ECON 203
Final on Costs, Supply, and Equilibrium

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 18:20 and end at 20:00

1. *(12 points)* **Honor Statement:** Please read and sign the following statement:

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2. *(15 points)* About Pareto efficiency.

   (a) *(4 points)* What does it mean when we say allocation A Pareto dominates (or Pareto improves on) allocation B?

   (b) *(3 points)* Using the concept of Pareto dominance, define Pareto efficiency.

   (c) *(4 points)* Give an alternative definition of Pareto efficiency that does not (explicitly) rely on Pareto dominance.

(d) *(4 points)* What property of the utility function makes Pareto efficiency the only formal welfare concept in Economics? Explain.

3. *(24 points total)* Consider a general equilibrium exchange economy. Person 1 has the utility function $u_1(C, F) = C^5F$ and the initial endowment $(C_{01}, F_{01}) = (2, 8)$, and person 2 has the utility function $u_2(C, F) = CF^2$ and the initial endowment $(C_{02}, F_{02}) = (5, 13)$.

   (a) *(6 points)* Find the marginal rate of substitution for both people.

   (b) *(6 points)* Find the contract curve or the set of Pareto efficient outcomes for this economy.

(c) *(6 points)* Show that when you are maximizing the utility function $u(C,F) = C^{\alpha}F^{\beta}$ subject to $p_cC + p_fF \leq I$ that the solution is $C(p_c,p_f,I) = \frac{\alpha}{\alpha+\beta}\frac{I}{p_c}$ and $F(p_c,p_f,I) = \frac{\beta}{\alpha+\beta}\frac{I}{p_f}$. You may use any technique you to chose to, formal derivation of all steps is not required. Note: You may assume the technical conditions hold. Namely that $C(p_c,p_f,I) > 0$ and $F(p_c,p_f,I) > 0$.

(d) *(6 points)* Find the Walrasian or competitive equilibrium prices. You may use the answers given in the last part, assume that the optimal consumptions are all strictly positive, and that both prices are strictly positive.

4. *(10 points total)* About the oil crisis.

(a) *(6 points)* Empirically speaking is there any evidence there ever has been a (natural) oil crisis, or that there ever will be?

(b) *(4 points)* If there has been, when did it start and how bad has it become. If not explain why not.

5. *(25 points total)* Consider a market where all firms use the same technology: $c(q) = 2q + \frac{1}{2}q^2 + 200$ but there are 2 type a firms that have zero start up costs and 2 type b firms that have a fixed sunk cost of 192. Throughout the question the demand will be: $Q = 142 - P$.

   (a) *(3 points)* Which type, a or b, have older capital? (I.e. it has been longer since they reinvested.)

   (b) *(4 points)* Find the marginal cost and average cost of a typical firm, and the average variable cost for each type.

(c) *(8 points)* Find the short run supply curve of each type of firm and the industry.

(d) *(3 points)* Find the short run equilibrium.

(e) *(4 points)* In the long run what price will be charged and how much will each firm produce?

(f) *(3 points)* Find the equilibrium quantity in the long run and the equilibrium number of firms.

6. *(14 points total)* Define the cost function as:

$$C(w, r, Q) = \min_{L,K} \max_{\mu} wL + rK - \mu (f(L, K) - Q)$$

Note: You may assume all technical conditions hold. Namely the optimal values (L^*, K^*, μ^*) are all strictly positive, that all appropriate second order conditions hold, and that all derivatives are well defined.

(a) *(8 points)* Prove that $\frac{\partial C(w,r,Q)}{\partial w} = L^* = L(w, r, Q)$ by proving the envelope theorem for this problem.

(b) *(6 points)* What property of the cost function then guarantees that $\frac{\partial L(w,r,Q)}{\partial w} \leq 0$? Explain.

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(d) *(4 points)* What property of the utility function makes Pareto efficiency the only formal welfare concept in Economics? Explain.

3. *(24 points total)* Consider a general equilibrium exchange economy. Person 1 has the utility function $u_1(C, F) = C^3F$ and the initial endowment $(C_{01}, F_{01}) = (12, 7)$, and person 2 has the utility function $u_2(C, F) = C^3F$ and the initial endowment $(C_{02}, F_{02}) = (9, 7)$.

   (a) *(6 points)* Find the marginal rate of substitution for both people.

   (b) *(6 points)* Find the contract curve or the set of Pareto efficient outcomes for this economy.

(c) *(6 points)* Show that when you are maximizing the utility function $u(C,F) = C^{\alpha}F^{\beta}$ subject to $p_cC + p_fF \leq I$ that the solution is $C(p_c,p_f,I) = \frac{\alpha}{\alpha+\beta}\frac{I}{p_c}$ and $F(p_c,p_f,I) = \frac{\beta}{\alpha+\beta}\frac{I}{p_f}$. You may use any technique you to chose to, formal derivation of all steps is not required. Note: You may assume the technical conditions hold. Namely that $C(p_c,p_f,I) > 0$ and $F(p_c,p_f,I) > 0$.

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(a) *(6 points)* Empirically speaking is there any evidence there ever has been a (natural) oil crisis, or that there ever will be?

(b) *(4 points)* If there has been, when did it start and how bad has it become. If not explain why not.

5. *(25 points total)* Consider a market where all firms use the same technology: $c(q) = 4q + q^2 + 64$ but there are 2 type a firms that have zero start up costs and 4 type b firms that have a fixed sunk cost of 39. Throughout the question the demand will be: $Q = 84 - P$.

   (a) *(3 points)* Which type, a or b, have older capital? (I.e. it has been longer since they reinvested.)

   (b) *(4 points)* Find the marginal cost and average cost of a typical firm, and the average variable cost for each type.

(c) *(8 points)* Find the short run supply curve of each type of firm and the industry.

(d) *(3 points)* Find the short run equilibrium.

(e) *(4 points)* In the long run what price will be charged and how much will each firm produce?

(f) *(3 points)* Find the equilibrium quantity in the long run and the equilibrium number of firms.

6. *(14 points total)* Define the cost function as:

$$C(w, r, Q) = \min_{L,K} \max_{\mu} wL + rK - \mu (f(L, K) - Q)$$

Note: You may assume all technical conditions hold. Namely the optimal values (L^*, K^*, μ^*) are all strictly positive, that all appropriate second order conditions hold, and that all derivatives are well defined.

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(d) *(4 points)* What property of the utility function makes Pareto efficiency the only formal welfare concept in Economics? Explain.

3. *(24 points total)* Consider a general equilibrium exchange economy. Person 1 has the utility function $u_1(C, F) = C^4F$ and the initial endowment $(C_{01}, F_{01}) = (5, 5)$, and person 2 has the utility function $u_2(C, F) = CF^5$ and the initial endowment $(C_{02}, F_{02}) = (12, 9)$.

   (a) *(6 points)* Find the marginal rate of substitution for both people.

   (b) *(6 points)* Find the contract curve or the set of Pareto efficient outcomes for this economy.

(c) *(6 points)* Show that when you are maximizing the utility function $u(C,F) = C^{\alpha}F^{\beta}$ subject to $p_cC + p_fF \leq I$ that the solution is $C(p_c,p_f,I) = \frac{\alpha}{\alpha+\beta}\frac{I}{p_c}$ and $F(p_c,p_f,I) = \frac{\beta}{\alpha+\beta}\frac{I}{p_f}$. You may use any technique you to chose to, formal derivation of all steps is not required. Note: You may assume the technical conditions hold. Namely that $C(p_c,p_f,I) > 0$ and $F(p_c,p_f,I) > 0$.

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(a) *(6 points)* Empirically speaking is there any evidence there ever has been a (natural) oil crisis, or that there ever will be?

(b) *(4 points)* If there has been, when did it start and how bad has it become. If not explain why not.

5. *(25 points total)* Consider a market where all firms use the same technology: $c(q) = 6q + 2q^2 + 50$ but there are 4 type a firms that have zero start up costs and 4 type b firms that have a fixed sunk cost of 42. Throughout the question the demand will be: $Q = 58 - \frac{1}{2}P$.

   (a) *(3 points)* Which type, a or b, have older capital? (I.e. it has been longer since they reinvested.)

   (b) *(4 points)* Find the marginal cost and average cost of a typical firm, and the average variable cost for each type.

(c) *(8 points)* Find the short run supply curve of each type of firm and the industry.

(d) *(3 points)* Find the short run equilibrium.

(e) *(4 points)* In the long run what price will be charged and how much will each firm produce?

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Note: You may assume all technical conditions hold. Namely the optimal values (L^*, K^*, μ^*) are all strictly positive, that all appropriate second order conditions hold, and that all derivatives are well defined.

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(d) *(4 points)* What property of the utility function makes Pareto efficiency the only formal welfare concept in Economics? Explain.

3. *(24 points total)* Consider a general equilibrium exchange economy. Person 1 has the utility function $u_1(C, F) = CF^2$ and the initial endowment $(C_{01}, F_{01}) = (9, 6)$, and person 2 has the utility function $u_2(C, F) = CF^4$ and the initial endowment $(C_{02}, F_{02}) = (6, 17)$.

   (a) *(6 points)* Find the marginal rate of substitution for both people.

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(b) *(4 points)* If there has been, when did it start and how bad has it become. If not explain why not.

5. *(25 points total)* Consider a market where all firms use the same technology: $c(q) = 2q + q^2 + 36$ but there are 2 type a firms that have zero start up costs and 2 type b firms that have a fixed sunk cost of 27. Throughout the question the demand will be: $Q = 56 - P$.

   (a) *(3 points)* Which type, a or b, have older capital? (I.e. it has been longer since they reinvested.)

   (b) *(4 points)* Find the marginal cost and average cost of a typical firm, and the average variable cost for each type.

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