ECON 203
Midterm on Consumer Theory

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 17:40 and end at 19:20

1. *(19 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(22 points total)* About the Slutsky equation.

   (a) *(8 points)* Write down the Slutsky equation in elasticity form, defining each term.

   Solution 1

   $$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\,s_x$$

   where $e_x(p_x)$ *is the standard (Marshallian) elasticity of the demand for* x *with regard to the price of* x, $e_{h_x}(p_x)$ *is the elasticity of the Hicksian (Income compensated) demand curve,* $e_x(I)$ *is the elasticity of the Marshallian demand curve with regards to income and* $s_x = \frac{p_x X}{I}$ *is the share of your income spent on the good* x.

   (b) *(4 points)* What are *Giffen Goods*? Using the Slutsky equation explain how they can exist. Are there any real world examples of Giffen Goods?

   Solution 2 *A* Giffen Good *is one where* $e_x(p_x) > 0$, *or when the price of the good rises the demand for that good rises. Using the slutsky equation we can see this means that* $e_{h_x}(p_x) - e_x(I)\,s_x > 0$ *since we know that* $e_{h_x}(p_x) < 0$ *this requires that* $-e_x(I)\,s_x > -e_{h_x}(p_x) > 0$ *thus we must have:*

   i. $|e_{h_x}(p_x)|$ *is* small, *or this is a good that has few substitutes.*

   ii. $e_x(I) < 0$ *or this is an inferior good, and this effect should be large*

   iii. s_x *is large, or it is a good that consumers spend most of their income on.*

Because, in particular, of the last requirement the best examples will be staple foods. One example of dubious merit is potatoes during the Irish potato famine. Potatoes were only consumed by the poor, the well off would substitute to other (less health) options thus $e_x(I)$ *was strongly negative. The poor would consume basically only potatoes and thus* $|e_{h_x}(p_x)|$ *would be small and* s_x *was large for the same reason.*

Recently a paper in the AER has found strong support for this in extremely poor rural China:

Jensen, Robert T., and Nolan H. Miller. "Giffen Behavior and Subsistence Consumption." The American Economic Review, vol. 98, no. 4, American Economic Association, 2008, pp. 1553–77, http://www.jstor.org/stable/29730133.

To be precise they found strong evidence for rice in the Hunan province (South Eastern China)—particularly among the desperately poor. They found this evidence by subsidizing the price of rice for some rural poor, and found the strongest explanatory variable was the degree of their poverty. They also found weak evidence for wheat in Gansu (North Central China).

In short, I was wrong, Giffen Goods ***do*** *exist. But they are still very rare.*

While that article does find strong evidence for rice in Hunan they have to select a very specific class of consumers. They must be desperately poor but not so desperately poor that they need all the calories they can get. The "upper class" of the desperately poor.

A common goal of governments is to increase the consumption of one good or a class of goods—like lunch, bread, or margarine. The rest of the question is about how to do this.

(c) *(3 points)* One method they might want to use is to subsidize everyone's income. Explain how this might reduce the consumption of the good in question.

Solution 3 *If the good is inferior* ($e_x(I) < 0$) *this will cause everyone to reduce their consumption.*

(d) *(3 points)* A second method would be to give them a small amount of the good for free. Explain how this might reduce the consumption of the good in question.

Solution 4 *Let* $\bar{x}$ *be the subsidy and assume the good is inferior, then there will be two classes of people.*

i. If before the subsidy their consumption was $x < \bar{x}$ *these people will consume* $\bar{x}$ *for free and increase their consumption.*

ii. If before the subsidy $x > \bar{x}$ *then this will act as an income subsidy and since* $e_x(I) < 0$ *they will reduce their consumption.*

If the good is normal or a luxury then both groups will increase their consumption, with those whom had $x < \bar{x}$ possibly increasing it more than with a straight income subsidy.

(e) *(4 points)* A final method would be to subsidize the price of that good. Using the slutsky equation explain how this method is always superior to the last two.

Solution 5 *Since we are talking about price decreasing we can see from the slutsky equation there will be two effects. First is substitution, $e_{h_x}(p_x) < 0$ so this will increase consumption. Second is the income effect $-e_x(I)\, s_x$. For normal goods this will reinforce the substitution effect, for inferior goods it will work against it the substitution effect but this is the only program that will increase consumption at all.*

3. *(13 points total)* About *strict monotonicity* or *more is better*

(a) *(3 points)* Define this assumption using words or symbols.

Solution 6 *If consumption bundle A has at least as much of every good as consumption bundle B and strictly more of at least one good, then A is strictly better than B.*

(b) *(4 points)* Give an example that makes it obvious that this is a very bad assumption about human preferences.

Solution 7 *Any food, if consumed to excess, can kill a person. The simplest example is water. According to this assumption if you would rather have a 1/2 liter of water than zero then you would also rather have 6 liters (to be consumed immediately) than 2 liters. Unfortunately it is a medical fact that 3 to 4 liters of water is enough to kill someone.*

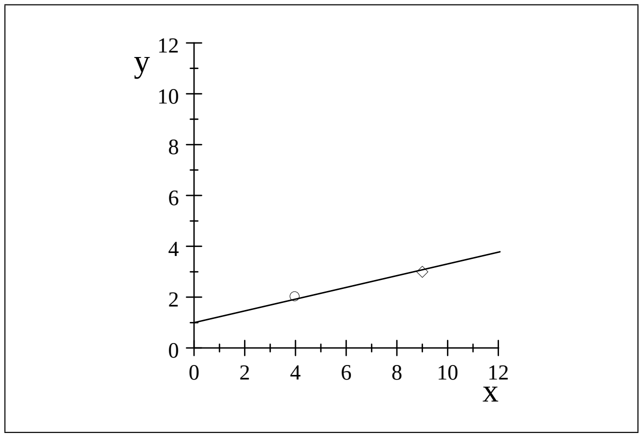
(c) *(2 points)* If it is such a bad assumption about human preferences, why do we make this assumption?

Solution 8 *The embarrassing fact is that it makes the math easy. With this assumption $MU_x > 0$ always, which greatly simplifies analysis. It is frankly hard to write down a mathematical function that satisfies the (more reasonable) assumption of local non-satiation that does not also satisfy at least weak monotonicity.*

It is also not "unreasonable" in analysis. We could, for example, say that you can store the water and then for all reasonable levels we would have strictly monotonic preferences over water.

(d) *(4 points)* Show what it rules out in indifference curves. Explain why.

Solution 9 *It rules out upward sloping indifference curves. For example in the picture below:*



The diamond has strictly more of all goods than the circle, thus by (weak) monotonicity it must be strictly better.

α	β	ρ	$\frac{\partial U}{\partial F}$	$\frac{\partial U}{\partial C}$	MRS	$C(p_f, p_c, F)$	$F(p_f, p_c, I)$	$C(p_f, p_c, I)$
1	16	1	$\frac{1}{F^2}$	$\frac{16}{C^2}$	$\frac{16}{C^2}F^2$	$4F\sqrt{\frac{1}{p_c}p_f}$	$\frac{1}{p_f+4\sqrt{p_c}\sqrt{p_f}}I$	$\frac{1}{p_f+4\sqrt{p_c}\sqrt{p_f}}I4\sqrt{\frac{1}{p_c}p_f}$
9	1	1	$\frac{9}{F^2}$	$\frac{1}{C^2}$	$\frac{1}{9C^2}F^2$	$\frac{1}{3}F\sqrt{\frac{1}{p_c}p_f}$	$\frac{1}{p_f+\frac{1}{3}\sqrt{p_c}\sqrt{p_f}}I$	$\frac{1}{p_f+\frac{1}{3}\sqrt{p_c}\sqrt{p_f}}I\frac{1}{3}\sqrt{\frac{1}{p_c}p_f}$
1	8	2	$\frac{2}{F^3}$	$\frac{16}{C^3}$	$\frac{8}{C^3}F^3$	$2F\sqrt[3]{\frac{1}{p_c}p_f}$	$\frac{I}{p_f+2p_c^{\frac{2}{3}}\sqrt[3]{p_f}}$	$2I\frac{\sqrt[3]{\frac{1}{p_c}p_f}}{p_f+2p_c^{\frac{2}{3}}\sqrt[3]{p_f}}$
8	1	2	$\frac{16}{F^3}$	$\frac{2}{C^3}$	$\frac{1}{8C^3}F^3$	$\frac{1}{2}F\sqrt[3]{\frac{1}{p_c}p_f}$	$2\frac{I}{2p_f+p_c^{\frac{2}{3}}\sqrt[3]{p_f}}$	$I\frac{\sqrt[3]{\frac{1}{p_c}p_f}}{2p_f+p_c^{\frac{2}{3}}\sqrt[3]{p_f}}$

4. *(34 points total)* Consider the utility function: $u(F, C) = -\alpha\frac{1}{F^\rho} - \beta\frac{1}{C^\rho}$. **NOTE:** There is significant partial credit to be gained for someone who can not answer all of the question.

   (a) *(5 points)* Establish this utility function is strictly monotone for $F > 0$ and $C > 0$, what does this tell us about someone maximizing this over a budget set: $p_f F + p_c C \leq I$.

   Solution 10 $U = -\alpha F^{-\rho} - \beta C^{-\rho}$ *so* $\frac{\partial U}{\partial F} = -(-\rho)\alpha F^{-\rho-1} = \frac{1}{F^{\rho+1}}\alpha\rho$ *and* $\frac{\partial U}{\partial C} = -(-\rho)\beta C^{-\rho-1} = \frac{1}{C^{\rho+1}}\beta\rho$ *thus it is strictly monotone. This means that the optimal consumptions will always be on the budget constraint, where* $p_f F + p_c C = I$.

   (b) *(2 points)* Establish this utility function is convex for $F > 0$ and $C > 0$.

   Solution 11 $MRS = \frac{\frac{\partial U}{\partial C}}{\frac{\partial U}{\partial F}} = \frac{\frac{1}{C^{\rho+1}}\beta\rho}{\frac{1}{F^{\rho+1}}\alpha\rho} = \frac{\beta}{\alpha}\frac{F^{\rho+1}}{C^{\rho+1}}$ *which is clearly decreasing in* C*, thus the preferences are convex.*

(c) *(2 points)* For utility maximization it is equivalent to $u(F,C) = \frac{1}{\alpha\frac{1}{F^\rho}+\beta\frac{1}{C^\rho}}$ for $F > 0$, $C > 0$.

Solution 12 *$u(F,C) = \left(\alpha\frac{1}{F^\rho}+\beta\frac{1}{C^\rho}\right)^{-1}$ and $u > 0$ if $F > 0$ and $C > 0$ thus $f(u) = -u^{-1}$ is a monotonic transformation of $u(F,C)$ and*

$$-u(F,C)^{-1} = -\left(\frac{1}{\alpha\frac{1}{F^\rho}+\beta\frac{1}{C^\rho}}\right)^{-1} = -\frac{1}{C^\rho}\beta - \frac{1}{F^\rho}\alpha$$

Now you will solve the problem of maximizing $u(F,C) = -\alpha\frac{1}{F^\rho} - \beta\frac{1}{C^\rho}$ over the budget set $p_fF + p_cC \leq I$ where $p_f > 0$, $p_c > 0$ and $I > 0$.

(d) *(2 points)* Set up the objective function.

Solution 13

$$L(F,C,\lambda) = -\alpha\frac{1}{F^\rho} - \beta\frac{1}{C^\rho} - \lambda(p_fF + p_cC - I)$$

(e) *(4 points)* Find the first order conditions.

$$\begin{aligned}
\frac{\partial L}{\partial F} &= \frac{1}{F^{\rho+1}}\alpha\rho - \lambda p_f = 0\\
\frac{\partial L}{\partial C} &= \frac{1}{C^{\rho+1}}\beta\rho - \lambda p_c = 0\\
\frac{\partial L}{\partial \lambda} &= -(p_fF + p_cC - I) = 0
\end{aligned}$$

(f) *(4 points)* Solve for the Bang for the Buck's and then find a function for C in terms of prices and F.

$$\begin{aligned}
\frac{1}{F^{\rho+1}}\alpha\rho - \lambda p_f &= 0\\
\lambda &= \frac{1}{F^{\rho+1}}\alpha\frac{\rho}{p_f}\\
\frac{1}{C^{\rho+1}}\beta\rho - \lambda p_c &= 0\\
\lambda &= \frac{1}{C^{\rho+1}}\beta\frac{\rho}{p_c}
\end{aligned}$$

$$\begin{aligned}
\frac{1}{F^{\rho+1}}\alpha\frac{\rho}{p_f} &= \frac{1}{C^{\rho+1}}\beta\frac{\rho}{p_c}\\
C^{\rho+1} &= \frac{1}{\frac{1}{F^{\rho+1}}\alpha\frac{\rho}{p_f}}\beta\frac{\rho}{p_c}\\
C^{\rho+1} &= F^{\rho+1}\frac{\beta}{\alpha}\frac{p_f}{p_c}\\
C &= F\left(\frac{\beta}{\alpha}\frac{p_f}{p_c}\right)^{\frac{1}{\rho+1}}
\end{aligned}$$

(g) *(3 points)* Find the demand curve for F.

Solution 14 $p_f F + p_c C = I$ *therefore* $p_f F + p_c \left(F \left(\frac{\beta}{\alpha} \frac{p_f}{p_c} \right)^{\frac{1}{\rho+1}} \right) = F \left(p_f + p_c \left(\frac{1}{\alpha} \frac{\beta}{p_c} p_f \right)^{\frac{1}{\rho+1}} \right) = I$ *or*

$$F = \frac{1}{\left(p_f + p_c \left(\frac{1}{\alpha} \frac{\beta}{p_c} p_f \right)^{\frac{1}{\rho+1}} \right)} I$$

This is the form I expect most of you will use, but it should be simplified.

$$\begin{aligned} F &= \frac{1}{\left(p_f + p_c^1 p_c^{-\frac{1}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha} \right)^{\frac{1}{\rho+1}} \right)} I \\ &= \frac{1}{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha} \right)^{\frac{1}{\rho+1}} \right)} I \end{aligned}$$

(h) *(6 points)* Establish the elasticity of food with respect to income is one ($e_f(I) = 1$). Why is this a problem? Why might it be a desirable characteristic for analysis of the behavior of rich and poor people?

Solution 15

$$\begin{aligned} F &= \frac{1}{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha} \right)^{\frac{1}{\rho+1}} \right)} I \\ \frac{\partial F}{\partial I} &= \frac{1}{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha} \right)^{\frac{1}{\rho+1}} \right)} \\ \frac{\partial F}{\partial I} * I &= \frac{1}{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha} \right)^{\frac{1}{\rho+1}} \right)} I = F \\ \frac{\partial F}{\partial I} \frac{I}{F} &= \left(\frac{\partial F}{\partial I} * I \right) \frac{1}{F} = F \frac{1}{F} = 1 \end{aligned}$$

this is a problem because it contradicts Engel's Law, that the income elasticity of food is strictly below one. It might be a desirable characteristic in analysis because it would mean that you are not assuming the rich and poor act differently, so any differences in behaviors would be because of something in your analysis—not an a-priori assumption.

(i) *(6 points)* Now find the demand curve for C using two different methods and that the results are the same either way.

Solution 16 *From equalizing the BfB's we see that:*

$$\begin{aligned}
\frac{1}{F^{\rho+1}}\alpha\frac{\rho}{p_f} &= \frac{1}{C^{\rho+1}}\beta\frac{\rho}{p_c} \\
C &= F\left(\frac{\beta}{\alpha}\frac{p_f}{p_c}\right)^{\frac{1}{\rho+1}} \\
&= \frac{1}{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\rho+1}}\right)} I \left(\frac{\beta}{\alpha}\frac{p_f}{p_c}\right)^{\frac{1}{\rho+1}}
\end{aligned}$$

From the budget constraint we find that:

$$\begin{aligned}
p_f F + p_c C &= I \\
p_f\left(\frac{1}{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\rho+1}}\right)} I\right) + p_c C &= I
\end{aligned}$$

$$\begin{aligned}
p_c C &= \left(1 - p_f\left(\frac{1}{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\rho+1}}\right)}\right)\right) I \\
C &= \frac{1}{p_c}\left(1 - p_f\left(\frac{1}{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\rho+1}}\right)}\right)\right) I \\
&= \frac{I}{p_c^{\frac{1}{\rho+1}}} \frac{p_f^{\frac{1}{\rho+1}}}{p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{1}{\alpha}\beta\right)^{\frac{1}{\rho+1}}} \left(\frac{1}{\alpha}\beta\right)^{\frac{1}{\rho+1}}
\end{aligned}$$

and yes, it is quite hard to prove they are equivalent. Sorry about that. Consider it "expert points." I guess I have to do it, don't I?

$$\frac{I}{p_c^{\frac{1}{\rho+1}}} \frac{p_f^{\frac{1}{\rho+1}}}{p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{1}{\alpha}\beta\right)^{\frac{1}{\rho+1}}} \left(\frac{1}{\alpha}\beta\right)^{\frac{1}{\rho+1}} = \frac{1}{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\rho+1}}\right)} I \left(\frac{\beta}{\alpha}\frac{p_f}{p_c}\right)^{\frac{1}{\rho+1}}$$

If we multiply both sides by $\frac{\left(p_f + p_c^{\frac{\rho}{\rho+1}} p_f^{\frac{1}{\rho+1}} \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\rho+1}}\right)}{I}$ *we get*

$$\frac{1}{p_c^{\frac{1}{\rho+1}}} p_f^{\frac{1}{\rho+1}} \left(\frac{1}{\alpha}\beta\right)^{\frac{1}{\rho+1}} = \left(\frac{1}{\alpha}\frac{\beta}{p_c} p_f\right)^{\frac{1}{\rho+1}}$$

and we can see that these are equal.

5. *(12 points)* Write down, explain, and give a real world example of two of the three (or four) great insights of rationality.

Solution 17 *I will write down four insights, though the last two can be combined into "it is the future that matters, not the past."*

(a) *It's the margin, not the average.*

When looking to decide whether to expand your firm the question you should ask is not "is my firm making a profit?" but rather "if I expand my firm will I increase or decrease the profit?". The first question is about the average profit, the second is about the margin.

Examples of this are everywhere. Prices are set by the margin, as explained by the Diamond/Water paradox (by Adam Smith). The price of a unit of water is very low, and of a diamond is very high even though the total value of water is nearly infinite and that of a diamond is nearly zero. The solution to this paradox is the marginal cost of each good. For water it is near zero (especially in Adam Smith's day when they didn't understand about clean water), for diamonds it either takes a lot of searching or work to produce one unit.

(b) *It's all relative*

It is not the "absolute" price of any good, but rather it's price relative to the prices of other goods, or my income.

One example of this is simply inflation, it is not my income but rather my income relative to the price of the bundle of goods I wish to purchase. Since that bundle is different from that of the "average" Turk I further have my own inflation rate.

Another example is the income relative to the cost of living. A job for $1500 a month sounds pretty good in Turkey, in the US it would hardly pay the bills, in Hong Kong it would be a poverty income.

A different class of examples are based on the question "How far away is Istanbul?" The relevant answer is how long it would take to get there. This could be 4 to 6 hours depending on traffic in Istanbul in the modern day. It could be 86 hours if you walk, which would have been a common option a hundred and fifty years ago. Of course even that answer is not sufficient, we should also consider the value of a unit of time relative to your income.

The next two insights can be joined into "it's the future that matters, not the past."

(c) *Sunk Costs are Sunk Costs*

A "sunk" cost is not recoverable. Whatever you used the money for the money now has been lost–you can not recover it no matter what you do.

The classic example of this is a ship that has sunk to the bottom of the ocean. Like the Titanic the ship would have value if you could get it to the surface but the cost is prohibitive.

My current favorite example is the Istanbul Canal from the Black Sea to the Marmaris. Most people do not want this, they do not own the land necessary to finish it, but they are still starting it. Why? Because once they spend the money it is a sunk cost, making finishing the canal lower and lower cost for future governments. The Turkish government is using the logic of sunk costs to make completing the canal more politically feasible in the future.

(d) *It's Opportunity Costs, not Accounting Costs, that we should pay attention to*

the "opportunity cost" of using a good is the value you could get from using the good for its "second best" purpose or simply selling it on the market.

For example say that you buy a machine for your factory. How much should you internally charge yourself for its use? The simple answer is the rate you would have to pay to rent the same machine, thus generally speaking r *is this rental rate. Simple accounting would say that in the year you purchase it you loose the cost of the item, and then afterwards it is free. (More advanced accounting would ameliorate the cost over several years, and might include the opportunity cost of selling it.)*

Another example, again, is salaries. Say you are offered $1500 per month, should you take the offer? It depends, if a competing firm is offering $1700 per month then you should not. Sure, $1500 per month is good, but the opportunity cost is all important.

Though please realize you might take the $1500 per month offer because the two jobs are not identical and the former is more to your taste.