ECON 203
Final on Costs, Supply, and Equilibrium

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 18:20 and end at 20:00

1. *(5 points)* **Honor Statement:** Please read and sign the following statement:

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2. *(8 points total)* About Famines:

   (a) *(4 points)* What did Malthus claim about famines? What basic fact of production did he base this on? Explain his reasoning.

   (b) *(4 points)* What did Amartya Sen prove about famines that implicitly contradicts his claim? Why does this make the Irish Potato famine especially tragic?

3. *(16 points total)* Assume that in an exchange economy, $U_1(X_1, Y_1) = X_1 Y_1^2$ and $U_2(X_2, Y_2) = \frac{3}{2}X_2 + \frac{1}{2}Y_2$, the initial endowments of person 1 are $\left(X_1^0, Y_1^0\right) = (5, 3)$ and for person 2 are $\left(X_2^0, Y_2^0\right) = (1, 33)$. Throughout you should only consider allocations where (X_1, Y_1, X_2, Y_2) are all strictly positive.

   (a) *(4 points)* Find the contract curve—or the set of Pareto efficient allocations—in this economy as a function of Y_1 in terms of X_1.

   Let p be the price of X and q be the price of Y.

   (b) *(2 points)* Find the optimal value for p/q in every Walrasian (or competitive) equilibrium.

   (c) *(6 points)* If person 1 has the initial income I_1 show that their optimal demands for (X_1, Y_1) are $\left(\frac{1}{3}\frac{I_1}{p}, \frac{2}{3}\frac{I_1}{q}\right)$.

   (d) *(4 points)* Find the final allocation in the Walrasian equilibrium, in other words find (X_1^*, Y_1^*).

4. *(31 points total)* In the market for widgets there is an exciting new technology with the total costs of $c_n(q) = \frac{1}{2}q^2 + 8$. Their are currently 4 firms in the industry using the old technology with total costs $c_o(q) = q^2 + 1$, and 2 firms decide to enter with the new technology. The new firms (type n) have no fixed sunk costs, and the old firms (type o) have no fixed start-up costs.

   (a) *(6 points)* Why is it reasonable to have the old firms have no fixed start-up costs? Why is it necessary to assume the new firms have no fixed sunk costs?

   (b) *(5 points)* Find the short run supply curve of both types of firms.

   Throughout the rest of the question let the demand curve be $Q = 36 - \frac{1}{2}P$.

   (c) *(4 points)* Find the short run equilibrium, you may assume all firms produce output.

   (d) *(2 points)* In the equilibrium you just found, show that the new firms have a higher total profit (when you subtract all fixed costs) than the old firms.

(e) *(4 points)* In the long run, find the minimal price old firms (type o) and new firms (type n) will have to charge to not exit the industry.

(f) *(6 points)* Find the long run equilibrium. How many new firms (type n) will there be? How many old firms (type o) will there be?

(g) *(4 points)* Explain the apparent contradiction between your answer in part d and part f of this question.

5. *(14 points total)* In a Robinson Crusoe Economy there is one person with the utility function $U(F, C)$ choosing their optimal F and C from a production possibilities set $g(F) + h(C) \leq T$. You may assume the indifference curves are strictly convex and that the production possibilities set is strictly convex.

   (a) *(6 points)* What does the *decentralization theorem* tell us about this problem? You may let p_f be the price of F and p_c be the price of C.

(b) *(4 points)* Other than a way to keep Robinson Crusoe from being bored, why does the decentralization theorem matter? I.e. why is useful for him to know his implicit price of food and clothing?

(c) *(4 points)* Why does Istanbul trade with Ankara? After all it is the industrial heart of Turkey and also (let us say) could produce more food. You should use the appropriate technical terms in your answer.

6. *(26 points total)* Define the short run cost function as $C^{SR}(w, v, r, K, q) = \min_{L,M}(wL + vM + rK)$ such that $f(L, M, K) \geq q$. Notice we call this the short run cost function because the amount of capital is fixed. You may assume that all inputs are always used in a strictly positive amount, also that $q > 0$, $K > 0$.

   (a) *(8 points)* Prove the Envelope Theorem by showing that $\frac{\partial C^{SR}}{\partial w} = L$.

(b) *(3 points)* According to the Envelope Theorem, what is $\frac{\partial C^{SR}}{\partial K}$?

Assume that $C^{SR}(w, r, K, q) = Kr + \frac{1}{K^4}q^6 w$.

(c) *(6 points)* Prove that this is a cost function, you do not need to prove it is concave in input prices.

(d) *(3 points)* Find the long run cost function. You do not need to simplify it. **WARNING:** Your answer will include irrational numbers.

(e) *(6 points)* In the real world, why is it important we can find the long run cost function from short run cost functions? You do not need to explain why we want to know the long run cost function.

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2. *(8 points total)* About Famines:

   (a) *(4 points)* What did Malthus claim about famines? What basic fact of production did he base this on? Explain his reasoning.

   (b) *(4 points)* What did Amartya Sen prove about famines that implicitly contradicts his claim? Why does this make the Irish Potato famine especially tragic?

3. *(16 points total)* Assume that in an exchange economy, $U_1(X_1, Y_1) = X_1 Y_1^3$ and $U_2(X_2, Y_2) = \frac{2}{3}X_2 + 2Y_2$, the initial endowments of person 1 are $\left(X_1^0, Y_1^0\right) = (25, 1)$ and for person 2 are $\left(X_2^0, Y_2^0\right) = (2, 26)$. Throughout you should only consider allocations where (X_1, Y_1, X_2, Y_2) are all strictly positive.

   (a) *(4 points)* Find the contract curve—or the set of Pareto efficient allocations—in this economy as a function of Y_1 in terms of X_1.

   Let p be the price of X and q be the price of Y.

   (b) *(2 points)* Find the optimal value for p/q in every Walrasian (or competitive) equilibrium.

   (c) *(6 points)* If person 1 has the initial income I_1 show that their optimal demands for (X_1, Y_1) are $\left(\frac{1}{4}\frac{I_1}{p}, \frac{3}{4}\frac{I_1}{q}\right)$.

   (d) *(4 points)* Find the final allocation in the Walrasian equilibrium, in other words find (X_1^*, Y_1^*).

4. *(31 points total)* In the market for widgets there is an exciting new technology with the total costs of $c_n(q) = q^2 + 16$. Their are currently 6 firms in the industry using the old technology with total costs $c_o(q) = 2q^2 + 2$, and 3 firms decide to enter with the new technology. The new firms (type n) have no fixed sunk costs, and the old firms (type o) have no fixed start-up costs.

   (a) *(6 points)* Why is it reasonable to have the old firms have no fixed start-up costs? Why is it necessary to assume the new firms have no fixed sunk costs?

   (b) *(5 points)* Find the short run supply curve of both types of firms.

   Throughout the rest of the question let the demand curve be $Q = 48 - P$.

   (c) *(4 points)* Find the short run equilibrium, you may assume all firms produce output.

   (d) *(2 points)* In the equilibrium you just found, show that the new firms have a higher total profit (when you subtract all fixed costs) than the old firms.

(e) *(4 points)* In the long run, find the minimal price old firms (type o) and new firms (type n) will have to charge to not exit the industry.

(f) *(6 points)* Find the long run equilibrium. How many new firms (type n) will there be? How many old firms (type o) will there be?

(g) *(4 points)* Explain the apparent contradiction between your answer in part d and part f of this question.

5. *(14 points total)* In a Robinson Crusoe Economy there is one person with the utility function $U(F, C)$ choosing their optimal F and C from a production possibilities set $g(F) + h(C) \leq T$. You may assume the indifference curves are strictly convex and that the production possibilities set is strictly convex.

   (a) *(6 points)* What does the *decentralization theorem* tell us about this problem? You may let p_f be the price of F and p_c be the price of C.

(b) *(4 points)* Other than a way to keep Robinson Crusoe from being bored, why does the decentralization theorem matter? I.e. why is useful for him to know his implicit price of food and clothing?

(c) *(4 points)* Why does Istanbul trade with Ankara? After all it is the industrial heart of Turkey and also (let us say) could produce more food. You should use the appropriate technical terms in your answer.

6. *(26 points total)* Define the short run cost function as $C^{SR}(w, v, r, K, q) = \min_{L,M}(wL + vM + rK)$ such that $f(L, M, K) \geq q$. Notice we call this the short run cost function because the amount of capital is fixed. You may assume that all inputs are always used in a strictly positive amount, also that $q > 0$, $K > 0$.

   (a) *(8 points)* Prove the Envelope Theorem by showing that $\frac{\partial C^{SR}}{\partial w} = L$.

(b) *(3 points)* According to the Envelope Theorem, what is $\frac{\partial C^{SR}}{\partial K}$?

Assume that $C^{SR}(w, r, K, q) = Kr + \frac{1}{\sqrt{K}} q^2 w$.

(c) *(6 points)* Prove that this is a cost function, you do not need to prove it is concave in input prices.

(d) *(3 points)* Find the long run cost function. You do not need to simplify it. **WARNING:** Your answer will include irrational numbers.

(e) *(6 points)* In the real world, why is it important we can find the long run cost function from short run cost functions? You do not need to explain why we want to know the long run cost function.

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3. *(16 points total)* Assume that in an exchange economy, $U_1(X_1, Y_1) = X_1^3 Y_1$ and $U_2(X_2, Y_2) = \frac{3}{2}X_2 + 3Y_2$, the initial endowments of person 1 are $\left(X_1^0, Y_1^0\right) = (6, 5)$ and for person 2 are $\left(X_2^0, Y_2^0\right) = (30, 1)$. Throughout you should only consider allocations where (X_1, Y_1, X_2, Y_2) are all strictly positive.

   (a) *(4 points)* Find the contract curve—or the set of Pareto efficient allocations—in this economy as a function of Y_1 in terms of X_1.

   Let p be the price of X and q be the price of Y.

   (b) *(2 points)* Find the optimal value for p/q in every Walrasian (or competitive) equilibrium.

   (c) *(6 points)* If person 1 has the initial income I_1 show that their optimal demands for (X_1, Y_1) are $\left(\frac{3}{4}\frac{I_1}{p}, \frac{1}{4}\frac{I_1}{q}\right)$.

   (d) *(4 points)* Find the final allocation in the Walrasian equilibrium, in other words find (X_1^*, Y_1^*).

4. *(31 points total)* In the market for widgets there is an exciting new technology with the total costs of $c_n(q) = \frac{1}{2}q^2 + 18$. Their are currently 3 firms in the industry using the old technology with total costs $c_o(q) = \frac{3}{4}q^2 + 3$, and 2 firms decide to enter with the new technology. The new firms (type n) have no fixed sunk costs, and the old firms (type o) have no fixed start-up costs.

   (a) *(6 points)* Why is it reasonable to have the old firms have no fixed start-up costs? Why is it necessary to assume the new firms have no fixed sunk costs?

   (b) *(5 points)* Find the short run supply curve of both types of firms.

   Throughout the rest of the question let the demand curve be $Q = 72 - 2P$.

   (c) *(4 points)* Find the short run equilibrium, you may assume all firms produce output.

   (d) *(2 points)* In the equilibrium you just found, show that the new firms have a higher total profit (when you subtract all fixed costs) than the old firms.

(e) *(4 points)* In the long run, find the minimal price old firms (type o) and new firms (type n) will have to charge to not exit the industry.

(f) *(6 points)* Find the long run equilibrium. How many new firms (type n) will there be? How many old firms (type o) will there be?

(g) *(4 points)* Explain the apparent contradiction between your answer in part d and part f of this question.

5. *(14 points total)* In a Robinson Crusoe Economy there is one person with the utility function $U(F, C)$ choosing their optimal F and C from a production possibilities set $g(F) + h(C) \leq T$. You may assume the indifference curves are strictly convex and that the production possibilities set is strictly convex.

   (a) *(6 points)* What does the *decentralization theorem* tell us about this problem? You may let p_f be the price of F and p_c be the price of C.

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(c) *(4 points)* Why does Istanbul trade with Ankara? After all it is the industrial heart of Turkey and also (let us say) could produce more food. You should use the appropriate technical terms in your answer.

6. *(26 points total)* Define the short run cost function as $C^{SR}(w, v, r, K, q) = \min_{L,M}(wL + vM + rK)$ such that $f(L, M, K) \geq q$. Notice we call this the short run cost function because the amount of capital is fixed. You may assume that all inputs are always used in a strictly positive amount, also that $q > 0$, $K > 0$.

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   (a) *(4 points)* Find the contract curve—or the set of Pareto efficient allocations—in this economy as a function of Y_1 in terms of X_1.

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   (c) *(6 points)* If person 1 has the initial income I_1 show that their optimal demands for (X_1, Y_1) are $\left(\frac{2}{5}\frac{I_1}{p}, \frac{3}{5}\frac{I_1}{q}\right)$.

   (d) *(4 points)* Find the final allocation in the Walrasian equilibrium, in other words find (X_1^*, Y_1^*).

4. *(31 points total)* In the market for widgets there is an exciting new technology with the total costs of $c_n(q) = \frac{1}{2}q^2 + 8$. Their are currently 8 firms in the industry using the old technology with total costs $c_o(q) = \frac{3}{4}q^2 + 3$, and 3 firms decide to enter with the new technology. The new firms (type n) have no fixed sunk costs, and the old firms (type o) have no fixed start-up costs.

   (a) *(6 points)* Why is it reasonable to have the old firms have no fixed start-up costs? Why is it necessary to assume the new firms have no fixed sunk costs?

   (b) *(5 points)* Find the short run supply curve of both types of firms.

   Throughout the rest of the question let the demand curve be $Q = 62 - 2P$.

   (c) *(4 points)* Find the short run equilibrium, you may assume all firms produce output.

   (d) *(2 points)* In the equilibrium you just found, show that the new firms have a higher total profit (when you subtract all fixed costs) than the old firms.

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