

ECON 203

Final Exam

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 18:40 and end at 20:20

All answers on this exam are integers except relative prices.

1. (12 points) **Honor Statement:** Please read and sign the following statement:

I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

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2. (14 points total) Consider an exchange economy, person one has Leontief preferences and the utility function: $U_1(C_1, F_1) = \min\{C_1, 2F_1\}$, person two has linear preferences with the utility function: $U_2(C_2, F_2) = 2C_2 + F_2$. Person one has the initial endowment $(C_1^e, F_1^e) = (9, 12)$ and person two has the initial endowment $(C_2^e, F_2^e) = (5, 3)$. You may assume throughout that both people will consume a positive amount of both goods.

- (a) (4 points) Find the Contract Curve, or the set of Pareto Efficient allocations. Explain your reasoning.

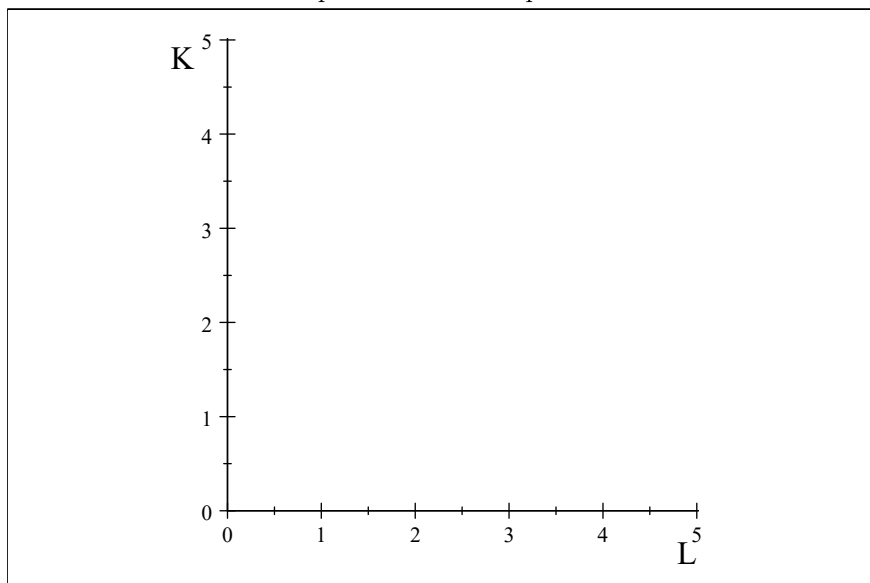
- (b) (4 points) Find the prices in all competitive equilibria. Explain your reasoning.

- (c) (6 points) Find the allocation, or the quantities of food and clothing that person one will consume.

3. (24 points total) In this question I want you to prove that for a specific production function and in general that the cost function is homogeneous of degree one in input prices, or $C(tw, tr, Q) = tC(w, r, Q)$ for a firm that has two inputs, K or capital and L or labor.

- (a) (15 points total) For the production function $Q = K^{\frac{1}{6}} L^{\frac{1}{3}}$.

- i. (4 points) In the graph below graph an isoquant when $Q = 1$. Be sure to list at least three points on this isoquant below.



- ii. (4 points) In the same graph find the cost minimizing isocost curve when $w = 8$ and $r = 4$.

- iii. (2 points) Now in the same graph find the cost minimizing isocost curve when $w = 4$ and $r = 2$.
- iv. (5 points) Show in the graph that this means $C(4, 2, 1) = \frac{1}{2}C(8, 4, 1)$. Explain how you could generalize this result to show that $\tilde{C}(tw, tr, Q) = tC(w, r, Q)$.
- (b) (9 points in total.) Now I want you to show the same thing for general production processes. Let (K^*, L^*) be the cost minimizing bundle at the input prices (w, r) and output Q .
- i. (3 points) If arbitrary (K, L) produce at least Q units of output is $wL^* + rK^* \geq wL + rK$, $wL^* + rK^* \leq wL + rK$, or are we unable to tell? Explain.
- ii. (6 points) Prove that the cost function is homogenous of degree one in input prices, or that for $t > 0$, $tC(w, r, Q) = C(tw, tr, Q)$.

4. (29 points total) Consider the following economy. Robinson Crusoe's utility function is: $U(F, C) = FC^2$ and the production possibility set is given by $18F + 6C \leq 54$.

(a) (4 points) Find the Marginal Rate of Transformation and the Marginal Rate of Substitution in this economy.

(b) (6 points) Find the Pareto Efficient outcome in this economy.

(c) (3 points) Find the implicit price of food in this economy, you may set the price of clothing to one.

(d) (16 points total) Assume that now Robinson Crusoe is considering opening up to world trade. The price of food (p_f) in the world economy is 4 ($p_f = 4$) and the price of food is one ($p_c = 1$).

i. (4 points) Find the amount they will produce, and the revenue they will receive. **HINT:** The answer is both simple and somewhat weird.

ii. (4 points) Find the amount they will consume.

iii. (2 points) Is Robinson better off with free trade? How do you know this?

iv. (3 points) When will Robinson not benefit from free trade? Explain.

v. (3 points) It is possible for the production possibilities frontier (PPF) to be shaped so that an economy will never change its production because of free trade. The PPF must be not upward sloping, either graph or give a mathematical function for the PPF such that this economy would never benefit from free trade. (**HINT:** The choice of what to produce will be very simple, think about perfect compliments.)

5. (21 points total) Assume that the cost of a firm in a given industry is $c(q) = 4q + q^2 + 36$, and the fixed sunk costs are 27.
- (a) (6 points) Find the Marginal cost, average variable costs, and the supply curve of a firm.
- (b) (3 points) If there are 5 firms in the industry what is the short run supply curve?
- (c) (3 points) Find the price at which firms will enter this industry.
- (d) (3 points) Find the medium run supply curve.
- (e) (6 points) If the demand in the industry is $Q = 74 - \frac{1}{2}P$ find the market price and quantity in the short and the medium run.

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2. (14 points total) Consider an exchange economy, person one has Leontief preferences and the utility function: $U_1(C_1, F_1) = \min\{2C_1, F_1\}$, person two has linear preferences with the utility function: $U_2(C_2, F_2) = 5C_2 + F_2$. Person one has the initial endowment $(C_1^e, F_1^e) = (6, 5)$ and person two has the initial endowment $(C_2^e, F_2^e) = (10, 2)$. You may assume throughout that both people will consume a positive amount of both goods.

- (a) (4 points) Find the Contract Curve, or the set of Pareto Efficient allocations. Explain your reasoning.

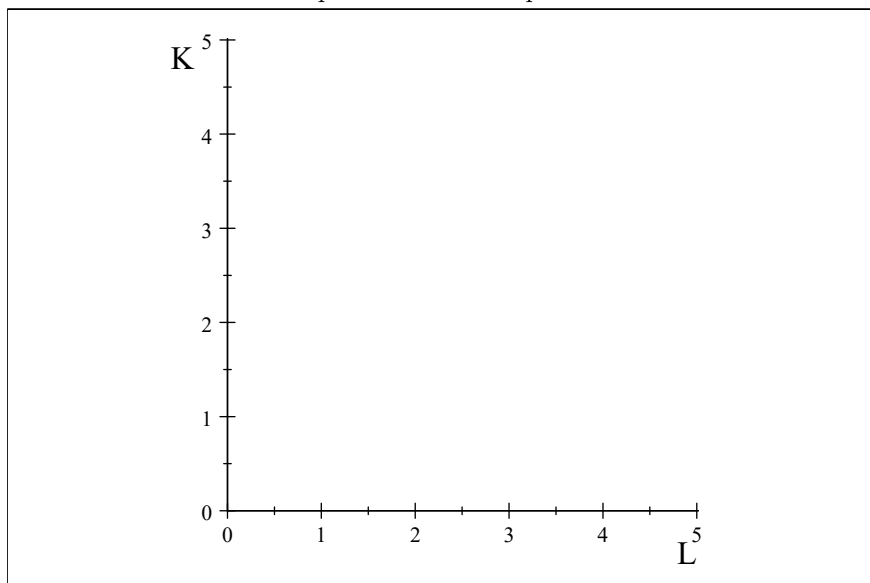
- (b) (4 points) Find the prices in all competitive equilibria. Explain your reasoning.

- (c) (6 points) Find the allocation, or the quantities of food and clothing that person one will consume.

3. (24 points total) In this question I want you to prove that for a specific production function and in general that the cost function is homogeneous of degree one in input prices, or $C(tw, tr, Q) = tC(w, r, Q)$ for a firm that has two inputs, K or capital and L or labor.

- (a) (15 points total) For the production function $Q = K^{\frac{2}{5}}L^{\frac{1}{5}}$.

- i. (4 points) In the graph below graph an isoquant when $Q = 1$. Be sure to list at least three points on this isoquant below.



- ii. (4 points) In the same graph find the cost minimizing isocost curve when $w = 2$ and $r = 4$.

- iii. (2 points) Now in the same graph find the cost minimizing isocost curve when $w = 4$ and $r = 8$.

- iv. (5 points) Show in the graph that this means $C(4, 8, 1) = 2C(2, 4, 1)$. Explain how you could generalize this result to show that $C(tw, tr, Q) = tC(w, r, Q)$.

- (b) (9 points in total.) Now I want you to show the same thing for general production processes. Let (K^*, L^*) be the cost minimizing bundle at the input prices (w, r) and output Q .
 - i. (3 points) If arbitrary (K, L) produce at least Q units of output is $wL^* + rK^* \geq wL + rK$, $wL^* + rK^* \leq wL + rK$, or are we unable to tell? Explain.

 - ii. (6 points) Prove that the cost function is homogenous of degree one in input prices, or that for $t > 0$, $tC(w, r, Q) = C(tw, tr, Q)$.

4. (29 points total) Consider the following economy. Robinson Crusoe's utility function is: $U(F, C) = F^3C$ and the production possibility set is given by $18F + 3C \leq 48$.

(a) (4 points) Find the Marginal Rate of Transformation and the Marginal Rate of Substitution in this economy.

(b) (6 points) Find the Pareto Efficient outcome in this economy.

(c) (3 points) Find the implicit price of food in this economy, you may set the price of clothing to one.

(d) (16 points total) Assume that now Robinson Crusoe is considering opening up to world trade. The price of food (p_f) in the world economy is 4 ($p_f = 4$) and the price of food is one ($p_c = 1$).

i. (4 points) Find the amount they will produce, and the revenue they will receive. **HINT:** The answer is both simple and somewhat weird.

ii. (4 points) Find the amount they will consume.

iii. (2 points) Is Robinson better off with free trade? How do you know this?

iv. (3 points) When will Robinson not benefit from free trade? Explain.

v. (3 points) It is possible for the production possibilities frontier (PPF) to be shaped so that an economy will never change its production because of free trade. The PPF must be not upward sloping, either graph or give a mathematical function for the PPF such that this economy would never benefit from free trade. (**HINT:** The choice of what to produce will be very simple, think about perfect compliments.)

5. (21 points total) Assume that the cost of a firm in a given industry is $c(q) = q + q^2 + 25$, and the fixed sunk costs are 24.
- (a) (6 points) Find the Marginal cost, average variable costs, and the supply curve of a firm.
- (b) (3 points) If there are 6 firms in the industry what is the short run supply curve?
- (c) (3 points) Find the price at which firms will enter this industry.
- (d) (3 points) Find the medium run supply curve.
- (e) (6 points) If the demand in the industry is $Q = 93 - 3P$ find the market price and quantity in the short and the medium run.

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- (a) (4 points) Find the Contract Curve, or the set of Pareto Efficient allocations. Explain your reasoning.

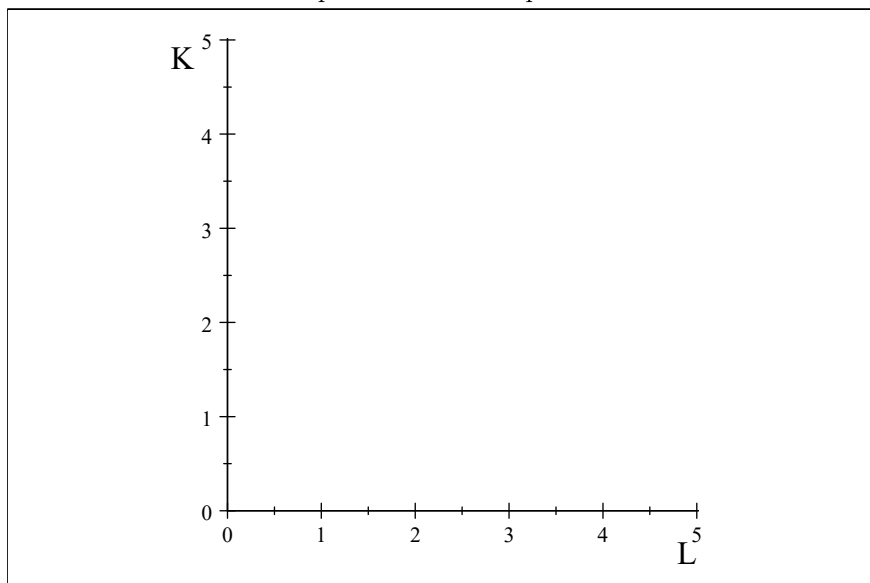
- (b) (4 points) Find the prices in all competitive equilibria. Explain your reasoning.

- (c) (6 points) Find the allocation, or the quantities of food and clothing that person one will consume.

3. (24 points total) In this question I want you to prove that for a specific production function and in general that the cost function is homogeneous of degree one in input prices, or $C(tw, tr, Q) = tC(w, r, Q)$ for a firm that has two inputs, K or capital and L or labor.

- (a) (15 points total) For the production function $Q = K^{\frac{1}{3}}L^{\frac{1}{6}}$.

- i. (4 points) In the graph below graph an isoquant when $Q = 1$. Be sure to list at least three points on this isoquant below.



- ii. (4 points) In the same graph find the cost minimizing isocost curve when $w = 4$ and $r = 8$.

- iii. (2 points) Now in the same graph find the cost minimizing isocost curve when $w = 2$ and $r = 4$.
- iv. (5 points) Show in the graph that this means $C(2, 4, 1) = \frac{1}{2}C(4, 8, 1)$. Explain how you could generalize this result to show that $\tilde{C}(tw, tr, Q) = tC(w, r, Q)$.
- (b) (9 points in total.) Now I want you to show the same thing for general production processes. Let (K^*, L^*) be the cost minimizing bundle at the input prices (w, r) and output Q .
- i. (3 points) If arbitrary (K, L) produce at least Q units of output is $wL^* + rK^* \geq wL + rK$, $wL^* + rK^* \leq wL + rK$, or are we unable to tell? Explain.
- ii. (6 points) Prove that the cost function is homogenous of degree one in input prices, or that for $t > 0$, $tC(w, r, Q) = C(tw, tr, Q)$.

4. (29 points total) Consider the following economy. Robinson Crusoe's utility function is: $U(F, C) = F^2C$ and the production possibility set is given by $8F + 4C \leq 48$.

(a) (4 points) Find the Marginal Rate of Transformation and the Marginal Rate of Substitution in this economy.

(b) (6 points) Find the Pareto Efficient outcome in this economy.

(c) (3 points) Find the implicit price of food in this economy, you may set the price of clothing to one.

(d) (16 points total) Assume that now Robinson Crusoe is considering opening up to world trade. The price of food (p_f) in the world economy is 4 ($p_f = 4$) and the price of food is one ($p_c = 1$).

i. (4 points) Find the amount they will produce, and the revenue they will receive. **HINT:** The answer is both simple and somewhat weird.

ii. (4 points) Find the amount they will consume.

iii. (2 points) Is Robinson better off with free trade? How do you know this?

iv. (3 points) When will Robinson not benefit from free trade? Explain.

v. (3 points) It is possible for the production possibilities frontier (PPF) to be shaped so that an economy will never change its production because of free trade. The PPF must be not upward sloping, either graph or give a mathematical function for the PPF such that this economy would never benefit from free trade. (**HINT:** The choice of what to produce will be very simple, think about perfect compliments.)

5. (21 points total) Assume that the cost of a firm in a given industry is $c(q) = q + q^2 + 36$, and the fixed sunk costs are 20.
- (a) (6 points) Find the Marginal cost, average variable costs, and the supply curve of a firm.
- (b) (3 points) If there are 5 firms in the industry what is the short run supply curve?
- (c) (3 points) Find the price at which firms will enter this industry.
- (d) (3 points) Find the medium run supply curve.
- (e) (6 points) If the demand in the industry is $Q = 85 - P$ find the market price and quantity in the short and the medium run.

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- (a) (4 points) Find the Contract Curve, or the set of Pareto Efficient allocations. Explain your reasoning.

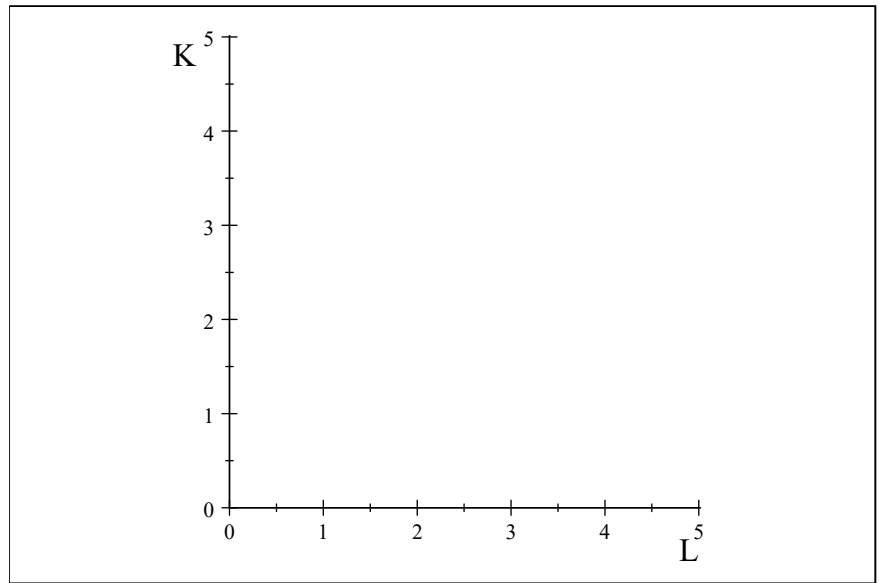
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- (a) (15 points total) For the production function $Q = K^{\frac{1}{4}}L^{\frac{1}{2}}$.

- i. (4 points) In the graph below graph an isoquant when $Q = 1$. Be sure to list at least three points on this isoquant below.



- ii. (4 points) In the same graph find the cost minimizing isocost curve when $w = 10$ and $r = 5$.

- iii. (2 points) Now in the same graph find the cost minimizing isocost curve when $w = 2$ and $r = 1$.
- iv. (5 points) Show in the graph that this means $C(2, 1, 1) = \frac{1}{5}C(10, 5, 1)$. Explain how you could generalize this result to show that $C(tw, tr, Q) = tC(w, r, Q)$.
- (b) (9 points in total.) Now I want you to show the same thing for general production processes. Let (K^*, L^*) be the cost minimizing bundle at the input prices (w, r) and output Q .
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4. (29 points total) Consider the following economy. Robinson Crusoe's utility function is: $U(F, C) = FC^3$ and the production possibility set is given by $6F + 2C \leq 48$.

(a) (4 points) Find the Marginal Rate of Transformation and the Marginal Rate of Substitution in this economy.

(b) (6 points) Find the Pareto Efficient outcome in this economy.

(c) (3 points) Find the implicit price of food in this economy, you may set the price of clothing to one.

(d) (16 points total) Assume that now Robinson Crusoe is considering opening up to world trade. The price of food (p_f) in the world economy is 2 ($p_f = 2$) and the price of food is one ($p_c = 1$).

i. (4 points) Find the amount they will produce, and the revenue they will receive. **HINT:** The answer is both simple and somewhat weird.

- ii. (4 points) Find the amount they will consume.

- iii. (2 points) Is Robinson better off with free trade? How do you know this?

- iv. (3 points) When will Robinson not benefit from free trade? Explain.

- v. (3 points) It is possible for the production possibilities frontier (PPF) to be shaped so that an economy will never change its production because of free trade. The PPF must be not upward sloping, either graph or give a mathematical function for the PPF such that this economy would never benefit from free trade. (**HINT:** The choice of what to produce will be very simple, think about perfect compliments.)

5. (21 points total) Assume that the cost of a firm in a given industry is $c(q) = 4q + \frac{1}{2}q^2 + 18$, and the fixed sunk costs are 16.
- (a) (6 points) Find the Marginal cost, average variable costs, and the supply curve of a firm.
- (b) (3 points) If there are 6 firms in the industry what is the short run supply curve?
- (c) (3 points) Find the price at which firms will enter this industry.
- (d) (3 points) Find the medium run supply curve.
- (e) (6 points) If the demand in the industry is $Q = 80 - 2P$ find the market price and quantity in the short and the medium run.