ECON 203
Final on Costs, Supply, and Equilibrium

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 18:20 and end at 20:00

1. *(5 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person. I will also neither aid others nor use any electronic devices.

   Name and Surname: _
   Student ID: _
   Signature: _

2. *(8 points total)* About Famines:

   (a) *(4 points)* What did Malthus claim about famines? What basic fact of production did he base this on? Explain his reasoning.

   Solution 1 *Malthus claimed that famines were inevitable. Due to a diminishing marginal product of labor and an exponential population growth rate it was inevetible that people starve to death.*

   As an aside let me mention that this famous result was very convenient for the wealthy at the time. This was during the start of the Industrial revolution, the poor were flooding the cities for work. This result meant that charity for them was misplaced, after all they were going to starve themselves no matter what the rich did.

   (b) *(4 points)* What did Amartya Sen prove about famines that implicitly contradicts his claim? Why does this make the Irish Potato famine especially tragic?

   Solution 2 *Famines are the governments fault.*

   He found through data analysis that the food shortage was never that bad, and rather what happened is that the food shortage impoversished farmers, who then can not afford the food they need to survive.

   This means, in short, that the famine in Ireland was completely the fault of the politicians in London. If you study the history of the time this becomes even more obvious. The potato blight that caused the collapse of the potato harvest affected all of Europe, but outside *of Ireland only 100 thousand died, in Ireland a million died.*

α	β	μ	τ	χ	ϕ	ξ	θ	$Y_1(X_1)$	$\frac{p}{q}$	X_1	Y_1	X_1^*	Y_1^*
3	1	$\frac{3}{2}$	3	6	5	30	1	$\frac{1}{6}X_1$	$\frac{1}{2}$	$\frac{3}{4p}I_1$	$\frac{1}{4q}I_1$	12	2
1	2	$\frac{3}{2}$	$\frac{1}{2}$	5	3	1	33	$6X_1$	3	$\frac{1}{3p}I_1$	$\frac{2}{3q}I_1$	2	12
1	3	$\frac{2}{3}$	2	25	1	2	26	X_1	$\frac{1}{3}$	$\frac{1}{4p}I_1$	$\frac{3}{4q}I_1$	7	7
2	3	2	1	9	2	1	28	$3X_1$	2	$\frac{2}{5p}I_1$	$\frac{3}{5q}I_1$	4	12

3. *(16 points total)* Assume that in an exchange economy, $U_1(X_1, Y_1) = X_1^\alpha Y_1^\beta$ and $U_2(X_2, Y_2) = \mu X_2 + \tau Y_2$, the initial endowmenst of person 1 are $\left(X_1^0, Y_1^0\right) = (\chi, \phi)$ and for person 2 are $\left(X_2^0, Y_2^0\right) = (\xi, \theta)$. Throughout you should only consider allocations where (X_1, Y_1, X_2, Y_2) are all strictly positive.

   (a) *(4 points)* Find the contract curve—or the set of Pareto efficient allocations—in this economy as a function of Y_1 in terms of X_1.

$$MRS^1 = \frac{MU_x^1}{MU_y^1} = \frac{\alpha \frac{U_1}{X_1}}{\beta \frac{U_1}{Y_1}} = \frac{\alpha}{\beta X_1} Y_1 = \frac{\mu}{\tau} = \frac{MU_x^2}{MU_y^2} = MRS^2$$

$$Y_1 = \frac{1}{\alpha}\frac{\beta}{\tau}\mu X_1$$

   Let p be the price of X and q be the price of Y.

   (b) *(2 points)* Find the optimal value for p/q in every Walrasian (or competitive) equilibrium.

$$\frac{MU_x^2}{p} = \frac{MU_y^2}{q}$$

$$\frac{\mu}{p} = \frac{\tau}{q}$$

$$\frac{p}{q} = \frac{\mu}{\tau}$$

   (c) *(6 points)* If person 1 has the initial income I_1 show that their optimal demands for (X_1, Y_1) are $\left(\frac{\alpha}{\alpha+\beta}\frac{I_1}{p}, \frac{\beta}{\alpha+\beta}\frac{I_1}{q}\right)$.

$$\frac{MU_x^1}{p} = \frac{MU_y^1}{q}$$

$$\frac{\alpha \frac{U_1}{X_1}}{p} = \frac{\beta \frac{U_1}{Y_1}}{q}$$

$$Y_1 = \frac{p}{q\alpha}\beta X_1$$

$$
\begin{aligned}
pX_1 + qY_1 &= I_1 \\
pX_1 + q\left(\frac{p}{q\alpha}\beta X_1\right) &= I_1 \\
X_1 &= \frac{\alpha}{\alpha+\beta}\frac{I_1}{p}
\end{aligned}
$$

and then

$$
\begin{aligned}
Y_1 &= \frac{p}{q\alpha}\beta X_1 \\
&= \frac{p}{q\alpha}\beta\left(\frac{\alpha}{\alpha+\beta}\frac{I_1}{p}\right) \\
&= \frac{I_1}{q}\frac{\beta}{\alpha+\beta}
\end{aligned}
$$

or alternatively:

$$
\begin{aligned}
p\left(\frac{\alpha}{\alpha+\beta}\frac{I_1}{p}\right) + qY_1 &= I_1 \\
qY_1 + \alpha\frac{I_1}{\alpha+\beta} &= I_1 \\
qY_1 &= I_1 - \alpha\frac{I_1}{\alpha+\beta} = \beta\frac{I_1}{\alpha+\beta}
\end{aligned}
$$

which is the same result.

(d) *(4 points)* Find the final allocation in the Walrasian equilibrium, in other words find (X_1^*, Y_1^*).

Solution 3 *Since $I_1 = pX_1^0 + qY_1^0 = p\chi + q\phi$ from the last part:*

$$
X_1^* = \frac{\alpha}{\alpha+\beta}\frac{(p\chi + q\phi)}{p}
$$

inserting $p = \frac{\mu}{\tau}q$

$$
X_1^* = \frac{\alpha}{\alpha+\beta}\frac{\left(\frac{\mu}{\tau}q\chi + q\phi\right)}{\frac{\mu}{\tau}q} = \frac{1}{\mu}\frac{\alpha\tau\phi + \alpha\mu\chi}{\alpha+\beta}
$$

and then

$$
Y_1^* = \frac{\beta}{\alpha+\beta}\frac{(p\chi + q\phi)}{q} = \frac{\beta}{\alpha+\beta}\frac{\left(\frac{\mu}{\tau}q\chi + q\phi\right)}{q} = \frac{1}{\tau}\frac{\beta\tau\phi + \beta\mu\chi}{\alpha+\beta}
$$

c_n	F	n_o	c_o	G	n_n	a	b	$s_o(P)$	$s_n(P)$	q_n^{sd}	P_n^{sd}	q_{sd}^o	P_{sd}^o	π_n	π_o	P^*
1	16	6	2	2	3	48	1	$\frac{1}{4}P$	$\frac{1}{2}P$	4	8	1	4	20	16	12
$\frac{1}{2}$	8	8	$\frac{3}{4}$	3	3	62	2	$\frac{2}{3}P$	P	4	4	2	3	10	9	6
$\frac{1}{2}$	18	3	$\frac{3}{4}$	3	2	72	2	$\frac{2}{3}P$	P	6	6	2	3	54	45	12
$\frac{1}{2}$	8	4	1	1	2	36	$\frac{1}{2}$	$\frac{1}{2}P$	P	4	4	1	2	24	15	8

4. *(31 points total)* In the market for widgets their is an exciting new technology with the total costs of $c_n(q) = c_n q^2 + F$. Their are currently n_o firms in the industry using the old technology with total costs $c_o(q) = c_o q^2 + G$, and n_n firms decide to enter with the new technology. The new firms (type o) have no fixed sunk costs, and the old firms (type n) have no fixed start-up costs.

   (a) *(6 points)* Why is it reasonable to have the old firms have no fixed start-up costs? Why is it necessary to assume the new firms have no fixed sunk costs?

   Solution 4 *Since the firms are deciding to enter they have no factories or etcetera. Thus if they choose to produce zero output they are choosing not to enter and not to build the factories. This makes the fixed cost one they only have to pay on startup.*

   It is reasonable to assume that the old firms (which have entered) are in the reverse situation. They do not have to reinvest in their capital, so all of their fixed costs are sunk.

   (b) *(5 points)* Find the short run supply curve of both types of firms.

   Solution 5 *The old firms (type o) will produce when:*

$$
\begin{aligned}
P &= mc_o = 2c_o q \\
q &= \frac{P}{2c_o}
\end{aligned}
$$

   as long as $P \geq AVC = \frac{c_o q^2}{q} = qc_o$ *which will always be true. Type n firms will produce when*

$$
\begin{aligned}
P &= mc_n = 2c_n q \\
q &= \frac{P}{2c_n}
\end{aligned}
$$

   as long as $P \geq AVC = \frac{c_n q^2 + F}{q}$, *to find when this will be true define* q_{sd} *as the quantity where* $MC = AVC$, *this is:*

$$
\begin{aligned}
2c_n q &= \frac{c_n q^2 + F}{q} \\
q_{sd} &= \frac{1}{c_n}\sqrt{Fc_n}
\end{aligned}
$$

   and thus we must have $P \geq 2c_n q_{sd} = 2c_n \frac{1}{c_n}\sqrt{Fc_n} = 2\sqrt{Fc_n}$.

   Throughout the rest of the question let the demand curve be $Q = a - bP$.

   (c) *(4 points)* Find the short run equilibrium, you may assume all firms produce output.

Solution 6 *Since all firms produce output*

$$S(p) = n_o\left(\frac{P}{2c_o}\right) + n_n\left(\frac{P}{2c_n}\right)$$

$$\begin{aligned}
S(p) &= D(P) \\
n_o\left(\frac{P}{2c_o}\right) + n_n\left(\frac{P}{2c_n}\right) &= a - bP \\
P &= 2ac_o\frac{c_n}{c_on_n + c_nn_o + 2bc_oc_n} \\
Q &= a - b\left(2ac_o\frac{c_n}{c_on_n + c_nn_o + 2bc_oc_n}\right) \\
&= a\frac{c_on_n + c_nn_o}{c_on_n + c_nn_o + 2bc_oc_n} \\
Q &= n_o\left(\frac{P}{2c_o}\right) + n_n\left(\frac{P}{2c_n}\right) \\
&= \frac{1}{2}\frac{P}{c_oc_n}\left(c_on_n + c_nn_o\right) \\
&= \frac{1}{2}\frac{\left(2ac_o\frac{c_n}{c_on_n+c_nn_o+2bc_oc_n}\right)}{c_oc_n}\left(c_on_n + c_nn_o\right) \\
&= a\frac{c_on_n + c_nn_o}{c_on_n + c_nn_o + 2bc_oc_n}
\end{aligned}$$

which is the same answer.

(d) *(2 points)* In the equilibrium you just found, show that the new firms have a higher total profit (when you subtract all fixed costs) than the old firms.

$$\begin{aligned}
\pi_o &= Pq_o - \left(c_oq_o^2 + G\right) \\
&= P\left(\frac{P}{2c_o}\right) - \left(c_o\left(\frac{P}{2c_o}\right)^2\right) - G \\
&= \frac{1}{4c_o}\left(P^2 - 4Gc_o\right)
\end{aligned}$$

$$\begin{aligned}
\pi_n &= Pq_n - \left(c_nq^2 + F\right) \\
&= P\left(\frac{P}{2c_n}\right) - \left(c_n\left(\frac{P}{2c_n}\right)^2\right) - F \\
&= \frac{1}{4c_n}\left(P^2 - 4Fc_n\right)
\end{aligned}$$

and by making G high enough relative to F, we can achieve the desired result. It took a lot of work to make the examples above just right.

(e) *(4 points)* In the long run, find the minimal price old firms (type o) and new firms (type n) will have to charge to not exit the industry.

Solution 7 *Above we found out $P \geq 2\sqrt{Fc_n}$ was necessary for the new firms not to exit. For the old firms let we find the q where $MC = AC$ or:*

$$\begin{aligned} 2c_o q &= \frac{c_o q^2 + G}{q} \\ q &= \frac{1}{c_o}\sqrt{Gc_o} \\ P &\geq mc_o\left(\frac{1}{c_o}\sqrt{Gc_o}\right) = 2c_o\left(\frac{1}{c_o}\sqrt{Gc_o}\right) = 2\sqrt{Gc_o} \end{aligned}$$

(f) *(6 points)* Find the long run equilibrium. How many new firms (type n) will there be? How many old firms (type o) will there be?

Solution 8 *We notice that $P^n_{sd} > P^o_{sd}$ so their will be no new firms, and*

$$P = P^o_{sd} = 2\sqrt{Gc_o}$$

Thus

$$Q = a - b\left(2\sqrt{Gc_o}\right)$$

and each firm will produce $\frac{1}{c_o}\sqrt{Gc_o}$ thus the number of old technology firms is

$$n = \frac{a - b\left(2\sqrt{Gc_o}\right)}{\frac{1}{c_o}\sqrt{Gc_o}}$$

Remark 9 *I hate to say this, but I am afraid I am going to get a lot of "alternate reality" answers to this question. I.e. if everyone uses the new technology the price is x and if everyone uses the old then the price is y. These will not get any credit.*
To be clear, in the long run their cost function is actually:

$$c(q) = \min\left(c_n q^2 + F, c_o q^2 + G\right)$$

and they will use the old technology if:

$$\begin{aligned} c_o q^2 + G &\leq c_n q^2 + F \\ (c_o - c_n) q^2 &\leq F - G \\ q^2 &\leq \frac{F - G}{c_o - c_n} \\ q &\leq \sqrt{\frac{F - G}{c_o - c_n}} \end{aligned}$$

which is true for this exam.

(g) *(4 points)* Explain the apparent contradicition between your answer in part d and part f of this question.

Solution 10 *While it is surprising (I had such **FUN** figuring out this problem[1]) there is no contradiction. The new technology is more profitable at high prices, but price must keep high enough or they will need to shut down. The old technology might be less profitable at high prices, but the capital cost is lower so they can produce at lower prices. In the long run the second effect is the one that matters.*

Indeed I expect that what will happen is that all entering firms will use the exciting new technology until the price gets too low, then entering firms will switch to the old technology and new technology firms will either exit or switch to the old technology.

5. *(14 points total)* In a Robinson Crusoe Economy their is one person with the utility function $U(F,C)$ choosing their optimal F and C from a production possibilities set $g(F)+h(C)\leq T$. You may assume the indifference curves are strictly convex and that the production possibilities set is strictly convex.

(a) *(6 points)* What does the *decentralization theorem* tell us about this problem? You may let p_f be the price of F and p_c be the price of C.

Solution 11 *The decentralization theorem says we can set up two problems:*

i. Revenue maximization over the set: $g(F)+h(C)\leq T$

ii. Utility maximization given ther revenue from the last part of the problem as the income.

For the appropriate (p_f,p_c) *this will result in the Pareto efficient allocation being chosen.*

(b) *(4 points)* Other than a way to keep Robinson Crusoe from being bored, why does the decentralization theorem matter? I.e. why is useful for him to know his implicit price of food and clothing?

Solution 12 *Because if he gets an opportunity to trade with others he will then be able to see how it would be advantageous to him. All he has to do is look at his price vector and compare it to the price vector of those who want to trade. If they are different he can benefit from trade.*

(c) *(4 points)* Why does Istanbul trade with Ankara? After all it is the industrial heart of Turkey and also (let us say) could produce more food. You should use the appropriate technical terms in your answer.

[1] That is sarcasm, it literally took me hours to finally get four versions with nice results.

Solution 13 *In this question I made it clear that I am giving Istanbul an* absolute advantage *in trade. I.e. it can produce more of everything than Ankara. But this is absolutely irrelevant.*

Trade, between cities or nations, is based on comparative advantage, *and Istanbul can not—by definition—have a comparative advantage in everything.*

6. *(26 points total)* Define the short run cost function as $C^{SR}(w, v, r, K, q) = \min_{L,M}(wL + vM + rK)$ such that $f(L, M, K) \geq q$. Notice we call this the short run cost function because the amount of capital is fixed. You may assume that all inputs are always used in a strictly positive amount, also that $q > 0$.

   (a) *(8 points)* Prove the Envelope Theorem by showing that $\frac{\partial C^{SR}}{\partial w} = L$.

   $$C^{SR}(w, v, r, K, q) = \min_{L,M} \max_{\mu} (wL + vM + rK) - \mu (f(L, M, K) - q)$$

   and the first order conditions of this problem are:

   $$\begin{aligned} w - \mu MP_l &= 0 \\ v - \mu MP_M &= 0 \\ -(f(L, M, K) - q) &= 0 \end{aligned}$$

   $$\frac{\partial C^{SR}}{\partial w} = L + \frac{\partial L}{\partial w}(w - \mu MP_l) + \frac{\partial M}{\partial w}(v - \mu MP_M) - \frac{\partial \mu}{\partial w}(f(L, M, K) - q)$$

   and since $(w - \mu MP_l) = (v - \mu MP_M) = (f(L, M, K) - q) = 0$ we are left with the result.

   (b) *(3 points)* According to the Envelope Theorem, what is $\frac{\partial C^{SR}}{\partial K}$?

   Solution 14 *It will be the direct affect alone, so* $\frac{\partial C^{SR}}{\partial K} = r - \mu MP_k$.

α	β	$C^{SR}(w, r, K, q)$	$\frac{\partial C^{SR}}{\partial q}$	K^*	C^{LR}
$\frac{1}{6}$	$\frac{4}{6}$	$Kr + \frac{1}{K^4} q^6 w$	$\frac{6}{K^4} q^5 w$	$2^{\frac{2}{5}} \sqrt[5]{\frac{q^6}{r} w}$	$\frac{5}{4} 2^{\frac{2}{5}} r \sqrt[5]{\frac{q^6}{r} w}$
$\frac{4}{6}$	$\frac{1}{6}$	$Kr + \frac{1}{\sqrt[4]{K}} q^{\frac{3}{2}} w$	$\frac{3}{2\sqrt[4]{K}} \sqrt{q} w$	$\frac{1}{4} 2^{\frac{2}{5}} \left(\frac{q^{\frac{3}{2}}}{r} w\right)^{\frac{4}{5}}$	$\frac{5}{4} 2^{\frac{2}{5}} q^{\frac{3}{2}} \frac{w}{\sqrt[5]{\frac{q^{\frac{3}{2}}}{r} w}}$
$\frac{1}{4}$	$\frac{1}{2}$	$Kr + \frac{1}{K^2} q^4 w$	$\frac{4}{K^2} q^3 w$	$\sqrt[3]{2 \frac{q^4}{r} w}$	$\frac{3}{2} \sqrt[3]{2} r \sqrt[3]{\frac{q^4}{r} w}$
$\frac{1}{2}$	$\frac{1}{4}$	$Kr + \frac{1}{\sqrt{K}} q^2 w$	$\frac{2}{\sqrt{K}} q w$	$\frac{1}{2} \sqrt[3]{2} \left(\frac{q^2}{r} w\right)^{\frac{2}{3}}$	$\frac{3}{2} \sqrt[3]{2} r \left(\frac{q^2}{r} w\right)^{\frac{2}{3}}$

   Assume that $C^{SR}(w, r, K, q) = w \frac{q^{\frac{1}{\alpha}}}{K^{\frac{\beta}{\alpha}}} + rK$.

   (c) *(6 points)* Prove that this is a cost function, you do not need to prove it is concave in input prices.

Solution 15 *Their are three properties you might want to show, but I will give you three points for each one—so you really only need to show two.*

i. *Non-decreasing in input prices.* $\frac{\partial C^{SR}}{\partial w} = \frac{q^{\frac{1}{\alpha}}}{K^{\frac{\beta}{\alpha}}} > 0$, $\frac{\partial C^{SR}}{\partial r} = K > 0$

ii. *Homogeneous of degree one in input prices.* $C^{SR}(tw, tr, K, q) = tw\frac{q^{\frac{1}{\alpha}}}{K^{\frac{\beta}{\alpha}}} + trK = t\left(w\frac{q^{\frac{1}{\alpha}}}{K^{\frac{\beta}{\alpha}}} + rK\right) = tC^{SR}(w, r, K, q)$

iii. *Non-decreasing in output:* $\frac{\partial C^{SR}}{\partial q} = \left(\frac{1}{\alpha}\right) w\frac{q^{\frac{1}{\alpha}-1}}{K^{\frac{\beta}{\alpha}}} > 0$

(d) *(3 points)* Find the long run cost function. You do not need to simplify it. **WARNING:** Your answer will include irrational numbers.

$$\begin{aligned}
\frac{\partial C^{SR}(w, r, K, q)}{\partial K} &= \left(-\frac{\beta}{\alpha}\right) w\frac{q^{\frac{1}{\alpha}}}{K^{\frac{\beta}{\alpha}+1}} + r = 0 \\
r &= \frac{\beta}{\alpha} w\frac{q^{\frac{1}{\alpha}}}{K^{\frac{\beta}{\alpha}+1}} \\
K^{\frac{\beta}{\alpha}+1} &= \frac{\beta}{\alpha}\frac{w}{r}q^{\frac{1}{\alpha}} \\
K &= \left(\frac{\beta}{\alpha}\frac{w}{r}q^{\frac{1}{\alpha}}\right)^{\frac{1}{\frac{\beta}{\alpha}+1}} = \left(\frac{q^{\frac{1}{\alpha}}}{r}\frac{w}{\alpha}\beta\right)^{\frac{\alpha}{\alpha+\beta}}
\end{aligned}$$

$$C^{LR} = C^{SR}(K^*) = w\frac{q^{\frac{1}{\alpha}}}{\left(\left(\frac{q^{\frac{1}{\alpha}}}{r}\frac{w}{\alpha}\beta\right)^{\frac{\alpha}{\alpha+\beta}}\right)^{\frac{\beta}{\alpha}}} + r\left(\left(\frac{q^{\frac{1}{\alpha}}}{r}\frac{w}{\alpha}\beta\right)^{\frac{\alpha}{\alpha+\beta}}\right)$$

and it does simplify elegantly, but I would not make you suffer through that.

(e) *(6 points)* In the real world, why is it important we can find the long run cost function from short run cost functions? You do not need to explain why we want to know the long run cost function.

Solution 16 *In reality* ***many*** *inputs are fixed in the short run. For example universities can not actually adjust the number of professors that quickly (unless they decide to shut down.)*

Furthermore every firm always operates in the short run. They probably do long run planning, but they will get their through a series of short run plans.

Thus if we did not know how to find the Long run cost function from the short run cost function it would simply be unobservable—but also critical for the long run planning of the firm and the industry.