ECON 203

Final on Costs, Supply, and Equilibrium

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 18:20 and end at 20:00

1. *(12 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _

   Student ID: _

   Signature:

   _

2. *(15 points)* About Pareto efficiency.

   (a) *(4 points)* What does it mean when we say allocation A Pareto dominates (or Pareto improves on) allocation B?

   Definition 1 *We say that A* Pareto dominates *B if everyone likes A better and some think it is strictly better.*

   (b) *(3 points)* Using the concept of Pareto dominance, define Pareto efficiency.

   Definition 2 *An allocation is* Pareto efficient *if there is no feasible allocation that Pareto dominates it.*

   (c) *(4 points)* Give an alternative definition of Pareto efficiency that does not (explicitly) rely on Pareto dominance.

   Definition 3 *An allocation is* Pareto efficient *if anything that makes some strictly better off makes others strictly worse off.*

   (d) *(4 points)* What property of the utility function makes Pareto efficiency the only formal welfare concept in Economics? Explain.

   Solution 4 *It is that preferences are "ordinal" or we can not measure your pleasure. This means we can not compare the happiness of one to another, and thus the only formal definition of "good" is to make everyone better or at least not worse off.*

δ	γ	χ	ϕ	κ	λ	ψ	ζ	MRS_1	MRS_2	$F_1(C_1)$	$\frac{p_f}{p_c}$
1	2	9	6	1	4	6	17	$\frac{1}{2C_1}F_1$	$\frac{1}{4C_2}F_2$	$23\frac{C_1}{30-C_1}$	2
3	1	12	7	3	1	9	7	$\frac{3}{C_1}F_1$	$\frac{3}{C_2}F_2$	$\frac{2}{3}C_1$	$\frac{1}{2}$
5	1	2	8	1	2	5	13	$\frac{5}{C_1}F_1$	$\frac{1}{2C_2}F_2$	$\frac{21C_1}{70-9C_1}$	$\frac{1}{3}$
4	1	5	5	1	5	12	9	$\frac{4}{C_1}F_1$	$\frac{1}{5C_2}F_2$	$\frac{14C_1}{340-19C_1}$	2

3. *(24 points total)* Consider a general equilibrium exchange economy. Person 1 has the utility function $u_1(C,F) = C^\delta F^\gamma$ and the initial endownment $(C_{01}, F_{01}) = (\chi, \phi)$, and person 2 has the utility function $u_2(C,F) = C^\kappa F^\lambda$ and the initial endownment $(C_{02}, F_{02}) = (\psi, \zeta)$.

   (a) *(6 points)* Find the marginal rate of substitution for both people.

$$
\begin{aligned}
MRS_1 &= \frac{MU_c^1}{MU_f^1} = \frac{\delta \frac{U_1}{C_1}}{\gamma \frac{U_1}{F_1}} = \frac{\delta}{\gamma}\frac{F_1}{C_1} \\
MRS_2 &= \frac{MU_c^2}{MU_f^2} = \frac{\kappa \frac{U_2}{C_2}}{\lambda \frac{U_2}{F_2}} = \frac{\kappa}{\lambda}\frac{F_2}{C_2}
\end{aligned}
$$

   (b) *(6 points)* Find the contract curve or the set of Pareto efficient outcomes for this economy.

$$
\begin{aligned}
MRS_1 &= MRS_2 \\
\frac{\delta}{\gamma}\frac{F_1}{C_1} &= \frac{\kappa}{\lambda}\frac{F_2}{C_2}
\end{aligned}
$$

$$
\begin{aligned}
F_1 + F_2 &= \phi + \zeta \\
C_1 + C_2 &= \chi + \psi
\end{aligned}
$$

$$
\frac{\delta}{\gamma}\frac{F_1}{C_1} = \frac{\kappa}{\lambda}\frac{(\phi + \zeta - F_1)}{(\chi + \psi - C_1)}
$$

$$
\begin{aligned}
\frac{\delta}{\gamma}F_1(\chi + \psi - C_1) &= \frac{\kappa}{\lambda}(\phi + \zeta - F_1)C_1 \\
\frac{\delta}{\gamma}F_1(\chi + \psi - C_1) &= -\frac{\kappa}{\lambda}C_1\zeta F_1 + \frac{\kappa}{\lambda}\phi C_1 \\
\frac{\delta}{\gamma}F_1(\chi + \psi - C_1) - \frac{\kappa}{\lambda}C_1\zeta F_1 &= \frac{\kappa}{\lambda}\phi C_1 \\
F_1 &= \frac{\frac{\kappa}{\lambda}\phi C_1}{\frac{\delta}{\gamma}(\chi + \psi - C_1) - \frac{\kappa}{\lambda}C_1\zeta}
\end{aligned}
$$

   (c) *(6 points)* Show that for the utility function $u(C,F) = C^\alpha F^\beta$ subject to $p_c C + p_f F \leq I$ that the solution is $C(p_c, p_f, I) = \frac{\alpha}{\alpha+\beta}\frac{I}{p_c}$ and $F(p_c, p_f, I) = \frac{\beta}{\alpha+\beta}\frac{I}{p_f}$. You may use any technique you to chose to, formal derivation of all steps is not required. Note: You may assume the technical conditions hold. Namely that $C(p_c, p_f, I) > 0$ and $F(p_c, p_f, I) > 0$.

   Solution 5 *I will use three different techniques.*

i. *Lagrange Method:*

$$L(C,F,\lambda) = C^{\alpha}F^{\beta} - \lambda(p_cC + p_fF - I)$$

$$\begin{aligned}
\alpha\frac{U}{C} - \lambda p_c &= 0 \\
\beta\frac{U}{F} - \lambda p_f &= 0 \\
-(p_cC + p_fF - I) &= 0
\end{aligned}$$

$$\frac{1}{C}U\frac{\alpha}{p_c} = \lambda = \frac{1}{F}U\frac{\beta}{p_f}$$

$$\frac{1}{C}U\frac{\alpha}{p_c} = \frac{1}{F}U\frac{\beta}{p_f}$$

$$C = F\frac{\alpha}{\beta p_c}p_f$$

$$-\left(p_c\left(F\frac{\alpha}{\beta p_c}p_f\right) + p_fF - I\right) = 0$$

$$\begin{aligned}
F &= \frac{I}{p_f + \frac{\alpha}{\beta}p_f} = \frac{1}{1+\frac{\alpha}{\beta}}\frac{I}{p_f} = \frac{\beta}{\alpha+\beta}\frac{I}{p_f} \\
C &= F\frac{\alpha}{\beta p_c}p_f = \left(\frac{\beta}{\alpha+\beta}\frac{I}{p_f}\right)\frac{\alpha}{\beta p_c}p_f \\
&= \alpha\frac{I}{p_c(\alpha+\beta)} = \frac{\alpha}{\alpha+\beta}\frac{I}{p_c}
\end{aligned}$$

ii. *MRS equals the ratio of the prices.*

$$\begin{aligned}
MRS &= \frac{\alpha\frac{U}{C}}{\beta\frac{U}{F}} = \frac{p_c}{p_f} \\
C &= F\frac{\alpha}{\beta p_c}p_f
\end{aligned}$$

and from this point on it would follow the derivation above.

iii. *Substituing out for* C *in the objective function. I doubt any of you used this, but*

$$\max_F C^{\alpha}F^{\beta} : C = \frac{1}{p_c}(I - Fp_f)$$

$$\max_F \left(\frac{1}{p_c}(I - Fp_f)\right)^{\alpha} F^{\beta}$$

$$
\begin{aligned}
-\alpha\left(\frac{1}{p_c}(I - Fp_f)\right)^{\alpha-1} F^{\beta} + \beta\left(\frac{1}{p_c}(I - Fp_f)\right)^{\alpha} F^{\beta-1} &= 0 \\
\beta\left(\frac{1}{p_c}(I - Fp_f)\right)^{\alpha} F^{\beta-1} &= \alpha\left(\frac{1}{p_c}(I - Fp_f)\right)^{\alpha-1} F^{\beta} \\
\beta\left(\frac{1}{p_c}(I - Fp_f)\right)^{\alpha-(\alpha-1)} &= \alpha F^{\beta-(\beta-1)} \\
\frac{\beta}{p_c}(I - Fp_f) &= \alpha F \\
F &= \beta\frac{I}{\alpha p_c + \beta p_f}
\end{aligned}
$$

Crazy, but it is a technique some like.

(d) *(6 points)* Find the Walrasian or competitive equilibrium prices. You may use the answers given in the last part, assume that the optimal consumptions are all strictly positive, and that both prices are strictly positive.

Solution 6 *We only need to make sure one market clears due to Walras's Law, so for example I could say:*

$$
\begin{aligned}
I_1 &= p_c\chi + p_f\phi \\
C_1 &= \frac{\delta}{\delta+\gamma}\frac{p_c\chi + p_f\phi}{p_c} \\
I_2 &= p_c\psi + p_f\zeta \\
C_2 &= \frac{\kappa}{\kappa+\lambda}\frac{p_c\psi + p_f\zeta}{p_c}
\end{aligned}
$$

$$
\begin{aligned}
C_1 + C_2 &= \chi + \psi \\
\frac{\delta}{\delta+\gamma}\frac{p_c\chi + p_f\phi}{p_c} + \frac{\kappa}{\kappa+\lambda}\frac{p_c\psi + p_f\zeta}{p_c} &= \chi + \psi
\end{aligned}
$$

$$
p_f = p_c\frac{\kappa\gamma\chi + \lambda\gamma\chi + \lambda\gamma\psi + \lambda\delta\psi}{\zeta\kappa\gamma + \zeta\kappa\delta + \kappa\delta\phi + \lambda\delta\phi}
$$

Or I could choose:

$$
\begin{aligned}
F_1 &= \frac{\gamma}{\delta+\gamma}\frac{p_c\chi + p_f\phi}{p_f} \\
F_2 &= \frac{\lambda}{\kappa+\lambda}\frac{p_c\psi + p_f\zeta}{p_f}
\end{aligned}
$$

$$
\frac{\gamma}{\delta+\gamma}\frac{p_c\chi + p_f\phi}{p_f} + \frac{\lambda}{\kappa+\lambda}\frac{p_c\psi + p_f\zeta}{p_f} = \phi + \zeta
$$

and get exactly the same answer. And of course one can normalize a price to one.

4. *(10 points total)* About the oil crisis.

   (a) *(6 points)* Empirically speaking is there any evidence there ever has been a (natural) oil crisis, or that there ever will be?

   Solution 7 *No, other than a blip in the late 70s and early 80s caused by OPEC the price of Oil per MWH has been essentially constant for the last 100 years. At the current time, most oil producers are expecting the peak in demand (highest quantity in history) by—at the latest—2040, thus it seems there will never be an oil crisis.*

   (b) *(4 points)* If there has been, when did it start and how bad has it become. If not explain why not.

   Solution 8 *There has not been one, and the basic reason is improvements in technology. This has had a two fold effect.*

   First, improved technology has made oil that was previously too expensive to extract now cost effective, thus increasing the supply of oil when the more traditional sources are running dry.

   Second, lowering the cost of alternative energy sources. As an example, the price of solar cells has fallen 90% in the last decade, making it now cheaper to (for example) run passenger ferries off of solar power than of diesel (oil).

   Won't it be cool when all the ferries in Istanbul are replaced by completely silent solar ferries? Or will you miss the smell of polutants in the air and the noise?

c	d	F	ϕ	n	m	a	b	P^b_{sd}	P_{sr}	Q_{sr}	q^{lr}	P^{lr}	Q^{lr}	n^{lr}
2	$\frac{1}{2}$	200	8	2	2	142	1	6	30	112	20	22	120	6
4	1	64	25	2	4	84	1	14	24	60	8	20	64	8
2	1	36	9	2	2	56	1	8	20	36	6	14	42	7
6	2	50	8	4	4	58	$\frac{1}{2}$	14	28	44	5	26	45	9

5. *(25 points total)* Consider a market where all firms use the same technology: $c(q) = cq + dq^2 + F$ but there are n type a firms that have zero start up costs and m type b firms that have a fixed sunk cost of $F - \phi$. Throughout the question the demand will be: $Q = a - bP$.

   (a) *(3 points)* Which type, a or b, have older capital? (I.e. it has been longer since they reinvested.)

   Solution 9 *If a firm's fixed sunk costs are high, it means they just bought their capital and it does not need replacement. Thus type a just reinvested and type b are older.*

(b) *(4 points)* Find the marginal cost and average cost of a typical firm, and the average variable cost for each type.

$$\begin{aligned} MC &= c + 2dq \\ AC &= \frac{cq + dq^2 + F}{q} \\ AVC_1 &= \frac{cq + dq^2}{q} \\ AVC_2 &= \frac{cq + dq^2 + \phi}{q} \end{aligned}$$

(c) *(8 points)* Find the short run supply curve of each type of firm and the industry.

Solution 10 *First we need to find the shut down quantity and price for each type.*
Type a:

$$\begin{aligned} c + 2dq &= \frac{cq + dq^2}{q} \\ q^a_{sd} &= 0 \\ P^a_{sd} &= c + 2d\,(0) = c \end{aligned}$$

Type b:

$$\begin{aligned} c + 2dq &= \frac{cq + dq^2 + \phi}{q} \\ q^b_{sd} &= \frac{1}{d}\sqrt{d\phi} \\ P^b_{sd} &= c + 2d\left(\frac{1}{d}\sqrt{d\phi}\right) = c + 2\sqrt{d\phi} \end{aligned}$$

If they produce their supply will be:

$$P = c + 2dq$$

$$q = \frac{1}{2d}(P - c)$$

Thus

$$\begin{aligned} s_a(P) &= \begin{cases} \frac{1}{2d}(P-c) & P \geq c \\ 0 & P \leq c \end{cases} \\ s_b(P) &= \begin{cases} \frac{1}{2d}(P-c) & P \geq c + 2\sqrt{d\phi} \\ 0 & P \leq c + 2\sqrt{d\phi} \end{cases} \end{aligned}$$

And

$$S(P) = ns_a(P) + ms_b(P) = \begin{cases} (n+m)\frac{1}{2d}(P-c) & P \geq c + 2\sqrt{d\phi} \\ n\frac{1}{2d}(P-c) & P \leq c + 2\sqrt{d\phi} \\ 0 & P < c \end{cases}$$

(d) *(3 points)* Find the short run equilibrium.

Solution 11 *There are three possible cases (all supply, only a, or neither) but the actual case is all supply, the simplest.*

$$\begin{aligned}(n+m)\frac{1}{2d}(P-c) &= a-bP \\ P &= \frac{2ad+cm+cn}{m+n+2bd} > c+2\sqrt{d\phi} \\ Q &= a-b\left(\frac{2ad+cm+cn}{m+n+2bd}\right)\end{aligned}$$

(e) *(4 points)* In the long run what price will be charged and how much will each firm produce?

Solution 12 *These are determined by*

$$\begin{aligned}MC &= AC \\ c+2dq &= \frac{cq+dq^2+F}{q} \\ q^{lr} &= \frac{1}{d}\sqrt{Fd} \\ P^{lr} &= c+2d\left(\frac{1}{d}\sqrt{Fd}\right) = c+2\sqrt{Fd}\end{aligned}$$

(f) *(3 points)* Find the equilibrium quantity in the long run and the equilibrium number of firms.

$$\begin{aligned}Q^{lr} &= a-bP^{lr} = a-b\left(c+2\sqrt{Fd}\right) \\ n^{lr} &= \frac{a-b\left(c+2\sqrt{Fd}\right)}{\frac{1}{d}\sqrt{Fd}}\end{aligned}$$

6. *(14 points total)* Define the cost function as:

$$C(w,r,Q) = \min_{L,K}\max_{\mu} wL+rK-\mu(f(L,K)-Q)$$

Note: You may assume all technical conditions hold. Namely the optimal values (L^*, K^*, μ^*) are all strictly positive, that all appropriate second order conditions hold, and that all derivatives are well defined.

(a) *(8 points)* Prove that $\frac{\partial C(w,r,Q)}{\partial w} = L^* = L(w,r,Q)$ by proving the envelope theorem for this problem.

Proof. The complete derivative is:

$$\frac{\partial C(w,r,Q)}{\partial w} = L^* + w\frac{\partial L}{\partial w} + r\frac{\partial K}{\partial w} - \frac{\partial \mu}{\partial w}(f(L,K)-Q) - \mu\frac{\partial f}{\partial L}\frac{\partial L}{\partial w} - \mu\frac{\partial f}{\partial k}\frac{\partial K}{\partial w}$$

The first order conditions when $(L,K,\mu) >> 0$ are:

$$\begin{aligned} w - \mu\frac{\partial f}{\partial L} &= 0 \\ r - \mu\frac{\partial f}{\partial k} &= 0 \\ -(f(L,K)-Q) &= 0 \end{aligned}$$

This makes it clear that $\frac{\partial \mu}{\partial w}(f(L,K)-Q) = 0$, too show the other two indrect effects are zero we rewrite the remaining phrase:

$$\frac{\partial C(w,r,Q)}{\partial w} = L^* + \left(w - \mu\frac{\partial f}{\partial L}\right)\frac{\partial L}{\partial w} + \left(r - \mu\frac{\partial f}{\partial k}\right)\frac{\partial K}{\partial w}$$

and now the first and the second condition give us that

$$\left(w - \mu\frac{\partial f}{\partial L}\right)\frac{\partial L}{\partial w} = 0 = \left(r - \mu\frac{\partial f}{\partial k}\right)\frac{\partial K}{\partial w}$$

leaving

$$\frac{\partial C(w,r,Q)}{\partial w} = L^*$$

■

(b) *(6 points)* What property of the cost function then guarantees that $\frac{\partial L(w,r,Q)}{\partial w} \leq 0$? Explain.

Solution 13 *We know that the cost function is concave with regards to prices because of the third property of cost functions. A necessary condition for a function to be concave is* $\frac{\partial^2 C}{\partial w^2} \leq 0$. *Since*

$$\frac{\partial C}{\partial w} = L^* \Rightarrow \frac{\partial^2 C}{\partial w^2} = \frac{\partial L^*}{\partial w} \leq 0 \ .$$