# ECON 203
# Midterm on Consumer Theory

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 19:40 and end at 21:20

1. *(16 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

2. *(7 points)* Define a *corner solution* to a utility maximization problem. Are corner solutions common or rare in reality? Explain your answer. (Hard) Why do we usually ignore corner solutions in our analysis? (It might help to think about perfect substitutes when answering this.)

   **Solution 1** *A* corner solution *is a situation where one or more goods a person can consume have a consumption level of zero.*

   *These are* common *in the real world.*

   *This is because there are too many goods, for example there are too many foreign countries for me to visit, there are too many types of jeans for me to buy them all (or want to.)*

   *We usually ignore them in our analysis because they make the math hard. To be precise when we might have goods that are at a corner solution we have to use the more complicated Kuhn-Tucker method instead of the Lagrange method. To put the same things in simpler terms—we have to check for each and every corner solution as a seperate exercise. Turning a problem with two goods into a three case problem, a problem with three goods into a six case problem. What a nightmare.*

   *Notice we do have to do this with perfect substitutes. In that problem we know that most of the time we will be at a corner solution, so we have to solve for which corner solution is better. Let me go into detail to explain how we do this (without using bang for the buck).*

   *We essentially can consider two extreme choices, either spending all our money on food (F) or clothing (C). Let the utility function be $U(F,C) = \alpha F + \beta C$. If we spend all on food we will have:*

$$U\left(\frac{I}{p_f}, 0\right) = \alpha \frac{I}{p_f} \ ,$$

   *on clothing:*

$$U\left(0, \frac{I}{p_c}\right) = \beta \frac{I}{p_c}.$$

*then we choose food if :*

$$\alpha \frac{I}{p_f} > \beta \frac{I}{p_c}$$
$$\frac{\alpha}{p_f} > \frac{\beta}{p_c}.$$

*which is the bang for the buck condition. Starting our analysis with this condition has the added benefit of knowing that the solution will (probably) be a corner solution, so it is better to start with this one, but we could use the other one.*

3. *(10 points)* Explain the *motivation* for Economists to assume rationality. Give an example of how not assuming rationality can lead to bad analysis and/or decisions. (You do not have to use the one I gave in class. It probably would be easier to think up one on your own.)

**Solution 2** *The motivation for assuming rationality is the most important "definition" of rationality. It is, simply put, that we should not assume people are stupid. We should not assume that the people we are studying are simply doing something because they are dumb, and can't see an obvious advantage.*

*The example my Devlopment economics -Econ professor gave when he was explaining this was factories in less developed nations. In general while these factories offered better income than farming people were not rushing to fill these jobs. The conclusion of many development economists is that "people were stupid" or they simply didn't respond to the same incentives as the rest of us. However one crazy man decided to locate his factory close to the farmers. The result? Everyone wanted to work in his factory and he made a fortune. It wasn't that people weren't responding to the incentives but that the risk of moving to that factories' location was too high.*

*My favorite example is how US slave owners optimally starved and then over-fed their slaves. Resulting in robust (physical) specimens even though modern nutritionists could not tell you the full impact of such a diet.*

**Remark 3** *I am going to change the next question so that the answers coincide with the question I meant to ask. I will then grade based on the obvious mistakes I made in writing up the orginal version*

| | | *Original Question* | | | *Correct Question* | | |
|---|---|---|---|---|---|---|---|
| $\alpha$ | $\beta$ | $\frac{L}{R}$ | $L(p_l, p_r, I)$ | $R(p_l, p_r, I)$ | $\frac{L}{R}$ | $L(p_l, p_r, I)$ | $R(p_l, p_r, I)$ |
| 1 | 2 | 2 | $2\frac{I}{2p_l+p_r}$ | $\frac{I}{2p_l+p_r}$ | $\frac{1}{2}$ | $\frac{I}{2p_l+p_r}$ | $2\frac{I}{2p_l+p_r}$ |
| 1 | 3 | 3 | $3\frac{I}{3p_l+p_r}$ | $\frac{I}{3p_l+p_r}$ | $\frac{1}{3}$ | $\frac{I}{3p_l+p_r}$ | $3\frac{I}{3p_l+p_r}$ |
| 4 | 1 | $\frac{1}{4}$ | $\frac{I}{p_l+4p_r}$ | $4\frac{I}{p_l+4p_r}$ | 4 | $4\frac{I}{p_l+4p_r}$ | $\frac{I}{p_l+4p_r}$ |
| 3 | 1 | $\frac{1}{3}$ | $\frac{I}{p_l+3p_r}$ | $3\frac{I}{p_l+3p_r}$ | 3 | $3\frac{I}{p_l+3p_r}$ | $\frac{I}{p_l+3p_r}$ |

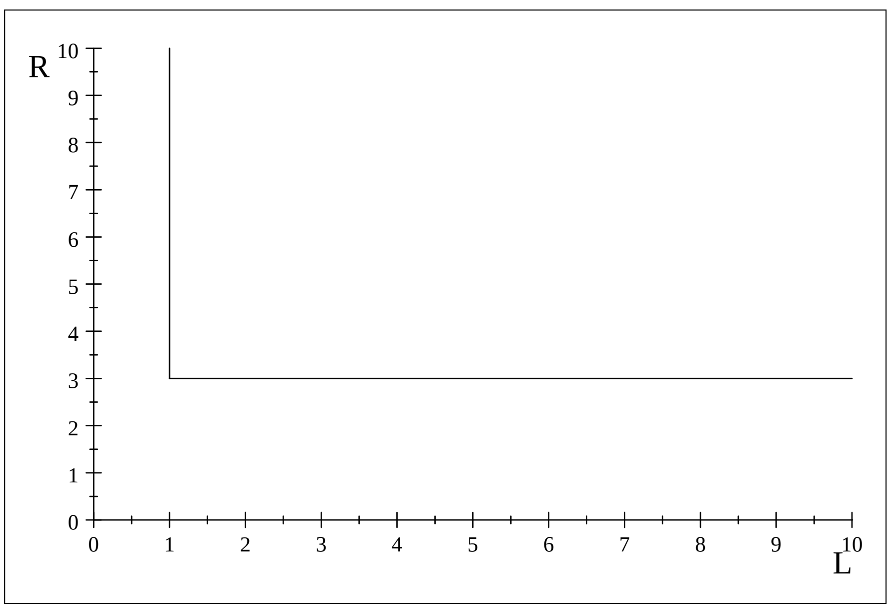
4. *(19 points total)* Lefty Wrightson is an odd fellow because he has $\alpha$ Left feet and $\beta$ Right feet. Fortunately he's Scottish so he just wears kilts all the time, but he has a real problem with finding shoes. He has a budget of $I$ euros to spend on shoes, and each left shoe costs $p_l$ and each right shoe costs $p_r$.

   (a) *(3 points)* Explain why we can represent his utility for shoes as $U(L,R) = \min\left(\frac{L}{\alpha}, \frac{R}{\beta}\right)$ where $L$ is the number of left shoes he buys and $R$ is the number of right shoes.

   **Solution 4** *What he really cares about is the number of matching sets of shoes he has. Thus for each $\alpha$ left shoes he buys he must by $\beta$ right shoes, and having more left or right shoes (alone) than this precise balance will not benefit him at all. The function that expresses having excess shoes of one type gives no benefit is the minimum.*

   (b) *(3 points)* In the graph below carefully draw an indifference curve for Lefty where $U(L,R) = \min\left(\frac{L}{\alpha}, \frac{R}{\beta}\right) = 1$. The number is associated with the line to the right or above.

   **Solution 5** *Using my computer program I can precisely graph it for $\alpha = 1$ and $\beta = 3$, it is:*




   *What I really care about is first whether you drew an indifference curve, second did you get the shape right, and finally for whatever funciton you graphed did you get the corner correct.*

(c) *(5 points)* Since the price of both left and right shoes is strictly positive, what do we know about the ratio of left to right shoes he will buy?

**Solution 6** *Since buying too many of one shoe will give them no benefit they will always buy when* $\frac{L}{\alpha} = \frac{R}{\beta}$*, or* $\frac{L}{R} = \frac{\alpha}{\beta}$*.*

(d) *(8 points)* Find his demand for both right and left shoes.

**Solution 7** *In this case setting up the objective function gives no benefit. We have two conditions:*

$$\begin{aligned} L &= \frac{\alpha}{\beta}R \\ p_l L + p_r R &= I \end{aligned}$$

$$\begin{aligned} p_l\left(\frac{\alpha}{\beta}R\right) + p_r R &= I \\ p_l \alpha R + \beta p_r R &= \beta I \\ R &= \frac{\beta I}{p_l \alpha + \beta p_r} \end{aligned}$$

*using the equivalent of equalizing the bang for the bucks we can then find that:*

$$L = \frac{\alpha}{\beta}R = \frac{\alpha}{\beta}\left(\frac{\beta I}{p_l \alpha + \beta p_r}\right) = \frac{\alpha I}{\alpha p_l + \beta p_r}$$

*or using the budget constraint we find:*

$$\begin{aligned} p_l L + p_r\left(\frac{\beta I}{p_l \alpha + \beta p_r}\right) &= I \\ p_l L &= I - \left(\frac{\beta p_r}{p_l \alpha + \beta p_r}\right) I \\ p_l L &= \frac{p_l \alpha}{p_l \alpha + \beta p_r} I \\ L &= \frac{\alpha I}{\alpha p_l + \beta p_r} \end{aligned}$$

*and with this problem obviously the first method is easier.*

5. *(8 points)* For **either** the axiom of preferences *montonicity* or *convexity.* **(Warning: If you try to answer both axioms I can choose whichever one I want to grade.)**

**MONOTONICITY**

(a) *(2 points)* Define the axiom.

**Solution 8** *More is better, or if I give you a little more of everything then you must be happier. To be precise mathematically is a little harder. If for every i* $A_i > B_i$ *then* $A \succ B$. *(Notice this is the weak axiom.)*

(b) *(3 points)* Give a real world example showing that preferences do not always satisfy this axiom.

**Solution 9** *This is the infinite glasses of X axiom, where X can be raki or water. In either case drinking too much of this good can kill you, thus strong montonicity is rejected. Weak montonicity is rejected because giving you a little of everything (say money) with each glass of raki will not change the fact that you will be dead by the end of the night.*

(c) *(3 points)* Explain what it implies in terms of indifference curves, including showing how the axiom fails if the indifference curve does not satisfy your restriction.

**Solution 10** *This means that indifference curves must be (weakly) downward sloping. If and indifference curve slopes up this means, by definition, that we can find two bundles on the same indifference curve and one has strictly more of every good than the other.*

**CONVEXITY**

(a) (*2 points)* Define the axiom.

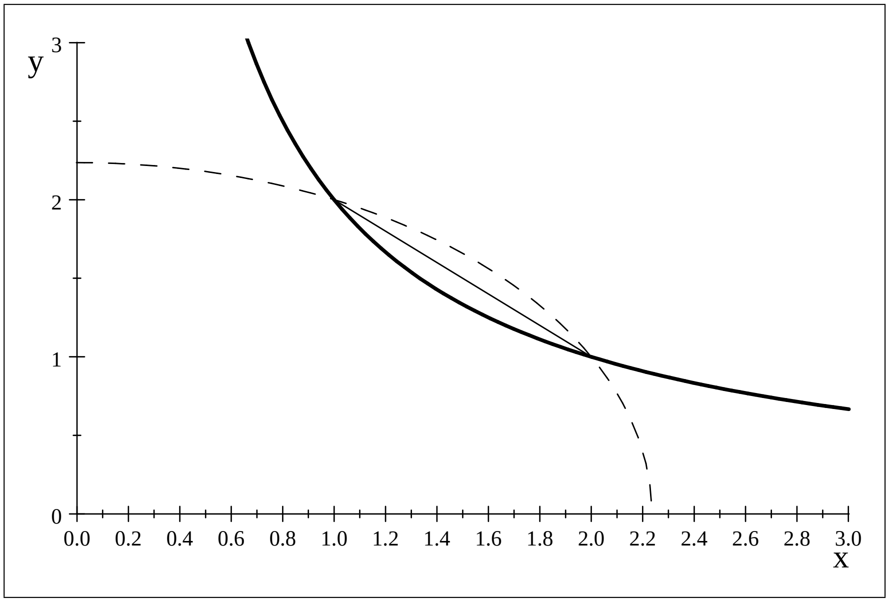
**Solution 11** *For this axiom it is easier to be precise with mathematical language. For* $0 \leq \lambda \leq 1$ *if* $A \succeq C$ *and* $B \succeq C$ *then* $\lambda A + (1 - \lambda) B \succeq C$. *In words "a little of both is better than all of one." But that is a lot less precise.*

(b) *(3 points)* Give a real world example showing that preferences do not always satisfy this axiom.

**Solution 12** *Say that you are indifferent between going to Bilkent and Sabanci. This would imply that you are indifferent between a dorm room at Sabanci and one at Bilkent. So that would mean that you would at least weakly prefer a dorm room at Bilkent for Sunday through Wednesday, and Sabanci for the rest of the week. There are some goods where "a little of both" is clearly worse than all of one, and the best example is where you live. Your family might have a "weekend house" but that is in addition to your city house. You would be strictly worse off if you had to move to the weekend house every weekend.*

(c) *(3 points)* Explain what it implies in terms of indifference curves, including showing how the axiom fails if the indifference curve does not satisfy your restriction.

**Solution 13** *In terms of indifference curves it is best to consider* $A \sim B$ *and* $C = A$. *Then for all* $\lambda$ $\lambda A + (1-\lambda) B$ *this is a line between* $A$ *and* $B$, *and all points on that line must be on a higher indifference curve. Or it should be "bowed away from the origin." Graphically the dashed curve can not be an indifference curve, because the solid thin line (between* $(1,2)$ *and* $(2,1)$*) is always below it. On the other hand the heavy dark curve can be because everything on the solid thin line is better than the end points. Notice the solid thin line (extended) would be fine as well.*




6. *(26 points total)* The duality theorem tells us that $X(p_x, p_y, I(p_x, p_y, U)) = h_x(p_x, p_y, U)$ where $X(p_x, p_y, I)$ is the Marshallian or normal demand for $X$, $h_x(p_x, p_y, U)$ is the Hicksian or income compensated demand for $X$, and $I(p_x, p_y, U)$ is the optimal expenditure to reach a utility level of $U$.

   (a) (*6 points)* Derive the Slutsky equation in elasticity form with regards to $p_y$ (NOT $p_x$). You may use the envelope theorem, which tells us that $\partial I(p_x, p_y, U) / \partial p_x = X$ and $\partial I(p_x, p_y, U) / \partial p_y = Y$.

**Solution 14**

$$\begin{aligned}
X\left(p_x, p_y, I\left(p_x, p_y, U\right)\right) &= h_x\left(p_x, p_y, U\right) \\
\frac{\partial X}{\partial p_y} + \frac{\partial X}{\partial I}\frac{\partial I}{\partial p_y} &= \frac{\partial h_x}{\partial p_y} \\
\left(\frac{\partial X}{\partial p_y} + \frac{\partial X}{\partial I}Y\right)\frac{p_y}{X} &= \left(\frac{\partial h_x}{\partial p_y}\right)\frac{p_y}{X} \\
e_x\left(p_y\right) + \frac{\partial X}{\partial I}Y\frac{p_y}{X} &= e_{h_x}\left(p_y\right) \\
e_x\left(p_y\right) + \frac{\partial X}{\partial I}\frac{p_y Y}{X}\frac{I}{I} &= e_{h_x}\left(p_y\right) \\
e_x\left(p_y\right) + \frac{\partial X}{\partial I}\frac{I}{X}\frac{p_y Y}{I} &= e_{h_x}\left(p_y\right) \\
e_x\left(p_y\right) + e_x\left(I\right)s_y &= e_{h_x}\left(p_y\right) \\
e_x\left(p_y\right) &= e_{h_x}\left(p_y\right) - e_x\left(I\right)s_y
\end{aligned}$$

(b) *(8 points)* Give the physical definition of each term in the Slutsky equation and discuss which terms illustrate which of the two effects on Marshallian demand.

**Solution 15** $e_x\left(p_y\right)$*—elasticity of the Marshallian demand with regard to the price of* $y$*.*

$e_x\left(I\right)$*—elasticity of the Marshallian demand with regard to income.*

$e_{h_x}\left(p_y\right)$*—elasticity of the Hicksian demand with regard to the price of* $y$*.*

$s_y$*—the share of income spent on* $y$*.*

| $\alpha$ | $\beta$ | $s_x$ | $s_y$ | $e_{h_x}\left(p_y\right)$ |
|---|---|---|---|---|
| $\frac{1}{4}$ | $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| $\frac{1}{5}$ | $\frac{1}{3}$ | $\frac{1}{5}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $\frac{1}{3}$ | $\frac{1}{5}$ | $\frac{1}{3}$ | $\frac{1}{5}$ | $\frac{1}{5}$ |

(c) *(12 points total)* Assume that a person has Cobb-Douglass preferences, and the demand for $X$ is $X = \frac{\alpha I}{p_x}$ and for $Y$ is $Y = \frac{\beta I}{p_y}$

i. *(8 points)* Find the elasticity of $X$ with regards to $p_y$, $I$, and the share of income this person spends on $X$. (The result with regards to $p_y$ might surprise you at first.) Do the same for $Y$, except find the elasticity of $Y$ with regards to the price of $X$.

**Solution 16**

$$\begin{aligned} X &= \frac{\alpha I}{p_x} \\ \frac{\partial X}{\partial p_y} &= 0 \\ e_x(p_y) &= \frac{\partial X}{\partial p_y}\frac{p_y}{X} = 0 \\ \frac{\partial X}{\partial I} &= \frac{\alpha}{p_x} \\ e_x(I) &= \frac{\partial X}{\partial I}\frac{I}{X} = \frac{\alpha}{p_x}\frac{I}{\frac{\alpha I}{p_x}} = 1 \\ s_x &= \frac{p_x X}{I} = \frac{p_x \frac{\alpha I}{p_x}}{I} = \alpha \end{aligned}$$

*using the same methodology one can quickly show that* $e_y(p_x) = 0$, $e_y(I) = 1$, $s_y = \beta$.

ii. *(4 points)* Using the Slutsky equation, derive a precise value for the Hicksian elasticity of $X$ with regards to the price of $Y$. Are $X$ and $Y$ net substitutes or compliments? Notice this is true for all Cobb-Douglass preferences.

**Solution 17**

$$\begin{aligned} e_x(p_y) + e_x(I)\, s_y &= e_{h_x}(p_y) \\ 0 + 1 * \beta &= e_{h_x}(p_y) \\ e_{h_x}(p_y) &= \beta \end{aligned}$$

*and since* $e_{h_x}(p_y) > 0$ *these are net substitutes, even though they are (in terms of Marshallian demand) neither substitutes nor compliments.*

7. *(8 points)* A friend of yours needs a new pair of jeans which cost 200 TL. However this friend also found an awesome shirt that he or she really wants for 300 TL. Using the concept of utility and the bang for the buck, explain to your friend what she or he needs to think about in order to decide which to buy. (See, Laman, utility theory is actually useful.)

**Solution 18** *The grading on this exam should really be the grade on this question. But... well... that's not really fair is it? *Sigh.* But if you can't use what you learned in the real world... then what was the point???*

*So let me think through the answer and then figure out what to say to this person. The bang for the buck condition in this case is quite simple, we just need to know which is higher. If* $\frac{MU_s}{p_s} > \frac{MU_j}{p_j}$ *then they should buy the shirt, or* $MU_s > \frac{p_s}{p_j} MU_j$ *or* $MU_s > 1.5 * MU_j$. *So I just have to explain what this means to this person.*

*But I will need to explain to them, or simply tell them, the usefullness of translating their desire for the shirt and their need for the jeans into numbers. Wants and needs are not, naturally, in the same scale, but if we can't put them into the same scale then decision making is much harder. So what would I say to them?*

*"While this might be hard, it would be great if you could compare your need for the jeans to your desire for the shirt in terms of numbers. If you can do this, then the answer to your question is simple. Do you want the shirt fifty percent more than you need the jeans? If yes, buy the shirt and have to wear run down, old jeans with it. If not buy the jeans and make do with your lame current collection of shirts."*

8. *(6 points)* As I explained in class, in both Microeconomics and Macroeconomics we almost always assume that people have Costant Elasticity of Subtitution (CES) utility functions. What is the income elasticity of all goods when a person has CES utility? Why might you want to make this implicit assumption? (It might help you to think about economic development in Macroeconomics.)

**Solution 19** *As explained in class, for CES utility functions* $X_i^* = f_i(P)\,I$ *where* $P$ *is the vector of prices. Thus (and this is all you need to answer) the income elasticity is one.*

*Also as explained in class, this means that your behavior will not change if you are rich versus being poor. The shares of income you spend on all goods will be constant.*

*To explain why this is useful consider someone who wants to argue that it is hypothesis* $X$ *which explains why countries develop or not. By assuming the rich (developed) and the poor (undeveloped) nations act the same in every other regard, if they find that* $X$ *does the job no one can complain about "confounding factors."*

*Say, for example, that I wanted to argue that complete credit markets were the key. Well if I didn't use CES I might implicitly be assuming that the rich save either more or less than the poor. If it was the former this could accelerate the effect of complete credit markets—but then some annoying graduate student would point this out to me and I'd be embarrassed.*

*In many ways, the assumption of CES is equivalent to "ceteris paribus" in our standard analysis. It may not be a reasonable assumption, but unless the focus of our study is on how CES fails it is a good standard assumption.*

**Remark 20** *In deference to Dr. Sang Seok Lee I should admit that this is not why we assume CES utility. Frankly speaking it's just simple to use. So simple, so lovely... la la la. But the reason it has not been dethroned is something like what I just pointed out. Many other things like quadratic loss functions are no longer cool because... well... they're silly. But CES is still the standard tool in all branches of economics.*