ECON 203
Final Exam

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 9:10 and end at 10:50

1. *(19 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _

   Student ID: _

   Signature:

   _

α	β	μ	δ	X	Y	L	M	$C_1(F_1)$	p_f	C_1^*	F_1^*
3	2	1	2	2	11	9	0	$\frac{3}{4}F_1$	$2p_c$	$\frac{72}{5}$	$\frac{24}{5}$
6	2	2	1	2	7	9	4	$6F_1$	$\frac{1}{2}p_c$	$\frac{33}{8}$	$\frac{11}{4}$
1	6	1	3	6	10	5	1	$\frac{1}{18}F_1$	$3p_c$	$\frac{36}{7}$	$\frac{72}{7}$
3	2	3	1	9	2	2	9	$\frac{9}{2}F_1$	$\frac{1}{3}p_c$	$\frac{29}{5}$	$\frac{58}{5}$
8	1	4	1	4	8	7	3	$32F_1$	$\frac{1}{4}p_c$	$\frac{16}{3}$	$\frac{8}{3}$

2. *(14 points total)* Consider an exchange economy, person one has the utility function: $U_1(C_1, F_1) = C_1^{\alpha} F_1^{\beta}$, person two has linear preferences with the utility function: $U_2(C_2, F_2) = \mu C_2 + \delta F_2$. Person one has the initial endowment $(C_1^e, F_1^e) = (X, Y)$ and person two has the initial endowment $(C_2^e, F_2^e) = (L, M)$. You may assume throughout that both people will consume a positive amount of both goods.

   (a) *(4 points)* Find the Contract Curve, or the set of Pareto Efficient allocations.

$$\begin{aligned} \frac{MU_F^1}{MU_C^1} &= \frac{MU_F^2}{MU_C^2} \\ \frac{\beta \frac{U_1}{F_1}}{\alpha \frac{U_1}{C_1}} &= \frac{\mu}{\delta} \\ C_1 &= \frac{\alpha}{\beta}\frac{\mu}{\delta} F_1 \end{aligned}$$

   (b) *(4 points)* Find the prices in all competitive equilibria.

$$\begin{aligned} \frac{MU_F^2}{p_f} &= \frac{MU_C^2}{p_c} \\ \frac{\delta}{p_f} &= \frac{\mu}{p_c} \end{aligned}$$

$$p_f = \frac{1}{\mu}\delta p_c$$

Solution 1 *You can and probably will normalize one of the prices to one, the obvious normalization is* $p_f = \delta$ *and* $p_c = \mu$*, but I wrote the answer in terms of* p_c *simply to allow for them to choose which they want to normalize.*

(c) *(6 points)* Find the allocation, or the quantities of food and clothing that person one will consume.

$$\frac{MU_F^1}{p_f} = \frac{MU_C^1}{p_c}$$

$$\frac{\beta\frac{U_1}{F_1}}{p_f} = \frac{\alpha\frac{U_1}{C_1}}{p_c}$$

$$p_f F_1 + p_c C_1 = p_c X + p_f Y$$

$$p_f = \frac{1}{\mu}\delta p_c$$

$$\frac{\beta\frac{U_1}{F_1}}{\frac{1}{\mu}\delta p_c} = \frac{\alpha\frac{U_1}{C_1}}{p_c}$$

$$\left(\frac{1}{\mu}\delta p_c\right) F_1 + p_c C_1 = p_c X + \left(\frac{1}{\mu}\delta p_c\right) Y$$

$$F_1 = \frac{1}{\alpha}\beta\frac{\mu}{\delta}C_1$$

$$\left(\frac{1}{\mu}\delta p_c\right)\left(\frac{1}{\alpha}\beta\frac{\mu}{\delta}C_1\right) + p_c C_1 = p_c X + \left(\frac{1}{\mu}\delta p_c\right) Y$$

$$\begin{aligned} C_1 &= \frac{1}{\alpha\mu + \beta\mu}\left(X\alpha\mu + Y\alpha\delta\right) \\ F_1 &= \frac{1}{\alpha}\beta\frac{\mu}{\delta}\left(\frac{1}{\alpha\mu + \beta\mu}\left(X\alpha\mu + Y\alpha\delta\right)\right) = \frac{1}{\delta\left(\alpha + \beta\right)}\left(X\beta\mu + Y\beta\delta\right) \end{aligned}$$

3. *(14 points)* What two facts do we know about the relationship between the long run average costs and short run average costs? How do we know these things? If the short run average costs are:

$$C^{SR}\left(w, r, K, q\right) = \min_L wL + rK - \mu\left(f\left(L, K\right) - Q\right)$$

what is $\partial C^{SR}/\partial K$ equal to and how do we know this?

Solution 2 *We know that the LRAC are always below the SRAC because by definition in the long run things are more flexible, thus your costs must be always lower than in the short run. We also know that the LRAC must be equal to the SRAC at some output level, because the firm must have been planning optimally for some given output level.*

$$\frac{\partial C^{SR}}{\partial K} = r - \lambda MP_k$$

we know this because of the envelope theorem.

4. *(8 points)* Define *Pareto efficiency.* Are suicide bombings Pareto efficient? Explain why or why not.

   Solution 3 *An allocation A is* Pareto efficient *if there is no feasible allocation B that* Pareto dominates *it.*

   An allocation A is Pareto efficient *if anything that makes some strictly happier makes others strictly worse off.*

   Suicide bombings are Pareto efficient, the proof of this is that someone is willing to give up their life to kill innocents, people who probably don't even understand why they are dying. On the other hand, we certainly prefer a world without suicide bombers, so not suicide bombing is also Pareto efficient.

5. *(12 points)* What is the difference between a fixed start up cost and a fixed sunk cost? Give an example of each. If two firms are identical except that one of them has strictly higher fixed sunk costs, what is the most likely explanation of this?

   Solution 4 *A fixed start up cost must be paid only if output is produced, a fixed sunk cost must be paid regardless of whether output is produced.*

   An example of a fixed start up cost is the resale value of your plants and equipment. An example of a sunk cost is the difference between the replacement cost of your plants and equipment and it's resale value.

   Thus the newer your plants and equipment the higher their resale value, and your fixed startup costs are higher or your fixed sunk costs are lower. Firms with high fixed sunk costs, everything else equal, are older firms.

6. *(4 points)* Define *Pareto dominance.*

   Solution 5 *An allocation A* Pareto dominates *an allocation B if everyone likes A as much as B and some people think A is strictly better.*

7. *(10 points)* Explain why in the long run we know that for a typical firm, $p = AC(q) = MC(q)$. This firm is making zero economic profits, will their accounting profits be zero? Why or why not?

Solution 6 *In the long run we have to have $p \geq AC(q)$ because otherwise firms would exit the industry. We must have $p \leq AC(q)$ because otherwise firms would want to enter the industry. Thus $p = AC(q)$, and $p = MC(q)$ because firms are profit maximizing.*

The accounting profits must be strictly positive because the owner of the firm must be making a fair income—enough so that they do not want to switch to a different industry. As well (if they own the capital) they must be getting a proper rate of return on the capital.

τ	κ	λ	π	I	χ	ϕ	F_{nt}	C_{nt}	*Produces*	F_t	C_t
1	1	2	3	12	$\frac{1}{3}$	1	2	3	F	2	6
1	2	8	2	12	1	$\frac{1}{6}$	2	1	C	3	1
1	3	9	1	12	1	$\frac{1}{3}$	3	1	F	3	3
2	1	1	6	9	$\frac{1}{2}$	1	1	3	C	3	3
3	1	1	6	8	$\frac{1}{12}$	1	1	2	F	1	4

8. *(20 points total)* Robinson Crusoe has the utility function $U(F, C) = F^\tau C^\kappa$ and a production possibilities set of $\lambda C + \pi F \leq I$.

   (a) *(6 points)* Find the amount of food and clothing he will consume and produce.

$$\frac{\tau \frac{U}{F}}{\kappa \frac{U}{C}} = \frac{\pi}{\lambda}$$
$$C = \pi F \frac{\kappa}{\lambda \tau}$$

$$\lambda \left(\pi F \frac{\kappa}{\lambda \tau} \right) + \pi F = I$$
$$F_{nt} = \frac{I}{\pi + \pi \frac{\kappa}{\tau}}$$
$$C_{nt} = \pi \left(\frac{I}{\pi + \pi \frac{\kappa}{\tau}} \right) \frac{\kappa}{\lambda \tau} = \frac{\kappa}{\lambda} \frac{I}{\kappa + \tau}$$

   (b) *(2 points)* Find the implicit prices of food and clothing in this equilibrium, one of the prices can be normalized to one.

   Solution 7 *I know that*
$$\frac{p_f}{p_c} = \frac{\pi}{\lambda}$$
   thus it must be that $p_f = \pi$, $p_c = \lambda$ is a fine solution. Of course if you normalize one price to one then it might not be so neat.

   (c) *(12 points total)* Robinson Crusoe has now realized he can trade with the Native Americans on the next island. In their economy $p_c = \chi$ and $p_f = \phi$.

i. *(3 points)* How much food and clothing will he produce? **Hint:** He will either produce all food or all clothing.

Solution 8 *If he produces only food, he will earn* $\phi \frac{I}{\pi}$*, if he produces only clothing he will earn* $\chi \frac{I}{\lambda}$*, which is larger depends on your exam. The column "produces" above tells you which is the better one to produce in each case.*

ii. *(6 points)* Find out how much he will consume of food and clothing. Is he better off? Is he worse off? How do you know?

Solution 9 *I will analyze the problem using R for his new revenue, and then plug in the proper value for R in the answers above. In fact it's quite easy to write in as a general formula of*

$$R = \max\left(\phi \frac{I}{\pi}, \chi \frac{I}{\lambda}\right) = I \max\left(\frac{\phi}{\pi}, \frac{\chi}{\lambda}\right)$$

His new budget constraint is:

$$\phi F + \chi C \leq R$$

thus his bang for the buck condition is:

$$\frac{\tau \frac{U}{F}}{\phi} = \frac{\kappa \frac{U}{C}}{\chi}$$

which implies

$$C = F \frac{\kappa}{\tau} \frac{\phi}{\chi}$$

putting this into his new budget constraint:

$$\begin{aligned} \phi F + \chi \left(F \frac{\kappa}{\tau} \frac{\phi}{\chi}\right) &= R \\ F_t &= \frac{R}{\phi + \frac{\kappa}{\tau}\phi} \\ C_t &= R \frac{\kappa}{\chi(\kappa + \tau)} \end{aligned}$$

He is always strictly better off, the simplest way to see this is because in this problem he always ends up buying the same amount as before of one of the goods, and consuming strictly more of the other. Since the utility function is Cobb-Douglass and therefore strictly monotonic this is enough.
Other ways you can answer this is show that his no-trade consumption always costs strictly less than his revenue, or (shudder) put the equilibrium consumptions before and after trade into his utility function. Because of this I made sure that the good raised to a power greater than one before trade was always one. Hopefully none of you chose to use that method. The most primitive way you could use would be to show that his new budget set

strictly contains his old one near his previous equilibrium consumption (which is equivalent to showing his no trade consumption costs strictly less.)

iii. *(3 points)* Under what conditions on the prices (p_c, p_f) do you know he will be better off?

Solution 10 *As long as* $\frac{p_f}{p_c} \neq \frac{\pi}{\lambda}$ *then he will be better off because then his old consumption point will be strictly inside his new budget set, and this will mean he can consume more of both goods and be strictly better off. The only case this might be violarted is if the consumer has Leontief preferences and the Production Possibility Set is a perfect rectangle.*