0 \\ \frac{\partial L}{\partial C} &= \beta \frac{U}{C} - \lambda = 0 \end{aligned}$$

   (c) *(8 points)* Solve for the optimal amount of Boa Constrictors and Coconuts to consume.

$$\frac{C}{\beta} = \frac{U}{\lambda} = 2\frac{B^2}{\alpha}\tau$$

$$C = 2\frac{B^2}{\alpha}\beta\tau$$

$$\begin{aligned} \left(2\frac{B^2}{\alpha}\beta\tau\right) + \tau B^2 &= X \\ \frac{C}{\beta 2}(\alpha + \beta 2) &= X \\ B &= \frac{\alpha}{\alpha\tau + 2\beta\tau}\sqrt{\frac{X}{\alpha}\tau(\alpha + 2\beta)} \\ C &= 2X\frac{\beta}{\alpha + 2\beta} \end{aligned}$$

Now Robinson Crusoe has realized there is a very advanced native society living on the island right next to him.

(d) Being afraid that their vast technological superiority will put him at a disadvantage, he first decides he will only trade for Boa Constrictor meat and Coconuts. The price of Boa Constrictor meat is $p_b = \rho$ and the price of Coconuts is $p_c = \chi$.

i. *(1 point)* Set up the Lagrangian objective function that should determine how much he will produce of Boa Constrictors and Coconuts.

$$R(\lambda, B, C) = \rho B + \chi C - \lambda \left(C + \tau B^2 - X\right)$$

ii. *(3 points)* Find the first order conditions of this objective function.

$$\begin{aligned}
\frac{\partial R}{\partial \lambda} &= -\left(C + \tau B^2 - X\right) = 0 \\
\frac{\partial R}{\partial B} &= \rho - \lambda 2\tau B = 0 \\
\frac{\partial R}{\partial C} &= \chi - \lambda = 0
\end{aligned}$$

iii. *(4 points)* Find the optimal amount of Boa Constrictor and Coconuts for him to produce.

$$\begin{aligned}
\chi &= \lambda = \frac{1}{2B\tau}\rho = \chi \\
B &= \frac{1}{2\tau\chi}\rho \\
C + \tau\left(\frac{1}{2\tau\chi}\rho\right)^{(2)} &= X \\
C &= X - \frac{1}{4\tau\chi^2}\rho^2
\end{aligned}$$

$$\begin{aligned}
R &= \rho\left(\frac{1}{2\tau\chi}\rho\right) + \chi\left(X - \frac{1}{4\tau\chi^2}\rho^2\right) \\
&= \frac{1}{4\tau\chi}\left(4X\tau\chi^2 + \rho^2\right)
\end{aligned}$$

iv. *(1 point)* Set up his utility maximization problem when he can trade his Boa Constrictor meat and Coconuts.

$$L(\lambda, B, C) = B^\alpha C^\beta - \lambda\left(\rho B + \chi C - R\right)$$

v. *(3 points)* Find the first order conditions.

$$\begin{aligned}\frac{\partial L}{\partial \lambda} &= -(\rho B + \chi C - R) = 0\\ \frac{\partial L}{\partial B} &= \alpha\frac{U}{B} - \lambda\rho = 0\\ \frac{\partial L}{\partial C} &= \beta\frac{U}{C} - \lambda\chi = 0\end{aligned}$$

vi. *(4 points)* Find the optimal amount of Boa Constrictor and Coconuts for him to consume.

$$\begin{aligned}\frac{\rho B}{\alpha} &= \frac{U}{\lambda} = \frac{\chi C}{\beta}\\ \frac{\rho B}{\alpha} &= \frac{\chi C}{\beta}\\ B &= C\frac{\alpha}{\beta}\frac{\chi}{\rho}\end{aligned}$$

$$\begin{aligned}\rho\left(C\frac{\alpha}{\beta}\frac{\chi}{\rho}\right) + \chi C &= R\\ C &= R\frac{\beta}{\chi(\alpha+\beta)}\\ B &= \left(R\frac{\beta}{\chi(\alpha+\beta)}\right)\frac{\alpha}{\beta}\frac{\chi}{\rho}\\ &= R\frac{\alpha}{\rho(\alpha+\beta)}\end{aligned}$$

vii. *(3 points)* How can we be certain Robinson Crusoe is better off with trade?

Solution 5 *There are two ways to do it.*
One method is to point out that given the revenue earned will allow him to strictly afford his previous bundle. I.e. that

$$\rho\left(\frac{\alpha}{\alpha\tau + 2\beta\tau}\sqrt{\frac{X}{\alpha}\tau(\alpha+2\beta)}\right) + \chi\left(2X\frac{\beta}{\alpha+2\beta}\right) < \frac{1}{4\tau\chi}\left(4X\tau\chi^2 + \rho^2\right)$$

This indicates that he can consume his old bundle and still have money left to spend on more. Thus he must be strictly happier.
If you weren't able to solve for the optimal revenue, a second option is to rely on the fact that if all goods are consumed at a strictly positive level and the prices locally are not equal to the prices globally, then there will be a strict improvement. Thus we need to show that:

$$\frac{p_b^l}{p_c^l} = \frac{MU_b}{MU_c} = \frac{\alpha\frac{U}{B}}{\beta\frac{U}{C}} = \frac{1}{B}C\frac{\alpha}{\beta} = \frac{1}{\left(\frac{\alpha}{\alpha\tau+2\beta\tau}\sqrt{\frac{X}{\alpha}\tau(\alpha+2\beta)}\right)}\left(2X\frac{\beta}{\alpha+2\beta}\right)\frac{\alpha}{\beta} = \frac{2\sqrt{X\tau}}{\sqrt{\frac{1}{\alpha}(\alpha+2\beta)}} \neq$$

(e) *(4 points)* Explain to Robinson Crusoe why it would be better for him to also trade for other goods, rather than only Boa Constrictors and Coconuts.

Solution 6 *The simplest explanation is that it's not like anyone will force him to trade. If he wants (say) a loincloth and they have one for sale that he thinks is worth it how could he not be better off buying the loincloth? He's already seen that producing for export is worth it, now he just needs to realize that there's no reason to not buy what he wants. Sure, he'll be poor because he doesn't produce anthing that valuable for the Native society, but that's no reason to keep himself even poorer by not buying what he wants. Seems obvious, so why do so many countries freak out about it?*

ϕ	μ	β	α	τ	ν	χ	P_r^{sd}	$q_r(P)$	$q_o(P)$	$Q(P)$	P_d^{lr}	π_r^d	P_f^{lr}
8	2	6	16	4	12	72	4	$\frac{1}{4}P$	$\frac{1}{8}P$	$\begin{cases} 9P & P<4 \\ 12P & P>4 \end{cases}$	16	24	8
36	4	32	32	8	16	80	8	$\frac{1}{8}P$	$\frac{1}{16}P$	$\begin{cases} 5P & P<8 \\ 7P & P>8 \end{cases}$	32	28	24
9	1	5	24	$\frac{3}{2}$	14	60	4	$\frac{1}{2}P$	$\frac{1}{3}P$	$\begin{cases} 20P & P<4 \\ 27P & P>4 \end{cases}$	12	27	6
27	3	24	16	9	18	72	6	$\frac{1}{6}P$	$\frac{1}{18}P$	$\begin{cases} 4P & P<6 \\ 7P & P\geq 6 \end{cases}$	24	21	18

4. *(27 points total)* The land of Riverun is a vast land with only one river running through it. In the supply of watermelons, there are only ν firms that can locate by the river. These firms (type r) have an economic cost of $C_r(q) = \phi + \mu q^2$, the fixed sunk cost are $F_{su} = \beta$. There are an unlimited number of other firms who have to pipe water from the river to their farms. These firms (type o) have a cost of $C_o(q) = \alpha + \tau q^2$ and their fixed start up costs are zero.

(a) *(5 points)* Find the short run supply curve of the type r firms, with costs $C_r(q)$.

Solution 7

$$
\begin{aligned}
MC &= 2\mu q \\
AVC &= \frac{\phi + \mu q^2 - \beta}{q}
\end{aligned}
$$

$$
\begin{aligned}
MC &\geq AVC \\
2\mu q^2 &\geq \phi + \mu q^2 - \beta \\
\mu q^2 &\geq \phi - \beta \\
q &\geq \sqrt{\frac{\phi - \beta}{\mu}} \\
P &= MC\left(\sqrt{\frac{\phi - \beta}{\mu}}\right) \geq 2\mu\sqrt{\frac{\phi - \beta}{\mu}}
\end{aligned}
$$

If price is above this critical price the amount supplied will be

$$
\begin{aligned}
P &= 2\mu q(P) \\
q(P) &= \frac{P}{2\mu}
\end{aligned}
$$

thus the short run supply is:

$$
q_r(P) = \begin{cases} 0 & P \leq 2\mu\sqrt{\frac{\phi-\beta}{\mu}} \\ \frac{P}{2\mu} & P \geq 2\mu\sqrt{\frac{\phi-\beta}{\mu}} \end{cases}
$$

(b) *(5 points)* Find the short run supply curve of the type o firms, with costs $C_o(q)$.

Solution 8 *Since* $F_{st} = 0$ *we can see that:*

$$
\begin{aligned}
MC &= 2\tau q \\
AVC &= \frac{\tau q^2}{q} = q\tau
\end{aligned}
$$

and

$$
\begin{aligned}
MC &\geq AVC \\
2\tau q &\geq q\tau
\end{aligned}
$$

for all $q \geq 0$*. Thus*

$$
\begin{aligned}
P &= 2\tau q(P) \\
q_o(P) &= \frac{P}{2\tau}
\end{aligned}
$$

(c) *(4 points)* If there are $n_o = \chi$ type o firms, what will the industry short run supply curve be?

$$
Q(P) = \begin{cases} \chi\frac{P}{2\tau} & P < 2\mu\sqrt{\frac{\phi-\beta}{\mu}} \\ \chi\frac{P}{2\tau} + \nu\frac{P}{2\mu} & P \geq 2\mu\sqrt{\frac{\phi-\beta}{\mu}} \end{cases}
$$

(d) *(4 points)* Assuming that at least one type o firm produces, what will the long run price be in this market?

Solution 9 *In the long run the price will be the shut down price of the type o producers, the type r producers may not produce but since there is a limited supply of them and I said at least one type o producer produces it must be dependent on the type that has a potentially unlimited number.*

$$\begin{aligned} MC &= 2\tau q \\ AC &= \frac{\alpha + \tau q^2}{q} \end{aligned}$$

$$\begin{aligned} MC &= AC \\ 2\tau q &= \frac{\alpha + \tau q^2}{q} \\ q &= \frac{1}{\tau}\sqrt{\alpha\tau} \end{aligned}$$

$$\begin{aligned} P_d^{lr} &= MC\left(\frac{1}{\tau}\sqrt{\alpha\tau}\right) \\ &= 2\tau\left(\frac{1}{\tau}\sqrt{\alpha\tau}\right) = 2\sqrt{\alpha\tau} \end{aligned}$$

(e) *(3 points)* What will the profit of type r firms be? Explain the peculiar result you have just found.

Solution 10 *For type r*

$$\begin{aligned} MC_r &= 2\mu q \\ AC_r &= \frac{\phi + \mu q^2}{q} \end{aligned}$$

and thus they will produce when

$$\begin{aligned} 2\mu q &\geq \frac{\phi + \mu q^2}{q} \\ q &\geq \frac{1}{\mu}\sqrt{\phi\mu} \\ P &\geq 2\mu\left(\frac{1}{\mu}\sqrt{\phi\mu}\right) = 2\sqrt{\mu\phi} \end{aligned}$$

(I do not actually expect you to check this here, it will be checked in the next part of the question, but to be thorough I have.)

Thus the quantity they will produce is

$$q_r(P) = \frac{2\sqrt{\alpha\tau}}{2\mu}$$

and their revenue will be:

$$R(q) = 2\sqrt{\alpha\tau}\left(\frac{2\sqrt{\alpha\tau}}{2\mu}\right) = 2\alpha\frac{\tau}{\mu}$$

their costs are:

$$C_r(q) = \phi + \mu\left(\frac{2\sqrt{\alpha\tau}}{2\mu}\right)^2 = \frac{1}{\mu}(\alpha\tau + \mu\phi)$$

and their profits are:

$$\pi = 2\alpha\frac{\tau}{\mu} - \frac{1}{\mu}(\alpha\tau + \mu\phi) = \frac{1}{\mu}(\alpha\tau - \mu\phi) > 0$$

this is economic rent*, or excess profits these firms earn because they are better than other firms in the industry. In the long run only the marginal firm will have zero economic profit. Marginal firms might have strictly positive profits because they are better at their job.*

(f) *(4 points)* Now this country decides to import watermelons, the cost function of firms outside of Riverun is $C_r(q)$ and there are no transportation costs. What will be the long run price of watermelons in this case? (There is an unlimited number of foreign firms.)

Solution 11 *Just to restate what I did above:*

$$\begin{aligned} MC_r &= 2\mu q \\ AC_r &= \frac{\phi + \mu q^2}{q} \end{aligned}$$

and thus the quantity they produce will be:

$$\begin{aligned} 2\mu q &= \frac{\phi + \mu q^2}{q} \\ q &= \frac{1}{\mu}\sqrt{\phi\mu} \end{aligned}$$

and the price will be $P_f^{lr} = 2\mu\left(\frac{1}{\mu}\sqrt{\phi\mu}\right) = 2\sqrt{\mu\phi}$.

(g) *(2 points)* Explain why even firms of type o might object to the importing of watermelons.

Solution 12 *When we say a firm has zero profit that is zero economic profit. It is true that without imports in the long run type o firms are making zero economic profit, but that is still a comfortable living for the owner of the firms. Allowing imports takes away that comfortable living and while they can make zero economic profit elsewhere they will be upset at having their "sure paycheck" taken away from them.*

5. *(15 points total)* In this question you will prove that the cost function—$c(w, r, Q)$—is non-decreasing in input prices.

   (a) *(10 points)* Using only algebra and the definition of cost minimization, show that if $w \geq \tilde{w}$ and $r \geq \tilde{r}$ then $c(w, r, Q) \geq c(\tilde{w}, \tilde{r}, Q)$.

   Solution 13 *Let (L^*, K^*) be cost minimizing at (w, r, Q), then:*

   $$c(w, r, Q) = wL^* + rK^* \ .$$

   Now since $w \geq \tilde{w}$ and $r \geq \tilde{r}$ it is obvious that:

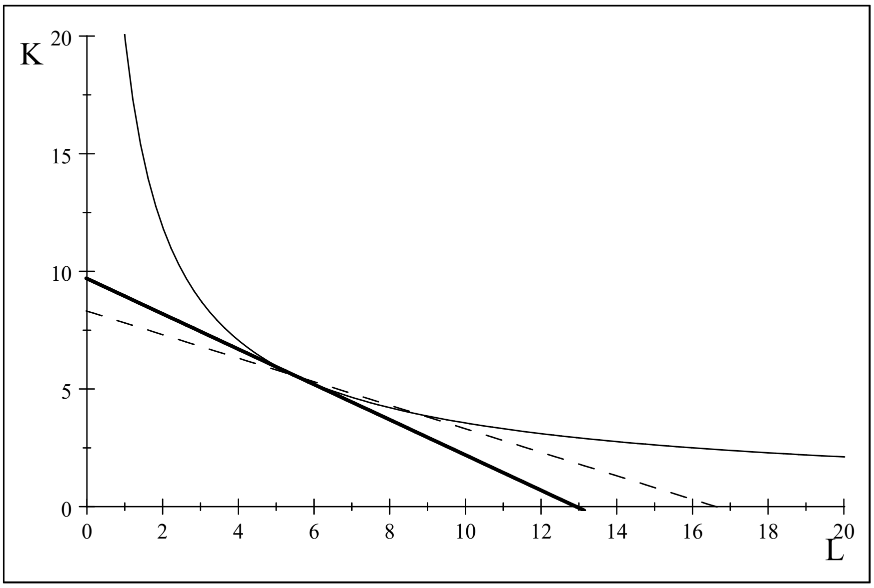
   $$wL^* + rK^* \geq \tilde{w}L^* + \tilde{r}K^* \ ,$$

   and by the definition of minimum it is obvious that:

   $$\tilde{w}L^* + \tilde{r}K^* \geq \min_{L,K: f(L,K) \geq Q} \tilde{w}L + \tilde{r}K = c(\tilde{w}, \tilde{r}, Q) \ .$$

   (b) *(5 points)* If r is held constant, show that if $w > \tilde{w}$ then $c(w, r, Q) \geq c(\tilde{w}, r, Q)$ using the following graph. The smooth curve is an isoquant.

   Solution 14



An Isocost line has the formula:

$$C = wL + rK$$

or graphically it has the formula:

$$K = \frac{C}{r} - \frac{w}{r}L$$

This formula tells us two things.

i. *When* $L = 0$ *(or it crosses the vertical axis) the value of the isocost curve is* $\frac{C}{r}$

ii. *The slope of an Isocost curve is* $-\frac{w}{r}$.

Given these two characteristics, we will have a tangency at the high price, like in the graph above (where $L = K = 20^{\frac{4}{7}} = 5.5392$ *and* $r = 4$, $w = 3$.*)*
Now since $w > \tilde{w}$ *then* $-\frac{w}{r} < -\frac{\tilde{w}}{r}$ *and so an isocost that goes through the old production point (*$L = K = 20^{\frac{4}{7}}$*) will cross the vertical axis at a lower point, suggesting that this passive production method will have a lower cost. (This is the dashed line above, which we will call passive costs at* $(\tilde{w}, r, Q)$*). Since, by definition the true costs must be lower we have*

$$\frac{c(\tilde{w}, r, Q)}{r} \leq \frac{PC(\tilde{w}, r, Q)}{r} \leq \frac{c(w, r, Q)}{r} .$$

In fact in this case, since the original L^* *was strictly positive, the second inequality is strict.*