ECON 203
Final on Costs, Supply, and Equilibrium

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 18:20 and end at 20:00

1. *(5 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(10 points)* Define the cost function as $C(w, r, Q) = \min_{L,K} wL + rK$ where (L, K) can produce at least Q units of output. Using only algebra and the definition of cost minimization show that if $w \geq \tilde{w}$, $r \geq \tilde{r}$ then $C(w, r, Q) \geq C(\tilde{w}, \tilde{r}, Q)$. Let (L^*, K^*) be the optimal labor and capital at (w, r).

   (a) *(3 points)* What relationship holds between $wL^* + rK^*$ and $\tilde{w}L^* + \tilde{r}K^*$? How do we know this?

   Solution 1 *Since $w \geq \tilde{w}$, $r \geq \tilde{r}$, $L^* \geq 0$ and $K^* \geq 0$ we know that:*

   $$wL^* + rK^* \geq \tilde{w}L^* + \tilde{r}K^*$$

   (b) *(3 points)* What relationship holds between $\tilde{w}L^* + \tilde{r}K^*$ and $C(\tilde{w}, \tilde{r}, Q)$? How do we know this?

   Solution 2 *(L^*, K^*) is optimal at (w, r) but it is only* ***a*** *way to produce Q units at $(\tilde{w}, \tilde{r})$ thus we must have*

   $$C(\tilde{w}, \tilde{r}, Q) \leq \tilde{w}L^* + \tilde{r}K^*$$

   (c) *(4 points)* Prove $C(w, r, Q) \geq C(\tilde{w}, \tilde{r}, Q)$.

   By assumption $C(w, r, Q) = wL^* + rK^*$, by part *a* $wL^* + rK^* \geq \tilde{w}L^* + \tilde{r}K^*$ and by part *b* $\tilde{w}L^* + \tilde{r}K^* \geq C(\tilde{w}, \tilde{r}, Q)$, bringing them together we get:

   $$C(w, r, Q) = wL^* + rK^* \geq \tilde{w}L^* + \tilde{r}K^* \geq C(\tilde{w}, \tilde{r}, Q)$$

c	d	a	b	τ	Q^*	P^*	$P_d(0)$	$P_s(0)$	$CS(0)$	$PS(0)$
26	2	19	1	3	4	15	19	13	8	4
8	1	13	$\frac{1}{2}$	12	6	14	26	8	36	18
1	$\frac{1}{2}$	15	$\frac{1}{2}$	4	7	16	30	2	49	49
3	1	30	2	6	8	11	15	3	16	32

c	d	a	b	τ	$Q(\tau)$	$P_s(\tau)$	$CS(\tau)$	$PS(\tau)$	DWL
26	2	19	1	3	2	14	2	1	3
8	1	13	$\frac{1}{2}$	12	2	10	4	2	24
1	$\frac{1}{2}$	15	$\frac{1}{2}$	4	6	14	36	36	2
3	1	30	2	6	4	7	34	8	12

3. *(31 points total)* In a market the supply curve is given by $Q_s = -c + dP_s$ where Q_s is the quantity supplied if the suppliers receive P_s, the demand curve is $Q_d = a - bP_d$ where Q_d is the quantity demanded if demanders pay P_d.

   (a) *(4 points)* Find the equilibrium quantity and price in this market.

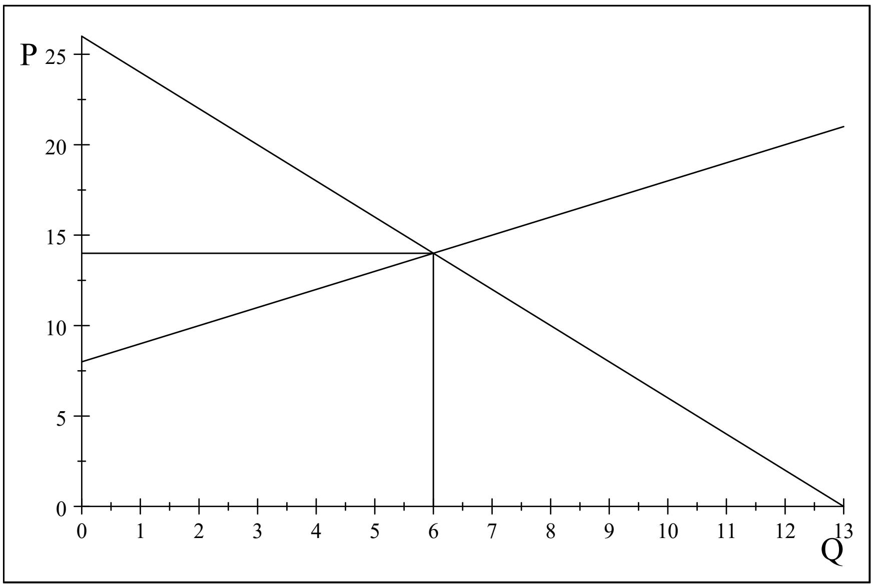
   Solution 3 *In equilibrium* $P_s = P_d = P^*$, *and* $Q_s = Q_d = Q^*$ *thus*

$$\begin{aligned} -c + dP &= a - bP \\ P &= \frac{a+c}{b+d} \end{aligned}$$

$$\begin{aligned} Q_d &= a - b\left(\frac{a+c}{b+d}\right) = \frac{ad-bc}{b+d} \\ Q_s &= -c + dP^* = -c + d\left(\frac{a+c}{b+d}\right) = \frac{ad-bc}{b+d} \end{aligned}$$

   (b) *(6 points)* In the graph below graph the supply and demand curve, mark the equilibrium and find the amount of Consumer and Producer surplus in this market.

   Solution 4 *I will use* $Q_s = -8 + P_s$ *and* $Q_d = 13 - \frac{1}{2}P_d$



Then for full credit you needed to show me how you found consumer and producer surplus,

$$\begin{aligned} CS &= \frac{1}{2}(26-14)\,6 = 36 \\ PS &= \frac{1}{2}(14-8)\,6 = 18 \end{aligned}$$

The general formulas are:

$$\begin{aligned} CS &= \frac{1}{2}(P_d(0) - P^*)\,Q^* \\ PS &= \frac{1}{2}(P^* - P_s(0))\,Q^* \end{aligned}$$

where $P_d(0)$ is the price at which the demand curve crosses the vertical axis, and the same for $P_s(0)$ with supply. but I was looking for answers based on the graph. The general solution is:

$$\begin{aligned} 0 &= a - bP_d \\ P_d(0) &= \frac{a}{b} \end{aligned}$$

$$CS = \frac{1}{2}\left(\frac{a}{b} - \frac{a+c}{b+d}\right)\frac{ad-bc}{b+d} = \frac{1}{2b}\frac{(ad-bc)^2}{(b+d)^2}$$

$$\begin{aligned} 0 &= -c + dP_s \\ P_s(0) &= \frac{c}{d} \end{aligned}$$

$$\begin{aligned} PS &= \frac{1}{2}\left(\frac{a+c}{b+d} - \frac{c}{d}\right)\frac{ad-bc}{b+d} \\ &= \frac{1}{2d}\frac{(ad-bc)^2}{(b+d)^2} \end{aligned}$$

(c) *(15 points)* Now the government has imposed a per unit tax of $t = \tau$ in this market. Find the new equilibrium and in the graph below draw demand and supply, and indicate Consumer Surplus, Producer Surplus, and Deadweight Loss in this new equilibrium. Also find the values of Consumer Surplus, Producer Surplus, and Deadweight Loss.

Solution 5 *Now $P_d = P_s + \tau$, but we still have $Q_d = Q_s$ thus we must have:*

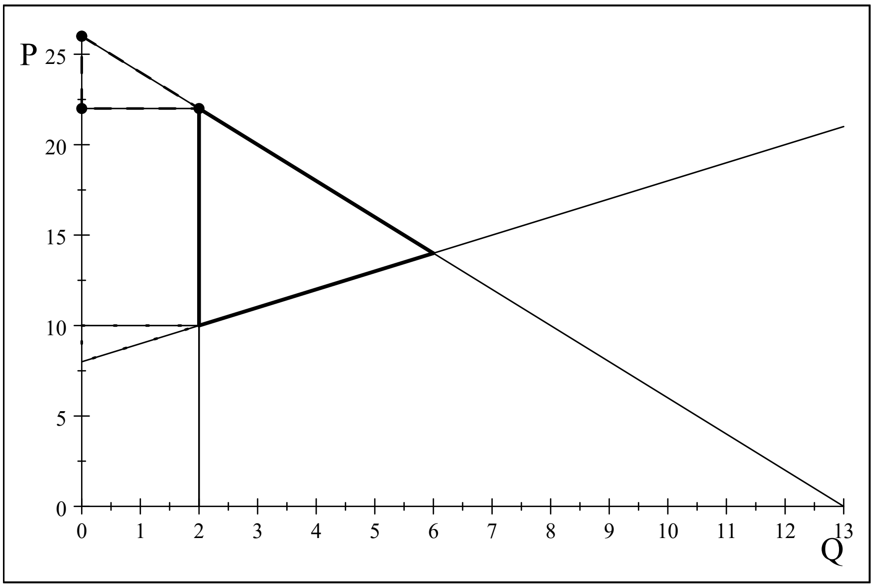
$$-c + dP_s = a - b(P_s + \tau)$$

$$
\begin{aligned}
P_s(\tau) &= \frac{1}{b+d}(a+c-b\tau) \\
Q(\tau) &= -c+dP_s(\tau) = -c+d\left(\frac{1}{b+d}(a+c-b\tau)\right) \\
&= -\frac{1}{b+d}(bc-ad+bd\tau)
\end{aligned}
$$

$$
\begin{aligned}
CS &= \frac{1}{2}(P_d(0)-P_d(\tau))\,Q(\tau) \\
PS &= \frac{1}{2}(P_s(\tau)-P_s(0))\,Q(\tau) \\
DWL &= \frac{1}{2}\tau(Q^*-Q(\tau))
\end{aligned}
$$

$$
\begin{aligned}
CS &= \frac{1}{2}\left(\frac{a}{b}-\left(\frac{1}{b+d}(a+c-b\tau)+\tau\right)\right)\left(-\frac{1}{b+d}(bc-ad+bd\tau)\right) \\
&= \frac{1}{2b(b+d)^2}(bc-ad+bd\tau)^2 \\
PS &= \frac{1}{2}\left(\frac{1}{b+d}(a+c-b\tau)-\frac{c}{d}\right)\left(-\frac{1}{b+d}(bc-ad+bd\tau)\right) \\
&= \frac{1}{2d(b+d)^2}(bc-ad+bd\tau)^2 \\
DWL &= \frac{1}{2}\tau\left(\frac{ad-bc}{b+d}-\left(-\frac{1}{b+d}(bc-ad+bd\tau)\right)\right) \\
&= \frac{1}{2}bd\frac{\tau^2}{b+d}
\end{aligned}
$$

Now for the graph I will continue using the same set of parameters, *and that means that the tax is* 12, $P_s^* = 10$, $P_d^* = 22$, $Q^* = 2$.



And in this graph you needed to indicate that the CS was the upper triangle (marked by dashes), the PS was the lower triangle (marked by dots) and the DWL was the right triangle (marked by a heavy dark line). Some of you noticed that the remaining box was government revenue, some of you tried to assign it to either CS or PS—which meant you were wrong. In this case then:

$$\begin{aligned} CS &= \frac{1}{2}(26-22)\,2 = 4 \\ PS &= \frac{1}{2}(10-8)\,2 = 2 \\ DWL &= \frac{1}{2}12\,(6-2) = 24 \end{aligned}$$

(d) *(6 points)* Why is the Deadweight loss a problem? Does this mean that the government should remove the tax in this market? Explain your answer.

Solution 6 *Most of you forgot to explain why it is a problem. There are several ways to express this, the simplest is to say that this is a social loss because for these units the marginal benefit is above the marginal cost but this potential gain is not captured. On a deeper level the first welfare theorem states that the competitive equilibrium is Pareto efficient and thus DWL indicates this market is not Pareto efficient.*

And no, it does not mean that government should remove the tax. What it does mean that their is a social cost in this market so that *social benefit should be higher than this. The government has to provide public goods and control externalities, and they need money to do this. Some of this benefit might be accrued here, because this market needs a legal system to exist—something not included in our simple model.*

4. *(12 points total)* About *Pareto efficiency.*

   (a) *(3 points)* Define *Pareto Dominance.*

   Solution 7 *An allocation A Pareto dominates an allocation B if everyone likes A better and some like it more.*

   (b) *(3 points)* Define *Pareto Efficiency.*

   Solution 8 *An allocation is Pareto efficient if there is no feasible allocation that Pareto dominates it (but you better have defined that term right to use this) or*
   if any change that makes some strictly better off makes others strictly worse off.

   (c) *(3 points)* Assume two people (a and b) are sharing a cookie, both always want more of the cookie. An allocation is a share for a (s_a) and a share for b (s_b) it is feasible if $s_a \geq 0$, $s_b \geq 0$ and $s_a + s_b \leq 1$. What is the set of Pareto efficient allocations? Explain.

   Solution 9 $s_a + s_b = 1$, *if* $s_a + s_b < 1$ *we can give each party* $\frac{1}{2}(1 - (s_a + s_b))$ *and make them strictly happier.*

   (d) *(3 points)* Describe something that is both morally disgusting to you and also Pareto efficient, explain why it is Pareto efficient.

   Solution 10 *I appreciated that some of you used different examples, and was generous with partial credit. However your example needed to be something original or your arguing good. One student offered war, but in most wars both sides say they wish their was a better way, so... not really.*
   The example I feel is strongest is a suicide bomber. She or he is giving up their life—all potential future utility—in exchange for murdering a bunch of helpless people who generally did absolutely nothing about the situation they are protesting. Gross, but obviously Pareto efficient because they bomber is stating this is their best outcome.

μ	δ	γ	χ	ρ	τ	$C_1(F_1), F_1(C_1)$	$\frac{p_c}{p_f}$	F_1^*	C_1^*
$\frac{1}{2}$	5	5	$\frac{1}{4}$	25	15	$-30\frac{C_1}{C_1-40}, 40\frac{F_1}{F_1+30}$	5	10	4
$\frac{1}{4}$	9	2	$\frac{1}{2}$	9	12	$28\frac{F_1}{F_1+18}, -18\frac{C_1}{C_1-28}$	3	3	4
$\frac{1}{3}$	10	2	$\frac{2}{3}$	25	10	$24\frac{F_1}{F_1+35}, -35\frac{C_1}{C_1-24}$	5	5	3
$\frac{3}{4}$	3	55	$\frac{1}{4}$	5	25	$-40\frac{F_1}{F_1-12}, 12\frac{C_1}{C_1+40}$	$\frac{1}{5}$	6	40

5. *(28 points total)* Consider an exchange economy, person 1 has the utility function: $U_1(C_1, F_1) = C_1 F_1^{\mu}$ and the initial endowment $\left(F_1^0, C_1^0\right) = (\delta, \gamma)$, person 2 has the utility function $U_2(C_2, F_2) = C_2 F_2^{\chi}$ and the initial endowment $\left(F_2^0, C_2^0\right) = (\rho, \tau)$.

   (a) *(6 points)* Find the marginal rate of substitution of both people.

   Solution 11

$$
\begin{aligned}
MRS_1 &= \frac{MU_f^1}{MU_c^1} = \frac{\mu \frac{U_1}{F_1}}{\frac{U_1}{C_1}} = \mu \frac{C_1}{F_1} \\
MRS_2 &= \frac{MU_f^2}{MU_c^2} \frac{\chi \frac{U_2}{F_2}}{\frac{U_2}{C_2}} = \chi \frac{C_2}{F_2}
\end{aligned}
$$

   (b) *(6 points)* Find the contract curve in this economy.

   Solution 12

$$
\begin{aligned}
\mu \frac{C_1}{F_1} &= \chi \frac{C_2}{F_2} \\
F_1 + F_2 &= \delta + \rho \\
C_1 + C_2 &= \gamma + \tau
\end{aligned}
$$

$$
\begin{aligned}
F_2 &= \delta + \rho - F_1 \\
C_2 &= \gamma + \tau - C_1
\end{aligned}
$$

$$
\mu \frac{C_1}{F_1} = \chi \frac{(\gamma + \tau - C_1)}{(\delta + \rho - F_1)}
$$

$$
\begin{aligned}
\mu C_1 (\delta + \rho - F_1) &= \chi (\gamma + \tau - C_1) F_1 \\
\mu C_1 (\delta + \rho - F_1) &= \chi F_1 + \gamma \chi F_1 - \chi C_1 F_1 \\
\mu C_1 (\delta + \rho - F_1) + \chi C_1 F_1 &= \chi F_1 + \gamma \chi F_1 \\
(\mu (\delta + \rho - F_1) + \chi F_1) C_1 &= \chi F_1 + \gamma \chi F_1 \\
C_1 &= \chi F_1 \frac{\gamma + 1}{\mu \delta + \mu \rho - \mu F_1 + \chi F_1}
\end{aligned}
$$

   (c) *(4 points)* Show that a conumer with Cobb-Douglass preferences of $U(F, C) = F^{\alpha} C^{\beta}$ ($\alpha > 0$, $\beta > 0$) maximizes utility over the budget set $p_f F + p_c C \leq I$ that the optimal consumption bundle is $F = \frac{\alpha}{\alpha+\beta} \frac{I}{p_f}$ and $C = \frac{\beta}{\alpha+\beta} \frac{I}{p_c}$. You may use this below even if you can not show it.

   Solution 13 *The primary point of this question is to make it feasible for them to answer the final part without deriving the demand curves.*

Their method is unimportant, so I am going to use the bang for the buck.

$$\frac{MU_f}{p_f} = \frac{\alpha \frac{U}{F}}{p_f} = \frac{\beta \frac{U}{C}}{p_c} = \frac{MU_c}{p_c}$$

Solving this we find: $C = \frac{F}{\alpha}\frac{\beta}{p_c}p_f$*, plugging this into the budget constraint gives us:*

$$\begin{aligned} p_f F + p_c C &= I \\ p_f F + p_c \left(\frac{F}{\alpha}\frac{\beta}{p_c} p_f \right) &= I \\ F &= \frac{I}{p_f + \frac{1}{\alpha}\beta p_f} \\ &= \left(\frac{I}{p_f + \frac{1}{\alpha}\beta p_f} \frac{p_f}{I} \right) \frac{I}{p_f} \\ &= \frac{\alpha}{\alpha + \beta} \frac{I}{p_f} \end{aligned}$$

And from this we can see that:

$$C = \frac{F}{\alpha}\frac{\beta}{p_c}p_f = \frac{\left(\frac{\alpha}{\alpha+\beta}\frac{I}{p_f}\right)}{\alpha}\frac{\beta}{p_c}p_f = \beta \frac{I}{p_c(\alpha+\beta)}$$

or that:

$$\begin{aligned} p_f F + p_c C &= I \\ p_f \left(\left(\frac{\alpha}{\alpha+\beta}\frac{I}{p_f} \right) \right) + p_c C &= I \\ C p_c + \alpha \frac{I}{\alpha+\beta} &= I \\ C &= \beta \frac{I}{\alpha p_c + \beta p_c} \\ &= \left(\beta \frac{I}{\alpha p_c + \beta p_c} \frac{p_c}{I} \right) \frac{I}{p_c} \\ &= \frac{\beta}{\alpha+\beta}\frac{I}{p_c} \end{aligned}$$

(d) *(2 points)* What are the incomes of person 1 and 2 in any Walrasian equilibrium in this economy. (The formula will include the price of food and clothing.)

Solution 14 $I_1 = p_f \delta + p_c \gamma$, $I_2 = p_f \rho + p_c \tau$

(e) *(10 points)* Find the Walrasian or Competitive equilibrium prices and the optimal consumptions of person 1 in this economy.

Solution 15 *One can solve for either the market for food or clothing, I will do both.*

$$\begin{aligned} F_1 &= \frac{\mu}{1+\mu}\frac{1}{p_f}\left(p_f\delta + p_c\gamma\right) \\ F_2 &= \frac{\chi}{1+\chi}\frac{1}{p_f}\left(p_f\rho + p_c\tau\right) \end{aligned}$$

Thus supply equals demand if:

$$\frac{\mu}{1+\mu}\frac{1}{p_f}\left(p_f\delta + p_c\gamma\right) + \frac{\chi}{1+\chi}\frac{1}{p_f}\left(p_f\rho + p_c\tau\right) = \delta + \rho$$

I will set $p_f = 1$ *for simplicity, and thus what I will be solving for is* p_c/p_f

$$\frac{\mu}{1+\mu}\frac{1}{1}\left(\delta + p_c\gamma\right) + \frac{\chi}{1+\chi}\frac{1}{1}\left(\rho + p_c\tau\right) = \delta + \rho$$

$$p_c = \frac{\delta + \rho + \delta\chi + \mu\rho}{\gamma\mu + \tau\chi + \tau\mu\chi + \gamma\mu\chi}$$

Then:

$$\begin{aligned} F_1 &= \frac{\mu}{1+\mu}\frac{1}{1}\left(\delta + \left(\frac{\delta + \rho + \delta\chi + \mu\rho}{\gamma\mu + \tau\chi + \tau\mu\chi + \gamma\mu\chi}\right)\gamma\right) = \frac{\gamma\mu\delta + \gamma\mu\rho + \tau\mu\delta\chi + \gamma\mu\delta\chi}{\gamma\mu + \tau\chi + \tau\mu\chi + \gamma\mu\chi} \\ C_1 &= \frac{1}{1+\mu}\frac{1}{\left(\frac{\delta+\rho+\delta\chi+\mu\rho}{\gamma\mu+\tau\chi+\tau\mu\chi+\gamma\mu\chi}\right)}\left(\delta + \left(\frac{\delta + \rho + \delta\chi + \mu\rho}{\gamma\mu + \tau\chi + \tau\mu\chi + \gamma\mu\chi}\right)\gamma\right) = \frac{1}{\delta + \rho + \delta\chi + \mu\rho}(\gamma\delta + \end{aligned}$$

Alternatively I could solve for the market for clothing:

$$\begin{aligned} C_1 &= \frac{1}{1+\mu}\frac{1}{p_c}\left(p_f\delta + p_c\gamma\right) \\ C_2 &= \frac{1}{1+\chi}\frac{1}{p_c}\left(p_f\rho + p_c\tau\right) \end{aligned}$$

$$\frac{1}{1+\mu}\frac{1}{p_c}\left(p_f\delta + p_c\gamma\right) + \frac{1}{1+\chi}\frac{1}{p_c}\left(p_f\rho + p_c\tau\right) = \gamma + \tau$$

At this point I would set $p_c = 1$ *and so:*

$$\frac{1}{1+\mu}\left(p_f\delta + \gamma\right) + \frac{1}{1+\chi}\left(p_f\rho + \tau\right) = \gamma + \tau$$

$$p_f = \frac{1}{\delta + \rho + \delta\chi + \mu\rho}\left(\gamma\mu + \tau\chi + \tau\mu\chi + \gamma\mu\chi\right)$$

This will, of course, be the inverse of the answer found in the answer key above. Then:

$$\begin{aligned}
F_1 &= \frac{\mu}{1+\mu}\frac{1}{\left(\frac{1}{\delta+\rho+\delta\chi+\mu\rho}\left(\gamma\mu+\tau\chi+\tau\mu\chi+\gamma\mu\chi\right)\right)}\left(\left(\frac{1}{\delta+\rho+\delta\chi+\mu\rho}\left(\gamma\mu+\tau\chi+\tau\mu\chi+\gamma\mu\right.\right.\right. \\
&= \frac{\gamma\mu\delta+\gamma\mu\rho+\tau\mu\delta\chi+\gamma\mu\delta\chi}{\gamma\mu+\tau\chi+\tau\mu\chi+\gamma\mu\chi} \\
C_1 &= \frac{1}{1+\mu}\left(\left(\left(\left(\frac{1}{\delta+\rho+\delta\chi+\mu\rho}\left(\gamma\mu+\tau\chi+\tau\mu\chi+\gamma\mu\chi\right)\right)\right)\delta+\gamma\right)\right. \\
&= \frac{1}{\delta+\rho+\delta\chi+\mu\rho}\left(\gamma\delta+\gamma\rho+\tau\delta\chi+\gamma\delta\chi\right)
\end{aligned}$$

One can easily verify that the two solutions are the same.

6. *(8 points)* Define the cost function as:

$$C\left(w,r,Q\right)=\min_{L,K}\max_{\mu}wL+rK-\mu\left(f\left(L,K\right)-Q\right)$$

You may assume all technical conditions hold. Namely the optimal values (L^*,K^*,μ^*) are all strictly positive, that all appropriate second order conditions hold, and that all derivatives are well defined. Prove that $\frac{\partial C(w,r,Q)}{\partial w}=L^*=L\left(w,r,Q\right)$ by proving the envelope theorem for this problem.

Solution 16

$$\frac{\partial C}{\partial w}=L+w\frac{\partial L}{\partial w}+r\frac{\partial K}{\partial w}-\frac{\partial \mu}{\partial w}\left(f\left(L,K\right)-Q\right)-\mu f_l\frac{\partial L}{\partial w}-\mu f_k\frac{\partial K}{\partial w}$$

However when we look at the first order conditions of this objective function we see that:

$$\begin{aligned}
w-\mu f_l &= 0 \\
r-\mu f_k &= 0 \\
f\left(L,K\right)-Q &= 0
\end{aligned}$$

thus we realize $\frac{\partial\mu}{\partial w}\left(f\left(L,K\right)-Q\right)=0$, *leaving us with*

$$\begin{aligned}
\frac{\partial C}{\partial w} &= L+w\frac{\partial L}{\partial w}+r\frac{\partial K}{\partial w}-\mu f_l\frac{\partial L}{\partial w}-\mu f_k\frac{\partial K}{\partial w} \\
&= L+\left(w-\mu f_l\right)\frac{\partial L}{\partial w}+\left(r-\mu f_k\right)\frac{\partial K}{\partial w}
\end{aligned}$$

and by rearranging terms we realize that it can be written this way, where it becomes obvious that since $w-\mu f_l=0$ $\left(w-\mu f_l\right)\frac{\partial L}{\partial w}=0$, *and since* $r-\mu f_k=0$ $\left(r-\mu f_k\right)\frac{\partial K}{\partial w}=0$.

7. *(6 points)* Assume that $\frac{\partial^2 C}{\partial q^2} > 0$, when marginal cost equals average cost what do we know about average cost? Explain your answer.

   Solution 17 *It means that average cost is at a minimum.*

   If the margin is below the average it will bring the average down, if the margin is above then it will bring it up. Since they are equal the average must be at its minimum.

   A mathematical way to show this is:

$$\begin{aligned}\frac{\partial}{q}\left(\frac{c(q)}{q}\right) &= -\frac{c(q)}{q^2} + \frac{c'(q)}{q} \\ &= \frac{1}{q}\left(-\frac{c(q)}{q} + c'(q)\right) \\ &= \frac{1}{q}(-AC + MC) \\ &= \frac{1}{q}(MC - AC)\end{aligned}$$

8. **Bonus Question** *(8 points total)*[1] Consider an exchange economy where $u_1(F_1, C_1) = \min\{F_1, C_1\}$ and $u_1(F_2, C_2) = \min\{F_2, C_2\}$, the total supply of food is Φ and the total supply of clothing is 2Φ.

   (a) *(5 points)* Find the contract curve or the set of Pareto efficient allocations, explain your reasoning.

   Solution 18 *People, this is a bonus question. I can not give points just for writing something down. Since these are Leontief (perfect compliments) utility functions person 1 is at their optimum whe $F_1 = C_1$, and person 2 if $F_2 = C_2$ however*

$$\begin{aligned}F_1 + F_2 &= \Phi \\ C_1 + C_2 &= 2\Phi\end{aligned}$$

   thus we can not achieve both right away. However notice we can also write this utility function as:

$$U_1 = \begin{cases} F_1 & F_1 \leq C_1 \\ C_1 & C_1 \geq F_1 \end{cases}$$

   and given this situation, neither will object *to getting too much clothing. Say that we want to give α units of food to person 1, how much clothing do they need?*

$$\alpha \leq C_1$$

[1] Since it is a bonus question grades on this question will not affect the adjustment. I will add it to your total after calculating the adjustment.

and how much clothing will the other person need?

$$\begin{aligned} C_2 &\geq \Phi - \alpha \\ 2\Phi - C_1 &\geq \Phi - \alpha \\ 2\Phi - (\Phi - \alpha) &\geq C_1 \\ \Phi + \alpha &\geq C_1 \end{aligned}$$

Thus the pareto set is $F_1 = \alpha \in [0, \Phi]$, $C_1 \in [\alpha, \Phi + \alpha]$.

(b) *(3 points)* If $\left(F_1^0, C_1^0\right) = (\phi, \Phi)$ what is the final allocation in the Walrasian equilibrium? What are the prices?

Solution 19 *At this allocation* $U_1 = F_1$ *and* $U_2 = F_2$*, thus they will not trade because they only want the same good. This means that* $p_c = 0$ *because if it was strictly positive they would want to buy more food—which they can not. Finally if* p_f *was zero then of course they would demand more food, so it can be anything as long as it is strictly positive: any* $p_f > 0$.