ECON 203
Final Exam

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 18:40 and end at 20:20

1. *(12 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _

   Student ID: _

   Signature:

   _

α	β	γ	δ	μ	π	η	ι	$C_1(F_1)$	$\frac{p_c}{p_f}$	(C_1^*, F_1^*)
1	2	2	1	9	12	5	3	$2F_1$	2	$(12, 6)$
1	3	3	1	7	9	4	12	$3F_1$	3	$(9, 3)$
5	1	1	4	19	11	1	15	$\frac{1}{5}F_1$	$\frac{1}{4}$	$(3, 15)$
2	1	5	1	6	5	10	2	$\frac{1}{2}F_1$	5	$(5, 10)$

2. *(14 points total)* Consider an exchange economy, person one has Leontief preferences and the utility function: $U_1(C_1, F_1) = \min\{\alpha C_1, \beta F_1\}$, person two has linear preferences with the utility function: $U_2(C_2, F_2) = \gamma C_2 + \delta F_2$. Person one has the initial endowment $(C_1^e, F_1^e) = (\mu, \pi)$ and person two has the initial endowment $(C_2^e, F_2^e) = (\eta, \iota)$. You may assume throughout that both people will consume a positive amount of both goods.

   (a) *(4 points)* Find the Contract Curve, or the set of Pareto Efficient allocations. Explain your reasoning.

   Solution 1 *Since person one has Leontief preferences, he will always consume where $\alpha C_1 = \beta F_1$, and person two has to adjust to make this possible. Thus the contract curve is*

   $$C_1 = \frac{\beta}{\alpha} F_1 \ .$$

   (b) *(4 points)* Find the prices in all competitive equilibria. Explain your reasoning.

Solution 2 *Since person two has perfect substitutes preferences* ***and*** *will consume both goods (assumed) we must have*

$$MRS_2 = \frac{MU_c^2}{MU_f^2} = \frac{\gamma}{\delta} = \frac{p_c}{p_f}$$

(c) *(6 points)* Find the allocation, or the quantities of food and clothing that person one will consume.

Solution 3 *We know that person one will consume* $C_1 = \frac{\beta}{\alpha} F_1$*, and that* $p_c = \gamma$*,* $p_f = \delta$ *so we have:*

$$\begin{aligned} \gamma C_1 + \delta F_1 &= \gamma\mu + \delta\pi \\ \gamma\left(\frac{\beta}{\alpha}F_1\right) + \delta F_1 &= \gamma\mu + \delta\pi \\ \frac{1}{\alpha}F_1\left(\alpha\delta + \beta\gamma\right) &= \gamma\mu + \delta\pi \\ F_1 &= \alpha\frac{\pi\delta + \gamma\mu}{\alpha\delta + \beta\gamma} \\ C_1 &= \frac{\beta}{\alpha}F_1 = \beta\frac{\pi\delta + \gamma\mu}{\alpha\delta + \beta\gamma} \end{aligned}$$

α	β	ω	ρ	κ	$C(\omega, \rho, 1)$	$C(\kappa\omega, \kappa\rho, 1)$
$\frac{2}{5}$	$\frac{1}{5}$	2	4	2	6	12
$\frac{1}{3}$	$\frac{1}{6}$	4	8	$\frac{1}{2}$	12	6
$\frac{1}{4}$	$\frac{1}{2}$	10	5	$\frac{1}{5}$	15	3
$\frac{1}{6}$	$\frac{1}{3}$	8	4	$\frac{1}{2}$	12	6

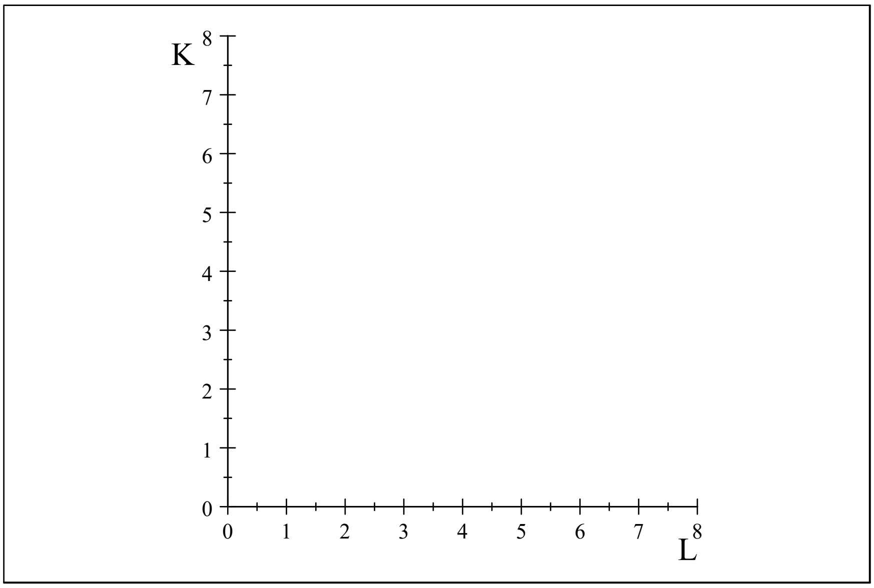
3. *(24 points total)* In this question I want you to prove that for a specific production function and in general that the cost function is homogeneous of degree one in input prices, or $C(tw, tr, Q) = tC(w, r, Q)$ for a firm that has two inputs, K or capital and L or labor.

   (a) *(15 points total)* For the production function $Q = K^\alpha L^\beta$.

      i. *(4 points)* In the graph below graph an isoquant when $Q = 1$. Be sure to list at least three points on this isoquant below.

      Solution 4 *We have* $1 = K^\alpha L^\beta$*, so* $K = L^{-\frac{\beta}{\alpha}}$ *and so the obious points to graph are:*

K	L
1	1
$2^{-\frac{\beta}{\alpha}}$	2
2	$2^{-\frac{\beta}{\alpha}}$



ii. *(4 points)* In the same graph find the cost minimizing isocost curve when $w = \omega$ and $r = \rho$.

Solution 5 *The solution will be* $(1,1)$ *which we can verify by*

$$\begin{aligned} \frac{MP_l}{MP_k} &= \frac{\beta\frac{Q}{L}}{\alpha\frac{Q}{K}} = \frac{\omega}{\rho} \\ L &= K\frac{\beta\rho}{\alpha\omega} \end{aligned}$$

: and since $\frac{\beta\rho}{\alpha\omega} = 1$ *the solution is the obvious one, where* $L = K = 1$ *and* $C = \rho + \omega$

iii. *(2 points)* Now in the same graph find the cost minimizing isocost curve when $w = \kappa\omega$ and $r = \kappa\rho$.

Solution 6 *Again the solution is still* $(1,1)$ *because*

$$\frac{MP_l}{MP_k} = \frac{\beta K}{L\alpha} = \frac{\kappa\omega}{\kappa\rho} = \frac{\omega}{\rho}$$

iv. *(5 points)* Show in the graph that this means $C(\kappa\omega, \kappa\rho, 1) = \kappa C(\omega, \rho, 1)$. Explain how you could generalize this result to show that $C(tw, tr, Q) = tC(w, r, Q)$.

Solution 7 *Since both prices have increased by a multiple of* κ*, this must be true, and we could figure it out also by noticing that if* $K = L = 1$ *then* $C(\kappa\omega, \kappa\rho, 1) = \kappa\omega + \kappa\rho = \kappa(\omega + \rho) = \kappa C(\omega, \rho, 1)$*. This also must be true more generally because it is the relative price—*w/r*—that determines the cost minimizing bundle and not the absolute value. Thus the optimal production method will not change and we will get the result.*

(b) *(9 points in total.)* Now I want you to show the same thing for general production processes. Let (K^*, L^*) be the cost minimizing bundle at the input prices (w, r) and output Q.

i. *(3 points)* If arbitrary (K, L) produce at least Q units of output is $wL^* + rK^* \geq wL + rK$, $wL^* + rK^* \leq wL + rK$, or are we unable to tell? Explain.

Solution 8 *By the definition of minimizing we must have* $wL^* + rK^* \leq wL + rK$

ii. *(6 points)* Prove that the cost function is homogenous of degree one in input prices, or that for $t > 0$, $tC(w, r, Q) = C(tw, tr, Q)$.

Solution 9 *We know that* $wL^* + rK^* \leq wL + rK$ *for any* (L, K) *that give us at least* Q *units, and this will continue to be true if we multiply both sides by any* $t > 0$ *or* $t(wL^* + rK^*) \leq t(wL + rK)$ *which is the same as saying* $twL^* + trK^* \leq twL + trK$*, or that* $C(tw, tr, Q) = twL^* + trK^* = t(wL^* + rK^*) = tC(w, r, Q)$*.*

α	β	a_f	a_c	T	p_f^w	(F^*, C^*)	p_f^*	(F_p, C_p)	R	(F_c, C_c)
1	2	18	6	54	4	(1, 6)	3	(3, 0)	12	(1, 8)
1	3	6	2	48	2	(2, 18)	3	(0, 24)	24	(3, 18)
3	1	18	3	48	4	(2, 4)	6	(0, 16)	16	(3, 4)
2	1	8	4	48	4	(4, 4)	2	(6, 0)	24	(4, 8)

4. *(29 points total)* Consider the following economy. Robinson Crusoe's utility function is: $U(F, C) = F^\alpha C^\beta$ and the production possibility set is given by $a_f F + a_c C \leq T$.

(a) *(4 points)* Find the Marginal Rate of Transformation and the Marginal Rate of Substitution in this economy.

$$MRT = \frac{a_f}{a_c}$$
$$MRS = \frac{MU_f}{MU_c} = \frac{\alpha \frac{U}{F}}{\beta \frac{U}{C}} = \frac{C}{F}\frac{\alpha}{\beta}$$

(b) *(6 points)* Find the Pareto Efficient outcome in this economy.

$$MRT = MRS$$
$$\frac{C}{F}\frac{\alpha}{\beta} = \frac{a_f}{a_c}$$
$$C = \frac{F}{\alpha}\frac{\beta}{a_c} a_f$$

$$
\begin{aligned}
a_f F + a_c C &= T \\
a_c C \frac{\alpha}{\beta} + a_c C &= T \\
\frac{C}{\beta} a_c (\alpha + \beta) &= T \\
C &= \frac{T}{a_c} \frac{\beta}{\alpha + \beta} \\
a_f F &= a_c C \frac{\alpha}{\beta} = a_c \left(\left(\frac{T}{a_c} \frac{\beta}{\alpha + \beta} \right) \right) \frac{\alpha}{\beta} = T \frac{\alpha}{\alpha + \beta} \\
F &= \frac{T}{a_f} \frac{\alpha}{\alpha + \beta}
\end{aligned}
$$

(c) *(3 points)* Find the implicit price of food in this economy, you may set the price of clothing to one.

$$
\begin{aligned}
MRT &= \frac{a_f}{a_c} = \frac{p_f}{p_c} \\
p_f &= \frac{a_f}{a_c}
\end{aligned}
$$

(d) *(16 points total)* Assume that now Robinson Crusoe is considering opening up to world trade. The price of food (p_f) in the world economy is τ ($p_f = \tau$) and the price of food is one ($p_c = 1$).

i. *(4 points)* Find the amount they will produce, and the revenue they will receive.

Solution 10 *Notice that $\tau \neq \frac{a_f}{a_c}$ so they will either produce all food or all clothing, depending on which maximizes their revenue. If they produce all clothing, they will earn $\frac{T}{a_c}$ if they produce all food they will earn $\tau \frac{T}{a_f}$, so their revenue will be:*

$$
R = \max \left\{ \frac{T}{a_c}, \frac{\tau T}{a_f} \right\}
$$

ii. *(4 points)* Find the amount they will consume.

Solution 11

$$
\begin{aligned}
\frac{C}{F} \frac{\alpha}{\beta} &= \tau \\
C &= \frac{F}{\alpha} \beta \tau
\end{aligned}
$$

$$\begin{aligned}
\tau F + C &= R \\
\tau F + \frac{F}{\alpha}\beta\tau &= R \\
F &= R\frac{\alpha}{\tau(\alpha+\beta)} \\
C &= \frac{F}{\alpha}\beta\tau = R\frac{\beta}{\alpha+\beta}
\end{aligned}$$

iii. *(2 points)* Is Robinson better off with free trade? How do you know this?

iv. *(3 points)* When will Robinson not benefit from free trade? Explain.

v. *(3 points)* It is possible for the production possibilities frontier (PPF) to be shaped so that an economy will never benefit from free trade. The PPF must be not upward sloping, either graph or give a mathematical function for the PPF such that this economy would never benefit from free trade. (**HINT**: The choice of what to produce will be very simple, think about perfect compliments.)

χ	δ	F	X	J	a	b	q_{sd}	P_{sd}	$s(P),\ P \geq P_{sd}$	$S(P),\ P > P_{sd}$	q_e	P_e	[illegible]
1	1	36	20	5	85	1	4	9	$\frac{1}{2}P - \frac{1}{2}$	$\frac{5}{2}P - \frac{5}{2}$	6	13	2
4	1	36	27	5	74	$\frac{1}{2}$	3	10	$\frac{1}{2}P - 2$	$\frac{5}{2}P - 10$	6	16	2
4	$\frac{1}{2}$	18	16	6	80	2	2	6	$P - 4$	$6P - 24$	6	10	1
1	1	25	24	6	93	3	1	3	$\frac{1}{2}P - \frac{1}{2}$	$3P - 3$	5	11	1

5. *(21 points total)* Assume that the cost of a firm in a given industry is $c(q) = \chi q + \delta q^2 + F$, and the fixed sunk costs are X.

   (a) *(6 points)* Find the Marginal cost, average variable costs, and the supply curve of a firm.

Solution 12 $MC = \chi + 2\delta q,\ AVC = \left(\chi q + \delta q^2 + F - X\right)/q,\ AC = \left(\chi q + \delta q^2 + F\right)/q$

$$\chi + 2\delta q = \frac{\chi q + \delta q^2 + F - X}{q}$$

$$q_{sd} = \frac{1}{\delta}\sqrt{F\delta - X\delta}$$

$$\begin{aligned}
P_{sd} &= MC(q_{sd}) = \chi + 2\delta\left(\frac{1}{\delta}\sqrt{F\delta - X\delta}\right) \\
&= \chi + 2\sqrt{F\delta - X\delta}
\end{aligned}$$

$$\begin{aligned}
P &= MC = \chi + 2\delta q \\
q &= \frac{1}{2\delta}(P - \chi)
\end{aligned}$$

$$s(P) = \begin{cases} \frac{1}{2\delta}(P-\chi) & P \geq \chi + 2\sqrt{F\delta - X\delta} \\ 0 & P \leq \chi + 2\sqrt{F\delta - X\delta} \end{cases}$$

(a) *(3 points)* If there are J firms in the industry what is the short run supply curve?

Solution 13 *I am going to be excessively precise here, I don't expect that much out of you.*

$$S(P) = Js(P) = \begin{cases} \frac{J}{2\delta}(P-\chi) & P > \chi + 2\sqrt{F\delta - X\delta} \\ \frac{K}{\delta}\sqrt{F\delta - X\delta} & P = \chi + 2\sqrt{F\delta - X\delta} \\ 0 & P < \chi + 2\sqrt{F\delta - X\delta} \end{cases}$$

where K*—*$0 \leq K \leq J$*—is an integer.*

(b) *(3 points)* Find the price at which firms will enter this industry.

Solution 14

$$\begin{aligned} P_e &= MC(q_e) = AC(q_e) \\ \chi + 2\delta q &= \frac{\left(\chi q + \delta q^2 + F\right)}{q} \\ q_e &= \frac{1}{\delta}\sqrt{F\delta} \\ P_e &= \chi + 2\delta\left(\frac{1}{\delta}\sqrt{F\delta}\right) \\ &= \chi + 2\sqrt{F\delta} \end{aligned}$$

(c) *(3 points)* Find the medium run supply curve.

Solution 15 *Again with the excess precision,*

$$S(P) = Js(P) = \begin{cases} \infty & P > \chi + 2\sqrt{F\delta} \\ \frac{N}{\delta}\sqrt{F\delta} & P = \chi + 2\sqrt{F\delta} \\ \frac{J}{2\delta}(P-\chi) & \chi + 2\sqrt{F\delta} > P > \chi + 2\sqrt{F\delta - X\delta} \\ \frac{K}{\delta}\sqrt{F\delta - X\delta} & P = \chi + 2\sqrt{F\delta - X\delta} \\ 0 & P < \chi + 2\sqrt{F\delta - X\delta} \end{cases}$$

where $0 \leq K \leq J \leq N$ *and all are integers.*

(d) *(6 points)* If the demand in the industry is $Q = a - bP$ find the market price and quantity in the short and the medium run.

Solution 16 *First we need to know the price in the short run, we assume all firms will produce and then we have:*

$$\begin{aligned} \frac{J}{2\delta}(P-\chi) &= a - bP \\ P &= \frac{J\chi + 2a\delta}{J + 2b\delta} \\ Q &= a - b\left(\frac{J\chi + 2a\delta}{J + 2b\delta}\right) = \frac{J(a - b\chi)}{J + 2b\delta} \end{aligned}$$

the price is strictly greater than average cost, so all firms will choose to produce. This latter fact, however, means that firms will enter in the medium run and the medium run price will be:

$$\begin{aligned} P &= \chi + 2\sqrt{F\delta} \\ Q &= a - 2b\sqrt{F\delta} - b\chi \end{aligned}$$