ECON 203
Midterm on Consumer Theory

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 17:40 and end at 19:20

1. *(19 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _

   Student ID: _

   Signature:

   _

2. *(24 points total)* About the normative definition of rationality.

   (a) *(9 points)* Write down the three axioms of preferences that make up this definition and define each axiom. Let $A \succsim B$ mean "the consumption bundle A is at least as good as the consumption bundle B."

   Solution 1 *Reflexivity:* $A \succsim A$

   Transitivity: $A \succsim B$ *and* $B \succsim C$ *implies* $A \succsim C$*, or there are no decision cycles.*

   Completeness: For all A *and* B*, either* $A \succsim B$ *or* $B \succsim A$*. Everything can be compared.*

   (b) *(9 points)* Give a counter example to each axiom. (These can not be about indifference curves.)

   Solution 2 *Reflexivity: Second Largest Slice of Cake preferences: If you always want the second largest slice of cake then out of* $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$ *you will only choose* $\frac{1}{3}$*, out of* $\left\{\frac{1}{3}, \frac{1}{4}\right\}$ *you will choose* $\frac{1}{4}$*. We know that both preferences are strict, and the former means* $\frac{1}{3} \succ \frac{1}{4}$ *while the latter means* $\frac{1}{4} \succ \frac{1}{3}$*, so transitivity implies* $\frac{1}{3} \succ \frac{1}{4} \succ \frac{1}{3}$*, violating reflexivity.*

   Transitivity: Any example of a decision cycle works here, that requires three items and you can not state which you would "choose" or "buy" because that negates the cycle.

   For example: Cell Phone A: Very cheap (hah, like that's true about any cell phone these days) and no features.

   Cell Phone B: Only a little more expensive and it's a basic smart phone.

Cell Phone C: Top of the line, and gosh, it seems not that much more expensive than cell phone B.
Thus we might say $B \succ A$, $C \succ B$, but then when we look at the price differential between C and A we might feel that $A \succ C$, creating a contradiction.
Completeness: Any two items that you really don't feel you can compare works here.
Some Classics: Being an ECON Professor or a Tibetan Monk, living on the Moon or living on Mars.
The most depressing example ever: Having my Mom die or having my Dad die.
But let me make clear, if YOU can not make the comparison it is a good example.

(c) *(6 points)* Two of these have clear implications in terms of indifference curves, draw indifference curves that are not allowed by each of these axioms and explain. (You may make all the other normal assumptions about preferences.)

Solution 3 *I marked you down if your indifference curves did not satisfy the standard axioms (obviously upward sloping being the prime example.) I do not have time to generate the pictures right now, but examples are in the answer keys.*

3. *(21 points)* About the income effect.

(a) *(7 points)* Given the Duality identity: $h_x(p_x, p_y, u) = X(p_x, p_y, I(p_x, p_y, u))$ and the Envelope theorem which implies $\partial I / \partial p_x = X$ derive the Slutsky equation in elasticity form: $e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x$.

Solution 4

$$\begin{aligned} \frac{\partial}{\partial p_x} h_x(p_x, p_y, u) &= \frac{\partial}{\partial p_x} X(p_x, p_y, I(p_x, p_y, u)) \\ \frac{\partial h_x}{\partial p_x} &= \frac{\partial X}{\partial p_x} + \frac{\partial X}{\partial I}\frac{\partial I}{\partial p_x} \end{aligned}$$

by the envelope theorem we know that $\partial I / \partial p_x = X$

$$\frac{\partial h_x}{\partial p_x} = \frac{\partial X}{\partial p_x} + \frac{\partial X}{\partial I} X$$

$$\begin{aligned} \frac{\partial h_x}{\partial p_x}\frac{p_x}{X} &= \frac{\partial X}{\partial p_x}\frac{p_x}{X} + \frac{\partial X}{\partial I} X \frac{p_x}{X} \\ e_{h_x}(p_x) &= e_x(p_x) + \frac{\partial X}{\partial I} X \frac{I}{I}\frac{p_x}{X} \\ e_{h_x}(p_x) &= e_x(p_x) + \frac{\partial X}{\partial I}\frac{I}{X}\frac{p_x X}{I} \\ e_{h_x}(p_x) &= e_x(p_x) + e_x(I)\, s_x \end{aligned}$$

(b) *(8 points)* Give the physical definition of each term in the Slutsky equation and state what we know either about it's sign or range. Also indicate which part is the substitution effect and which part is the Income effect.

Solution 5 *Rewriting the Slutsky equation we see that:*

$$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x$$

where

$e_x(p_x)$—*Marshallian or normal demand curves own price elasticity*

$e_{h_x}(p_x)$—*Hicksian or Income compensated demand curves own price elasticity,* $e_{h_x}(p_x) \leq 0$

$e_x(I)$—*Marshallian or normal demand curves income elasticity*

s_x—*the share of income spent on* X, $0 \leq s_x \leq 1$

$e_{h_x}(p_x)$—*the substitution effect*

$-e_x(I)\, s_x$—*the income effect, note that the negative is important but I didn't mark you down if you didn't include it.*

(c) *(6 points)* Assume that bread is an inferior good with downward sloping demand, the Turkish government wants to increase the total amount of bread Turks consume and make everyone happier. Which of the following three policies would accomplish this? Explain the problem with the other two policies.

i. Change in the price of bread (either raising or lowering the price.)

ii. Change in Income (either raising or lowering income.)

iii. Giving out free bread to every citizen.

Solution 6 *We are told* $e_B(I) < 0$, $e_B(p_B) < 0$ *and we want* $B \uparrow$ *while everyone is happier.*

Everyone being happier rules out either increasing the price or decreasing income.

Given this, increasing income will decrease B. *(Note decreasing* I *will work but will make people less happy.)*

Giving out free bread, like bread tickets, is simply an income transfer for most people (except those that consume almost no bread). Thus this will decrease B *unless the government starts buying almost all the bread.*

Thus the only strategy that will work is decreasing the price of bread, which since $e_B(p_B) < 0$ *will increase the consumption of bread.*

Note that this policy is the one that the Turkish government decided on.

4. *(4 points)* Explain why every model must be wrong by definition.

Solution 7 *A model, like a map, is supposed to capture only the key details of a phenomena. It is supposed to give us insight, not tell us what to do every moment. A map will intentionally leave details like elevation, road hazards, out simply because including them would lead to confusion.*

If you try to include every aspect of an interaction then it would just be a description, not able to tell us what would happen in a similar but different situation at all. For example $E = mc^2$ *is not correct, a more precise approximation is* $E^2 = \left(mc^2\right)^2 + (pc)^2$ *where p is momentum, and this is just a closer approximation.*

5. *(32 points total)* A *corner solution* is a case where one or more variables is at its constrained amount, in our case this is zero. In this question we are interested in when consumers might be at a corner solution, or consume zero of one or more goods. You may assume throughout that the consumer has normal preferences, i.e. her preferences are rational, continuous, monotonic, and convex.

   (a) *(4 points)* What assumption do we usually make about preferences that rules out corner solutions? Explain.

   Solution 8 *'We assume goods are essential, or that indifference curves never cross the axes. This guarantees that we consume some of all goods in our bundle.*

   (b) *(5 points)* In the real world, are corner solutions common or rare? Explain why you think this. (Note: A good argument is worth points even if your guess is wrong.)

   Solution 9 *Clearly they are very common, for example vegetarians eat no meat. Even if we label one of the goods "rice" then there is bread, pasta, potatoes, and etcetera that can be substituted for that good. Many people do not consume one of the goods in that class. On an extreme level most consumers would say that Pepsi cola and Coca cola are different goods, meaning that many of us consume almost exclusively one of them.*

   But, one could say, a restaurant will only have Coca cola or Pepsi cola, so in the end we end up consume almost all goods. Rice flour is used in many products you don't think of as rice, and even vegetarians often consume meat by accident.

α	β	$F(p_f, p_c)$	$C(p_c, I)$	$F \mid C = 0$	$C \leq 0$
11	1	$11\frac{p_c}{p_f}$	$\frac{I}{p_c} - 11$	$\frac{I}{p_f}$	$I \leq 11p_c$
10	2	$5\frac{p_c}{p_f}$	$\frac{I}{p_c} - 5$	$\frac{I}{p_f}$	$I \leq 5p_c$
9	3	$3\frac{p_c}{p_f}$	$\frac{I}{p_c} - 3$	$\frac{I}{p_f}$	$I \leq 3p_c$
8	4	$2\frac{p_c}{p_f}$	$\frac{I}{p_c} - 2$	$\frac{I}{p_f}$	$I \leq 2p_c$

(c) *(23 points total)* Consider the utility function $U(F,C) = \alpha \ln F + \beta C$, the price of food is p_f, of clothing is p_c, and the income is I. (In your answers below you can not use the cheating method I described in class.)

i. *(1 point)* What class of utility functions is this from?

Solution 10 *It is a quasi-linear utility function.*

ii. *(2 points)* Set up the Lagrangian objective function for this consumer, make the multiplier on the constraint λ.

$$\max_{F,C} \min_{\lambda} \alpha \ln F + \beta C - \lambda (p_f F + p_c C - I)$$

iii. *(3 points)* Find the three first order conditions **assuming the consumption of clothing is strictly positive.**

$$\begin{aligned} \frac{\alpha}{F} - \lambda p_f &= 0 \\ \beta - \lambda p_c &= 0 \\ -(p_f F + p_c C - I) &= 0 \end{aligned}$$

iv. *(4 points)* Find the bang for the buck for food and clothing **assuming the consumption of clothing is strictly positive.**

$$\begin{aligned} \frac{\alpha}{F} - \lambda p_f &= 0 \\ \lambda &= \frac{1}{F}\frac{\alpha}{p_f} \\ \beta - \lambda p_c &= 0 \\ \lambda &= \frac{\beta}{p_c} \end{aligned}$$

v. *(4 points)* Equalize the bang for the bucks and find the demand for food **assuming the consumption of clothing is strictly positive.**

$$\begin{aligned} \frac{1}{F}\frac{\alpha}{p_f} &= \frac{\beta}{p_c} \\ F &= \frac{\alpha}{\beta}\frac{p_c}{p_f} \end{aligned}$$

Remark 11 *A "demand curve" is the endogenous (F) in terms of the exogenous (p_f, p_c, I) generally it will have all three variables in it, but with quasi-linear one of the goods is not affected by Income. The precise wording of the question explicitly makes no mention of the budget constraint for a reason.*

Notice that with a Cobb-Douglass $F = \mu \frac{I}{p_f}$, so p_c does not appear. This is equally weird, though in this case you do need the budget constraint.

vi. *(2 points)* Find the demand for clothing **assuming the consumption of clothing is strictly positive.**

$$\begin{aligned}
(p_f F + p_c C - I) &= 0 \\
p_f \left(\frac{\alpha}{\beta}\frac{p_c}{p_f}\right) + p_c C &= I \\
\frac{\alpha}{\beta} p_c + p_c C &= I \\
C &= \frac{I}{p_c} - \frac{\alpha}{\beta}
\end{aligned}$$

vii. *(2 points)* If the optimal quantity of clothing is zero, how much food will this consumer buy? (Hint: This question is ridiculously easy.)

$$\begin{aligned}
p_f F + p_c C &= I, C = 0 \Rightarrow \\
p_f F &= I \\
F &= \frac{I}{p_f}
\end{aligned}$$

viii. *(5 points)* Find a condition under which the optimal consumption of clothing is zero.

Solution 12 *Sigh, there's a RIGHT way and a WRONG way to answer this question, and unfortunately they both give the same answer... so I guess I'll have to give full credit to the WRONG way. The way most of you probably answered this was just to find out when demand for C was weakly negative:*

$$\begin{aligned}
\frac{I}{p_c} - \frac{\alpha}{\beta} &\leq 0 \\
I &\leq \frac{\alpha}{\beta} p_c
\end{aligned}$$

This, however, is not the proper way of answering this question the proper way is to see when:

$$BFB_f = \frac{1}{F}\frac{\alpha}{p_f} \geq \frac{\beta}{p_c} = BFB_c$$

when $C = 0$ and $F = \frac{I}{p_f}$

$$\begin{aligned}
\frac{1}{\frac{I}{p_f}}\frac{\alpha}{p_f} &\geq \frac{\beta}{p_c} \\
\frac{\alpha}{I} &\geq \frac{\beta}{p_c} \\
\alpha p_c &\geq \beta I \\
\frac{\alpha}{\beta} p_c &\geq I
\end{aligned}$$

I am pleased that so many of you used the intuitive approach, and generally just didn't realize $F = \frac{I}{p_f}$ *in that case.*
The reason the former way is "wrong" is because you aren't using the primitive condition that can lead to a corner solution. But unfortunately both of the answers are the same. Sigh.