ECON 203
Final on Costs, Supply, and Equilibrium

Be sure to show your work for all answers, even if the work is simple.
This exam will begin at 9:30 and end at 11:10

1. *(5 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person. I will also not aid anyone else. Finally I will not use a calculator or other electronic aid for calculation.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(10 points)* Define the cost function as $C(w,r,Q) = \min_{L,K} wL + rK$ where (L,K) can produce at least Q units of output. Using only algebra and the definition of cost minimization show that if $w \geq \tilde{w}$, $r \geq \tilde{r}$ then $C(w,r,Q) \geq C(\tilde{w},\tilde{r},Q)$. Let (L^*,K^*) be the optimal labor and capital at (w,r).

   (a) *(3 points)* What relationship holds between $wL^* + rK^*$ and $\tilde{w}L^* + \tilde{r}K^*$? How do we know this?

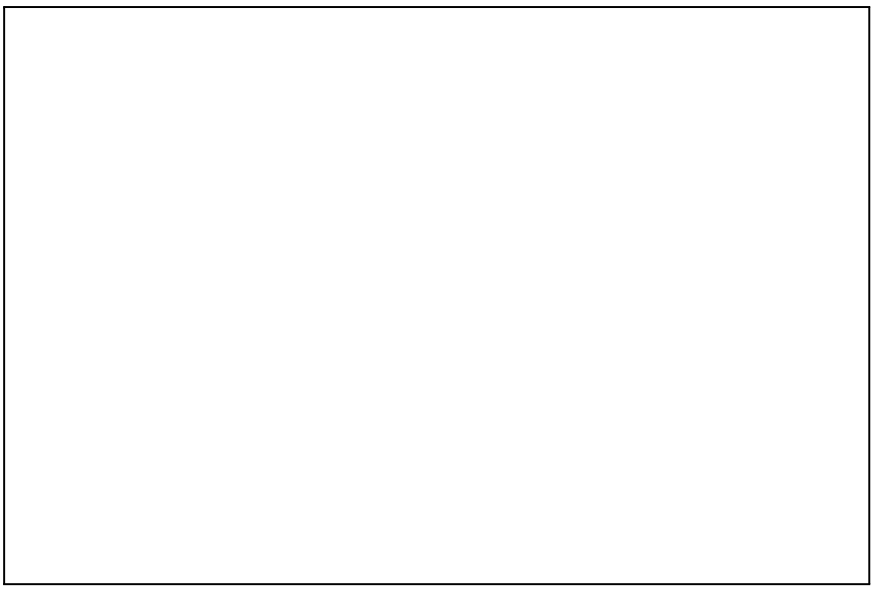
   (b) *(3 points)* What relationship holds between $\tilde{w}L^* + \tilde{r}K^*$ and $C(\tilde{w},\tilde{r},Q)$? How do we know this?

   (c) *(4 points)* Prove $C(w,r,Q) \geq C(\tilde{w},\tilde{r},Q)$.

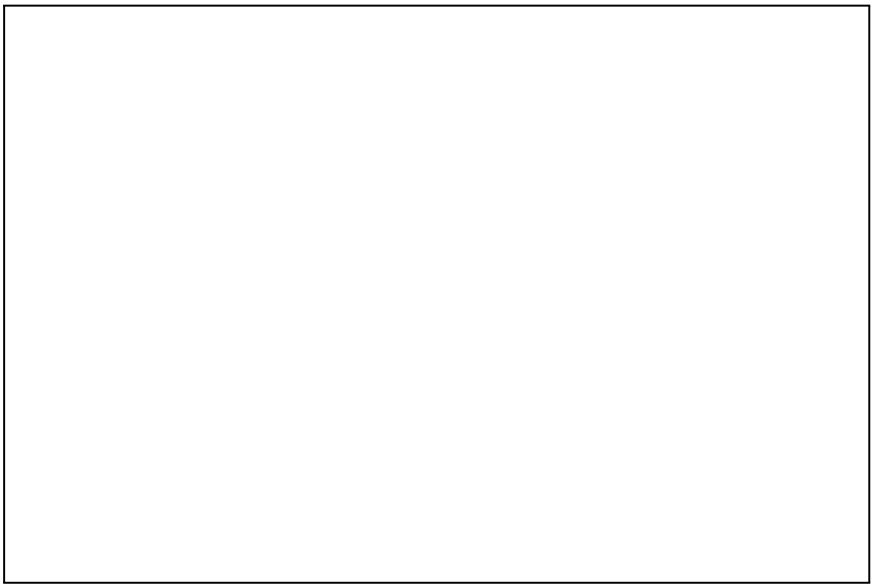
3. *(31 points total)* In a market the supply curve is given by $Q_s = -1 + \frac{1}{2}P_s$ where Q_s is the quantity supplied if the suppliers receive P_s, the demand curve is $Q_d = 15 - \frac{1}{2}P_d$ where Q_d is the quantity demanded if demanders pay P_d.

(a) *(4 points)* Find the equilibrium quantity and price in this market.

(b) *(6 points)* In the graph below graph the supply and demand curve, mark the equilibrium and find the amount of Consumer and Producer surplus in this market.



(c) *(15 points)* Now the government has imposed a per unit tax of $t = 4$ in this market. Find the new equilibrium and in the graph below draw demand and supply, and indicate Consumer Surplus, Producer Surplus, and Deadweight Loss in this new equilibrium. Also find the values of Consumer Surplus, Producer Surplus, and Deadweight Loss.



(d) *(6 points)* Why is the Deadweight loss a problem? Does this mean that the government should remove the tax in this market? Explain your answer.

4. *(12 points total)* About *Pareto efficiency.*

   (a) *(3 points)* Define *Pareto Dominance.*

(b) *(3 points)* Define *Pareto Efficiency.*

(c) *(3 points)* Assume two people (a and b) are sharing a cookie, both always want more of the cookie. An allocation is a share for a (s_a) and a share for b (s_b) it is feasible if $s_a \geq 0$, $s_b \geq 0$ and $s_a + s_b \leq 1$. What is the set of Pareto efficient allocations? Explain.

(d) *(3 points)* Describe something that is both morally disgusting to you and also Pareto efficient, explain why it is Pareto efficient.

5. *(28 points total)* Consider an exchange economy, person 1 has the utility function: $U_1(C_1, F_1) = C_1 F_1^{\frac{1}{4}}$ and the initial endowment $\left(F_1^0, C_1^0\right) = (9, 2)$, person 2 has the utility function $U_2(C_2, F_2) = C_2 F_2^{\frac{1}{2}}$ and the initial endowment $\left(F_2^0, C_2^0\right) = (9, 12)$.

   (a) *(6 points)* Find the marginal rate of substitution of both people.

   (b) *(6 points)* Find the contract curve in this economy.

(c) *(4 points)* Show that a consumer with Cobb-Douglas preferences of $U(F,C) = F^{\alpha}C^{\beta}$ ($\alpha > 0$, $\beta > 0$) maximizes utility over the budget set $p_f F + p_c C \leq I$ that the optimal consumption bundle is $F = \frac{\alpha}{\alpha+\beta}\frac{I}{p_f}$ and $C = \frac{\beta}{\alpha+\beta}\frac{I}{p_c}$. You may use this below even if you can not show it.

(d) *(2 points)* What are the incomes of person 1 and 2 in any Walrasian equilibrium in this economy. (The formula will include the price of food and clothing.)

(e) *(10 points)* Find the Walrasian or Competitive equilibrium prices and the optimal consumptions of person 1 in this economy.

6. *(8 points)* Define the cost function as:

$$C(w,r,Q) = \min_{L,K} \max_{\mu} wL + rK - \mu\left(f(L,K) - Q\right)$$

You may assume all technical conditions hold. Namely the optimal values (L^*, K^*, μ^*) are all strictly positive, that all appropriate second order conditions hold, and that all derivatives are well defined. Prove that $\frac{\partial C(w,r,Q)}{\partial w} = L^* = L(w,r,Q)$ by proving the envelope theorem for this problem.

7. *(6 points)* Assume that $\frac{\partial^2 C}{\partial q^2} > 0$, when marginal cost equals average cost what do we know about average cost? Explain your answer.

8. **Bonus Question** *(8 points total)* Consider an exchange economy where $u_1(F_1, C_1) = \min\{F_1, C_1\}$ and $u_1(F_2, C_2) = \min\{F_2, C_2\}$, the total supply of food is 7 and the total supply of clothing is 14.

Remark 1 *Since it is a bonus question grades on this question will not affect the adjustment. I will add it to your total after calculating the adjustment.*

(a) *(5 points)* Find the contract curve or the set of Pareto efficient allocations, explain your reasoning.

(b) *(3 points)* If $\left(F_1^0, C_1^0\right) = (2,7)$ what is the final allocation in the Walrasian equilibrium? What are the prices?

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2. *(10 points)* Define the cost function as $C(w,r,Q) = \min_{L,K} wL + rK$ where (L,K) can produce at least Q units of output. Using only algebra and the definition of cost minimization show that if $w \geq \tilde{w}$, $r \geq \tilde{r}$ then $C(w,r,Q) \geq C(\tilde{w},\tilde{r},Q)$. Let (L^*,K^*) be the optimal labor and capital at (w,r).

   (a) *(3 points)* What relationship holds between $wL^* + rK^*$ and $\tilde{w}L^* + \tilde{r}K^*$? How do we know this?

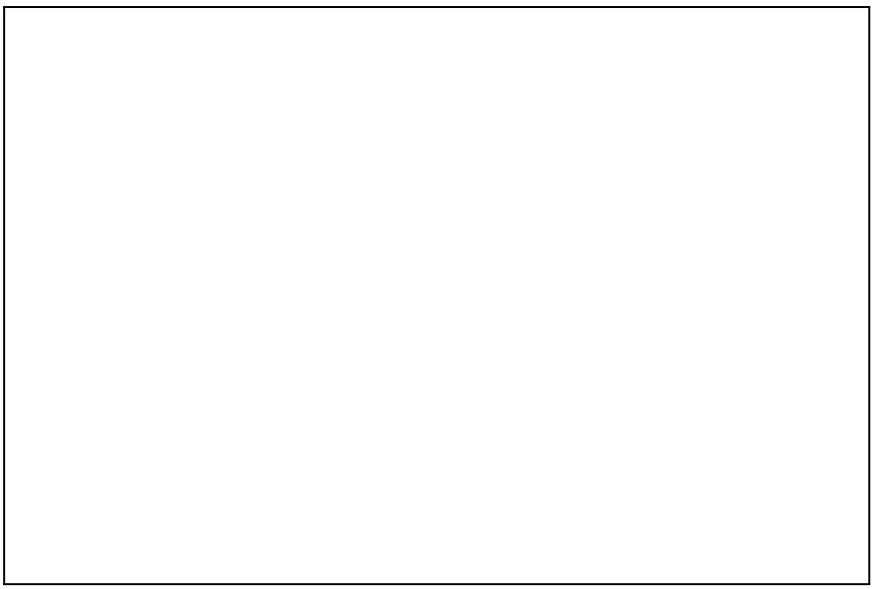
   (b) *(3 points)* What relationship holds between $\tilde{w}L^* + \tilde{r}K^*$ and $C(\tilde{w},\tilde{r},Q)$? How do we know this?

   (c) *(4 points)* Prove $C(w,r,Q) \geq C(\tilde{w},\tilde{r},Q)$.

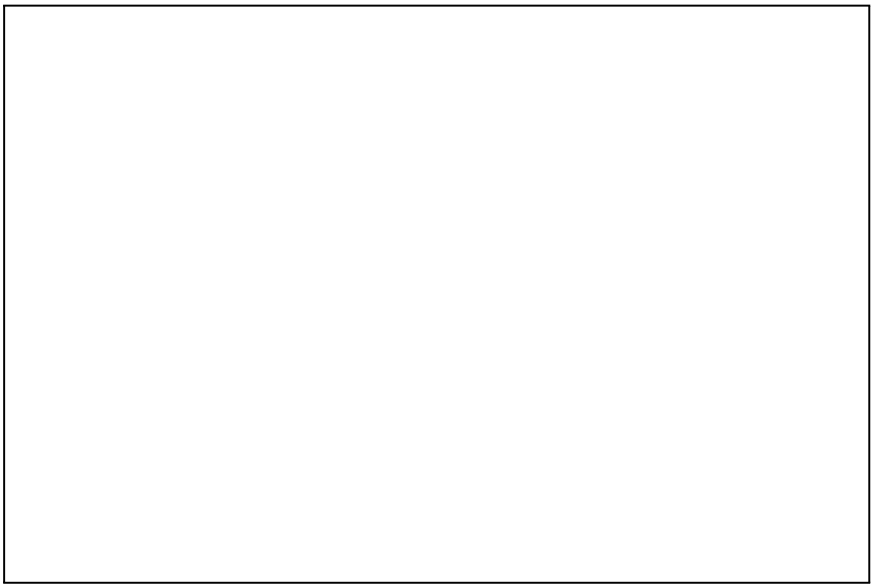
3. *(31 points total)* In a market the supply curve is given by $Q_s = -3 + P_s$ where Q_s is the quantity supplied if the suppliers receive P_s, the demand curve is $Q_d = 30 - 2P_d$ where Q_d is the quantity demanded if demanders pay P_d.

(a) *(4 points)* Find the equilibrium quantity and price in this market.

(b) *(6 points)* In the graph below graph the supply and demand curve, mark the equilibrium and find the amount of Consumer and Producer surplus in this market.



(c) *(15 points)* Now the government has imposed a per unit tax of $t = 6$ in this market. Find the new equilibrium and in the graph below draw demand and supply, and indicate Consumer Surplus, Producer Surplus, and Deadweight Loss in this new equilibrium. Also find the values of Consumer Surplus, Producer Surplus, and Deadweight Loss.



(d) *(6 points)* Why is the Deadweight loss a problem? Does this mean that the government should remove the tax in this market? Explain your answer.

4. *(12 points total)* About *Pareto efficiency.*

   (a) *(3 points)* Define *Pareto Dominance.*

(b) *(3 points)* Define *Pareto Efficiency.*

(c) *(3 points)* Assume two people (a and b) are sharing a cookie, both always want more of the cookie. An allocation is a share for a (s_a) and a share for b (s_b) it is feasible if $s_a \geq 0$, $s_b \geq 0$ and $s_a + s_b \leq 1$. What is the set of Pareto efficient allocations? Explain.

(d) *(3 points)* Describe something that is both morally disgusting to you and also Pareto efficient, explain why it is Pareto efficient.

5. *(28 points total)* Consider an exchange economy, person 1 has the utility function: $U_1(C_1, F_1) = C_1 F_1^{\frac{3}{4}}$ and the initial endowment $\left(F_1^0, C_1^0\right) = (3, 55)$, person 2 has the utility function $U_2(C_2, F_2) = C_2 F_2^{\frac{1}{4}}$ and the initial endowment $\left(F_2^0, C_2^0\right) = (5, 25)$.

   (a) *(6 points)* Find the marginal rate of substitution of both people.

   (b) *(6 points)* Find the contract curve in this economy.

(c) *(4 points)* Show that a consumer with Cobb-Douglas preferences of $U(F,C) = F^{\alpha}C^{\beta}$ ($\alpha > 0$, $\beta > 0$) maximizes utility over the budget set $p_f F + p_c C \leq I$ that the optimal consumption bundle is $F = \frac{\alpha}{\alpha+\beta}\frac{I}{p_f}$ and $C = \frac{\beta}{\alpha+\beta}\frac{I}{p_c}$. You may use this below even if you can not show it.

(d) *(2 points)* What are the incomes of person 1 and 2 in any Walrasian equilibrium in this economy. (The formula will include the price of food and clothing.)

(e) *(10 points)* Find the Walrasian or Competitive equilibrium prices and the optimal consumptions of person 1 in this economy.

6. *(8 points)* Define the cost function as:

$$C(w,r,Q) = \min_{L,K} \max_{\mu} wL + rK - \mu\left(f(L,K) - Q\right)$$

You may assume all technical conditions hold. Namely the optimal values (L^*, K^*, μ^*) are all strictly positive, that all appropriate second order conditions hold, and that all derivatives are well defined. Prove that $\frac{\partial C(w,r,Q)}{\partial w} = L^* = L(w,r,Q)$ by proving the envelope theorem for this problem.

7. *(6 points)* Assume that $\frac{\partial^2 C}{\partial q^2} > 0$, when marginal cost equals average cost what do we know about average cost? Explain your answer.

8. **Bonus Question** *(8 points total)* Consider an exchange economy where $u_1(F_1, C_1) = \min\{F_1, C_1\}$ and $u_1(F_2, C_2) = \min\{F_2, C_2\}$, the total supply of food is 9 and the total supply of clothing is 18.

   Remark 2 *Since it is a bonus question grades on this question will not affect the adjustment. I will add it to your total after calculating the adjustment.*

   (a) *(5 points)* Find the contract curve or the set of Pareto efficient allocations, explain your reasoning.

   (b) *(3 points)* If $\left(F_1^0, C_1^0\right) = (4, 9)$ what is the final allocation in the Walrasian equilibrium? What are the prices?

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2. *(10 points)* Define the cost function as $C(w,r,Q) = \min_{L,K} wL + rK$ where (L,K) can produce at least Q units of output. Using only algebra and the definition of cost minimization show that if $w \geq \tilde{w}$, $r \geq \tilde{r}$ then $C(w,r,Q) \geq C(\tilde{w},\tilde{r},Q)$. Let (L^*,K^*) be the optimal labor and capital at (w,r).

   (a) *(3 points)* What relationship holds between $wL^* + rK^*$ and $\tilde{w}L^* + \tilde{r}K^*$? How do we know this?

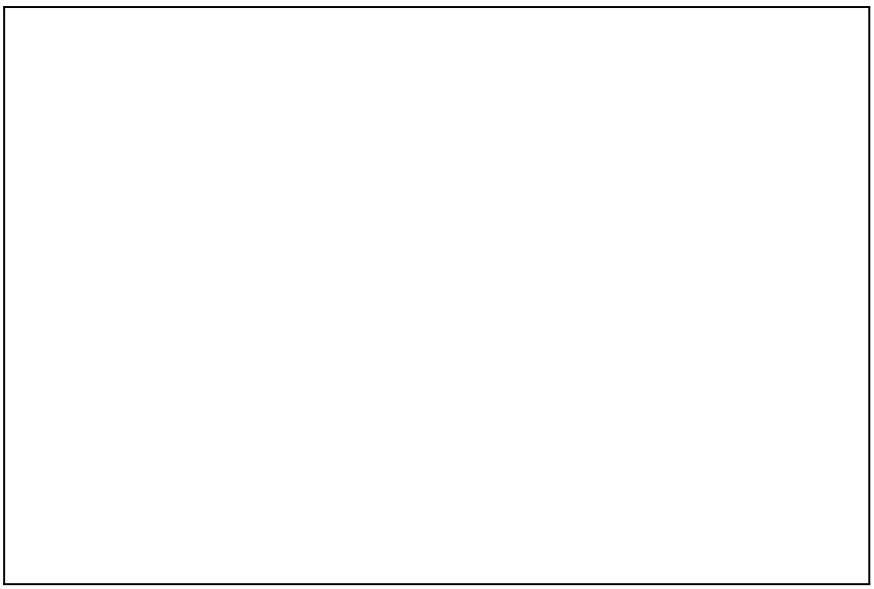
   (b) *(3 points)* What relationship holds between $\tilde{w}L^* + \tilde{r}K^*$ and $C(\tilde{w},\tilde{r},Q)$? How do we know this?

   (c) *(4 points)* Prove $C(w,r,Q) \geq C(\tilde{w},\tilde{r},Q)$.

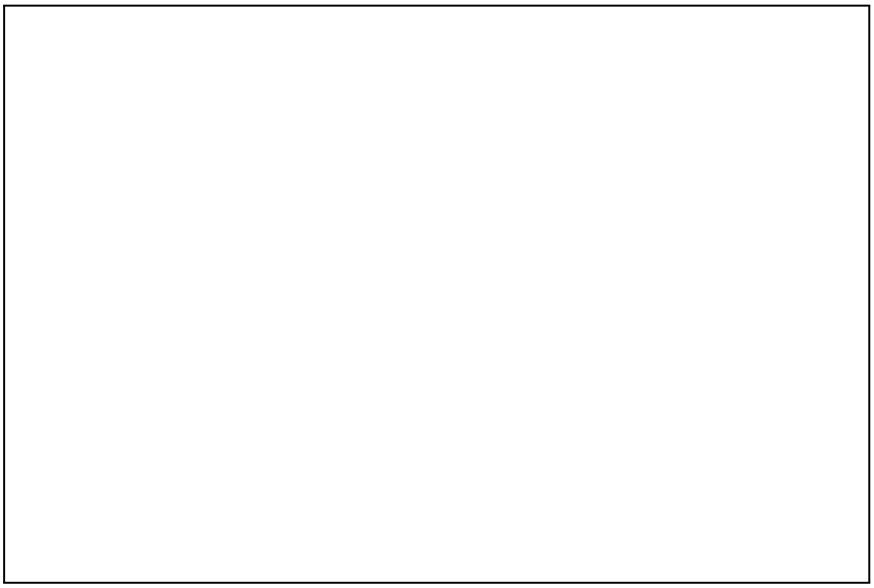
3. *(31 points total)* In a market the supply curve is given by $Q_s = -8 + P_s$ where Q_s is the quantity supplied if the suppliers receive P_s, the demand curve is $Q_d = 13 - \frac{1}{2}P_d$ where Q_d is the quantity demanded if demanders pay P_d.

(a) *(4 points)* Find the equilibrium quantity and price in this market.

(b) *(6 points)* In the graph below graph the supply and demand curve, mark the equilibrium and find the amount of Consumer and Producer surplus in this market.



(c) *(15 points)* Now the government has imposed a per unit tax of $t = 12$ in this market. Find the new equilibrium and in the graph below draw demand and supply, and indicate Consumer Surplus, Producer Surplus, and Deadweight Loss in this new equilibrium. Also find the values of Consumer Surplus, Producer Surplus, and Deadweight Loss.



(d) *(6 points)* Why is the Deadweight loss a problem? Does this mean that the government should remove the tax in this market? Explain your answer.

4. *(12 points total)* About *Pareto efficiency.*

   (a) *(3 points)* Define *Pareto Dominance.*

(b) *(3 points)* Define *Pareto Efficiency.*

(c) *(3 points)* Assume two people (a and b) are sharing a cookie, both always want more of the cookie. An allocation is a share for a (s_a) and a share for b (s_b) it is feasible if $s_a \geq 0$, $s_b \geq 0$ and $s_a + s_b \leq 1$. What is the set of Pareto efficient allocations? Explain.

(d) *(3 points)* Describe something that is both morally disgusting to you and also Pareto efficient, explain why it is Pareto efficient.

5. *(28 points total)* Consider an exchange economy, person 1 has the utility function: $U_1(C_1, F_1) = C_1 F_1^{\frac{1}{3}}$ and the initial endowment $\left(F_1^0, C_1^0\right) = (10, 2)$, person 2 has the utility function $U_2(C_2, F_2) = C_2 F_2^{\frac{2}{3}}$ and the initial endowment $\left(F_2^0, C_2^0\right) = (25, 10)$.

   (a) *(6 points)* Find the marginal rate of substitution of both people.

   (b) *(6 points)* Find the contract curve in this economy.

(c) *(4 points)* Show that a consumer with Cobb-Douglas preferences of $U(F,C) = F^{\alpha}C^{\beta}$ ($\alpha > 0$, $\beta > 0$) maximizes utility over the budget set $p_f F + p_c C \leq I$ that the optimal consumption bundle is $F = \frac{\alpha}{\alpha+\beta}\frac{I}{p_f}$ and $C = \frac{\beta}{\alpha+\beta}\frac{I}{p_c}$. You may use this below even if you can not show it.

(d) *(2 points)* What are the incomes of person 1 and 2 in any Walrasian equilibrium in this economy. (The formula will include the price of food and clothing.)

(e) *(10 points)* Find the Walrasian or Competitive equilibrium prices and the optimal consumptions of person 1 in this economy.

6. *(8 points)* Define the cost function as:

$$C(w,r,Q) = \min_{L,K} \max_{\mu} wL + rK - \mu\left(f(L,K) - Q\right)$$

You may assume all technical conditions hold. Namely the optimal values (L^*, K^*, μ^*) are all strictly positive, that all appropriate second order conditions hold, and that all derivatives are well defined. Prove that $\frac{\partial C(w,r,Q)}{\partial w} = L^* = L(w,r,Q)$ by proving the envelope theorem for this problem.

7. *(6 points)* Assume that $\frac{\partial^2 C}{\partial q^2} > 0$, when marginal cost equals average cost what do we know about average cost? Explain your answer.

8. **Bonus Question** *(8 points total)* Consider an exchange economy where $u_1(F_1, C_1) = \min\{F_1, C_1\}$ and $u_1(F_2, C_2) = \min\{F_2, C_2\}$, the total supply of food is 6 and the total supply of clothing is 12.

   Remark 3 *Since it is a bonus question grades on this question will not affect the adjustment. I will add it to your total after calculating the adjustment.*

   (a) *(5 points)* Find the contract curve or the set of Pareto efficient allocations, explain your reasoning.

   (b) *(3 points)* If $\left(F_1^0, C_1^0\right) = (1,6)$ what is the final allocation in the Walrasian equilibrium? What are the prices?

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2. *(10 points)* Define the cost function as $C(w,r,Q) = \min_{L,K} wL + rK$ where (L,K) can produce at least Q units of output. Using only algebra and the definition of cost minimization show that if $w \geq \tilde{w}$, $r \geq \tilde{r}$ then $C(w,r,Q) \geq C(\tilde{w},\tilde{r},Q)$. Let (L^*,K^*) be the optimal labor and capital at (w,r).

   (a) *(3 points)* What relationship holds between $wL^* + rK^*$ and $\tilde{w}L^* + \tilde{r}K^*$? How do we know this?

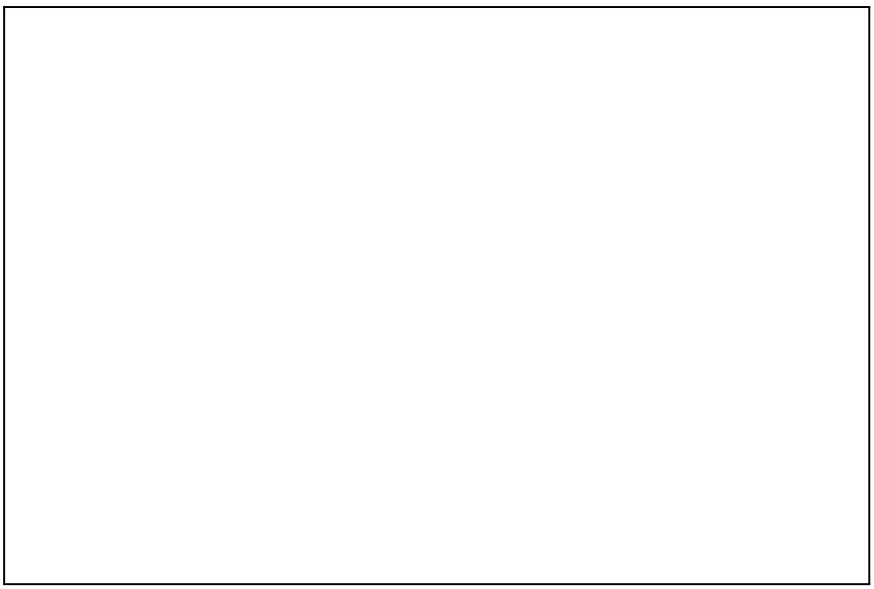
   (b) *(3 points)* What relationship holds between $\tilde{w}L^* + \tilde{r}K^*$ and $C(\tilde{w},\tilde{r},Q)$? How do we know this?

   (c) *(4 points)* Prove $C(w,r,Q) \geq C(\tilde{w},\tilde{r},Q)$.

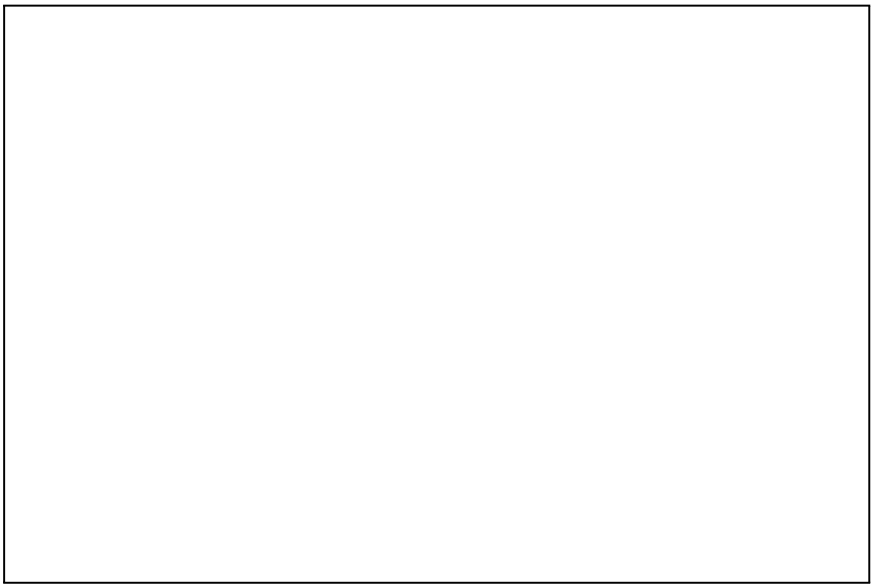
3. *(31 points total)* In a market the supply curve is given by $Q_s = -26 + 2P_s$ where Q_s is the quantity supplied if the suppliers receive P_s, the demand curve is $Q_d = 19 - P_d$ where Q_d is the quantity demanded if demanders pay P_d.

(a) *(4 points)* Find the equilibrium quantity and price in this market.

(b) *(6 points)* In the graph below graph the supply and demand curve, mark the equilibrium and find the amount of Consumer and Producer surplus in this market.



(c) *(15 points)* Now the government has imposed a per unit tax of $t = 3$ in this market. Find the new equilibrium and in the graph below draw demand and supply, and indicate Consumer Surplus, Producer Surplus, and Deadweight Loss in this new equilibrium. Also find the values of Consumer Surplus, Producer Surplus, and Deadweight Loss.



(d) *(6 points)* Why is the Deadweight loss a problem? Does this mean that the government should remove the tax in this market? Explain your answer.

4. *(12 points total)* About *Pareto efficiency.*

   (a) *(3 points)* Define *Pareto Dominance.*

(b) *(3 points)* Define *Pareto Efficiency.*

(c) *(3 points)* Assume two people (a and b) are sharing a cookie, both always want more of the cookie. An allocation is a share for a (s_a) and a share for b (s_b) it is feasible if $s_a \geq 0$, $s_b \geq 0$ and $s_a + s_b \leq 1$. What is the set of Pareto efficient allocations? Explain.

(d) *(3 points)* Describe something that is both morally disgusting to you and also Pareto efficient, explain why it is Pareto efficient.

5. *(28 points total)* Consider an exchange economy, person 1 has the utility function: $U_1(C_1, F_1) = C_1 F_1^{\frac{1}{2}}$ and the initial endowment $\left(F_1^0, C_1^0\right) = (5, 5)$, person 2 has the utility function $U_2(C_2, F_2) = C_2 F_2^{\frac{1}{4}}$ and the initial endowment $\left(F_2^0, C_2^0\right) = (25, 15)$.

   (a) *(6 points)* Find the marginal rate of substitution of both people.

   (b) *(6 points)* Find the contract curve in this economy.

(c) *(4 points)* Show that a consumer with Cobb-Douglas preferences of $U(F,C) = F^{\alpha}C^{\beta}$ ($\alpha > 0$, $\beta > 0$) maximizes utility over the budget set $p_f F + p_c C \leq I$ that the optimal consumption bundle is $F = \frac{\alpha}{\alpha+\beta}\frac{I}{p_f}$ and $C = \frac{\beta}{\alpha+\beta}\frac{I}{p_c}$. You may use this below even if you can not show it.

(d) *(2 points)* What are the incomes of person 1 and 2 in any Walrasian equilibrium in this economy. (The formula will include the price of food and clothing.)

(e) *(10 points)* Find the Walrasian or Competitive equilibrium prices and the optimal consumptions of person 1 in this economy.

6. *(8 points)* Define the cost function as:

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7. *(6 points)* Assume that $\frac{\partial^2 C}{\partial q^2} > 0$, when marginal cost equals average cost what do we know about average cost? Explain your answer.

8. **Bonus Question** *(8 points total)* Consider an exchange economy where $u_1(F_1, C_1) = \min\{F_1, C_1\}$ and $u_1(F_2, C_2) = \min\{F_2, C_2\}$, the total supply of food is 8 and the total supply of clothing is 16.

   Remark 4 *Since it is a bonus question grades on this question will not affect the adjustment. I will add it to your total after calculating the adjustment.*

   (a) *(5 points)* Find the contract curve or the set of Pareto efficient allocations, explain your reasoning.

   (b) *(3 points)* If $\left(F_1^0, C_1^0\right) = (3,8)$ what is the final allocation in the Walrasian equilibrium? What are the prices?