ECON 203
Midterm on Consumer Theory

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 17:40 and end at 19:20

1. *(12 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(26 points total)* About Corner solutions.

   (a) *(3 points)* Define the "bang for the buck" of a good.

   $$BfB_x = \frac{MU_x}{p_x}$$

   (b) *(6 points)* What is a Corner solution? Is it common or rare in the real world? Explain.

   Solution 1 *A* corner solution *is a case where a consumer consumes none of one or more goods. It is likely that for most consumers their are more goods that they are at a corner solution in than not. For example I own a Hyundai Tuscon from 2006 (gulp) so I am at a corner solution with regards to all other cars. I do rarely rent one, but that still leaves me at a corner solution with regards to most others. Likewise most of my clothing comes from Pull and Bear or Decatlon (don't judge me), so again I am at a corner solutions with regards to most brands of clothing.*

   But if you even look at classes of goods its quite common for people to be at corner solutions. Some people don't eat meat, some don't drink alcohol, some don't drive so they don't use cars at all. I personally do not own any dresses. Shocking, isn't it?

   I also don't eat liver, squash? Don't really like squash... fruit and chocolate, many of you love it but not me. And then hot peppers. I will eat jalepenos straight out of the jar, but many don't eat them at all.

   (c) *(3 points)* What do we know about the bang for the buck of a good in a corner solution? You may assume this consumer only consumes two goods.

Solution 2 *If someone is at a corner solution for y this means that* $\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$ *for every* $y > 0$.

τ	μ	$\frac{\partial L}{\partial X}$	$\frac{\partial L}{\partial Y}$	$\frac{MU_x}{p_x}$	$\frac{MU_y}{p_y}$
1	2	$1-\lambda p_x$	$2-\lambda p_y$	$\frac{1}{p_x}$	$\frac{2}{p_y}$
1	3	$1-\lambda p_x$	$3-\lambda p_y$	$\frac{1}{p_x}$	$\frac{3}{p_y}$
3	1	$3-\lambda p_x$	$1-\lambda p_y$	$\frac{3}{p_x}$	$\frac{1}{p_y}$
2	1	$2-\lambda p_x$	$1-\lambda p_y$	$\frac{2}{p_x}$	$\frac{1}{p_y}$

(d) *(14 points total)* Consider the utility function $u(X,Y) = \tau X + \mu Y$.

i. *(1 Points)* Set up the objective function for maximizing this function over the budget set $p_x X + p_y Y \leq I$ where $p_x > 0$ is the price of a unit of X, p_y is the same for Y, and $I > 0$ is the amount of income this person has.

$$L(X,y,\lambda) = \tau X + \mu Y - \lambda(p_x X + p_y Y - I)$$

ii. *(5 Points)* Find the first derivatives of this objective function and the bang for the bucks for both goods.

$$\frac{\partial L}{\partial X} = \tau - \lambda p_x$$
$$\frac{\partial L}{\partial Y} = \mu - \lambda p_y$$
$$\frac{\partial L}{\partial \lambda} = -(p_x X + p_y Y - I)$$

$$BfB_x = \frac{\tau}{p_x}$$
$$BfB_y = \frac{\mu}{p_y}$$

iii. *(8 Points)* Verify that for arbitrary p_x and p_y it is impossible to set all of the first derivatives to zero. If you cannot what should this consumer do?

Solution 3 *If* $\frac{\partial L}{\partial X} = 0$ *then* $\tau - \lambda p_x = 0$ *or* $\lambda = \frac{\tau}{p_x}$, *if we also have* $\frac{\partial L}{\partial Y} = 0$ *then* $\mu - \lambda(p_x)p_y = 0$ *or* $\mu - \frac{\tau}{p_x}p_y = 0$ *which requires the ratio of prices to be exactly right. When this is not true (usually) this person should consume whichever good has a higher bang for the buck, so if* $\frac{\tau}{p_x} > \frac{\mu}{p_y}$ *they should have* $X = \frac{I}{p_x}$ *and if* $\frac{\tau}{p_x} < \frac{\mu}{p_y}$ *they should have* $Y = \frac{I}{p_y}$.

3. *(30 points total)* About the income and substitution effect:

(a) *(12 points)* Write down the Slutsky equation in elasticity form, defining each term in isolation (if you wrote it down outside of the equation) and which term(s) indicate the income and substitution effect.

Solution 4 *the equation is:*

$$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x$$

where:

i. $e_x(p_x)$ *is the elasticity of the Marshallian or normal demand curve, the total effect.*

ii. $e_{h_x}(p_x)$ *is the elasticity of the Hicksian or Income Compensated demand curve, this is also the substitution effect and will always be negative.*

iii. $e_x(I)$ *is the income elasticity of the good.*

iv. $s_x = \frac{p_x X}{I}$ *is the share of income spent on this good.*

Finally $-e_x(I)\, s_x$ *is the total Income effect. It is negative because "real income" (your budget set) has shrunk, the income elasticity tells us how you will react to this, and the share of your income you spend on this good will tell us how much this change in income matters to you.*

(b) *(8 points)* Define a *Giffen good*, and using the Slutsky equation explain why the only known example of a Giffen good is an excellent candidate for being a Giffen good. (I.e. why we should expect Giffen goods to be like the only known Giffen good.)

Solution 5 *A Giffen good has an upward sloping demand curve, or* $e_x(p_x) > 0$. *Given that we know* $e_{h_x}(p_x) < 0$ *this means that* $-e_x(I)\, s_x > -e_{h_x}(p_x) > 0$. *This tells us several things.*

i. $e_x(I) < 0$ *so this good must be inferior, and preferably strongly so.*

ii. s_x *must be "large."*
This makes a staple like rice in Hunan province an excellent example. For most people in their data set rice is over half the calories they consume, thus they probably spend at least a quarter of their income on rice alone. Thus s_x *is large. We also know from anecdotal evidence that it is strongly inferior. It is the "filler" that the Chinese use to stuff their tummies at the end of the meal. This is causing a problem for the rich who do not eat enough rice and get heart problems. Thus it is both a staple good (a significant share of their income is spent on it) and inferior. This is why almost every candidate for a Giffen good is a staple food, it necessarily has a high share of income and it is often inferior.*

(c) *(10 points total)* Define (the change in) *Consumer Surplus* and *Compensating Variation*—I am not expecting precise mathematical precision in your answer, but of course precise mathematical precision would be nice.

Solution 6 *Consumer surplus is the area underneath the demand curve and above the price. To be mathematical and precise it is:*

$$CS = \int_{p^*}^{\infty} X(z, p_y, I)\, dz$$

and thus the change in price from p_x^o to p_x^n is:

$$\Delta CS = \int_{p_x^o}^{p_x^n} X(z, p_y, I)\, dz$$

Compensating Variation is much simpler to explain, it is how much you would have to give me to compensate me for the change in the price of x, or:

$$CV = I(p_x^n, p_y, u_0) - I(p_x^o, p_y, u_0)$$

where u_0 is my initial utility level. Using the first fundamental theorem of calculus we know that:

$$CV = \int_{p_x^o}^{p_x^n} h_x(z, p_y, I)\, dz$$

where $h_x(p_x, p_y, I)$ is the Hicksian or Income Compensated demand.

i. *(3 points)* Using the Slutsky equation explain why Consumer Surplus is not an accurate measure of Compensating variation, and thus the change in consumers' welfare.

   Solution 7 *From the Slutsky equation we know that:*

   $$e_{h_x}(p_x) = e_x(p_x) + e_x(I)\, s_x$$

   so the magnitude of the error depends the size of $e_x(I)\, s_x$. Since Consumer surplus has no clear and direct welfare interpretation and it is different from something that does, it means that the magnitude of Consumer surplus is not a good measure of consumer welfare.

ii. *(3 points)* For what types of goods do you expect this difference to be trivial? For what types do you expect this difference to be significant? Again use the Slutsky equation to explain your answer.

   Solution 8 *If $s_x\tilde{}0$ then $e_x(I)\, s_x\tilde{}0$ and we can use consumer surplus without much of an issue. For example if we looked at the market for bubble gum. On the other hand if we looked at housing, the class of goods food, or clothing then the difference is going to be significant, and using Consumer Surplus simply should not be done.*

4. *(8 points)* Google maps tells me that Galatasaray's stadium is 26 kilometers from Bagdat Caddesi. Which great insight of rationality tells us that this is almost irrelevant information? What is more important information for a decision maker? You should consider several different types of people.

Solution 9 *The insight I call "everything is relative." Distance is not too important for a rational decision maker. For example if I go by car between these two destinations the answer depends significantly on the time of day. At either 8:30AM or 5:30PM it will probably take an hour or more, at 1AM it will take less than a half an hour.*

But my answer also depends on what vehicle I have. If I have no car it will take me about an hour and a half. If I cannot afford public transport it will take me five hours.

And it also depends on the value of my time. If I am a busy business person an hour might be too much time, but if I am retired then five hours might just be a lovely way to spend the day.

α	β	$F(C, p_f, p_c)$	$C(p_f, p_c, I)$	$F(p_f, p_c, I)$
2	$\frac{1}{2}$	$2C\sqrt{\frac{p_c}{p_f}}$	$\frac{I}{p_c+2\sqrt{p_c}\sqrt{p_f}}$	$2I\frac{\sqrt{\frac{p_c}{p_f}}}{p_c+2\sqrt{p_c}\sqrt{p_f}}$
3	$\frac{1}{3}$	$3C\sqrt{\frac{p_c}{p_f}}$	$\frac{I}{p_c+3\sqrt{p_c}\sqrt{p_f}}$	$3I\frac{\sqrt{\frac{p_c}{p_f}}}{p_c+3\sqrt{p_c}\sqrt{p_f}}$
1	4	$\frac{1}{2}C\sqrt{\frac{p_c}{p_f}}$	$\frac{I}{p_c+\frac{1}{2}\sqrt{p_c}\sqrt{p_f}}$	$\frac{1}{2}I\frac{\sqrt{\frac{p_c}{p_f}}}{p_c+\frac{1}{2}\sqrt{p_c}\sqrt{p_f}}$
1	9	$\frac{1}{3}C\sqrt{\frac{p_c}{p_f}}$	$\frac{I}{p_c+\frac{1}{3}\sqrt{p_c}\sqrt{p_f}}$	$\frac{1}{3}I\frac{\sqrt{\frac{p_c}{p_f}}}{p_c+\frac{1}{3}\sqrt{p_c}\sqrt{p_f}}$

5. *(24 points total)* Consider the utility function $U(F, C) = -\frac{\alpha}{F} - \frac{\beta}{C}$.

   (a) *(2 points)* Find the first derivatives of this function, show that it is strictly monotonic.

   Solution 10
   $$U = -\alpha F^{-1} - \beta C^{-1}$$

   $$\begin{aligned}\frac{\partial U}{\partial F} &= (-\alpha)\left(-\frac{1}{F^2}\right) = \frac{1}{F^2}\alpha > 0 \\ \frac{\partial U}{\partial C} &= (-\beta)\left(-C^{-1-1}\right) = \frac{1}{C^2}\beta > 0\end{aligned}$$

   thus it is strictly monotonic.

   (b) *(2 points)* Is their any reason it is a problem that utility is always negative?

Solution 11 *No, not really, zero is just a number. It might seem strange, but otherwise we would have to work with the utility function* $U = \left(\frac{\alpha}{F} + \frac{\beta}{C}\right)^{-1} = \frac{CF}{C\alpha + F\beta}$ *which would be a royal pain.*

(c) *(2 points)* Set up the Lagrangian you would use to find the optimal consumptions over the budget set $p_f F + p_c C \leq I$, where $p_f > 0$ is the price of a unit of food, $p_c > 0$ is the price of a unit of clothing, and $I > 0$ is the total income.

$$L(F, C, \lambda) = -\frac{\alpha}{F} - \frac{\beta}{C} - \lambda(p_f F + p_c C - I)$$

(d) *(4 points)* Find the first order conditions of this objective function.

$$\begin{aligned} \frac{1}{F^2}\alpha - \lambda p_f &= 0 \\ \frac{1}{C^2}\beta - \lambda p_c &= 0 \\ -(p_f F + p_c C - I) &= 0 \end{aligned}$$

(e) *(4 points)* Solve for a function for F in terms of C and prices.

$$\begin{aligned} \frac{1}{F^2}\frac{\alpha}{p_f} &= \lambda = \frac{1}{C^2}\frac{\beta}{p_c} \\ C^2\frac{\alpha}{p_f} &= \frac{\beta}{p_c}F^2 \\ C^2\frac{\alpha}{\beta}\frac{p_c}{p_f} &= F^2 \\ C\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} &= F = F(C, p_f, p_c) \end{aligned}$$

(f) *(4 points)* Find the demand for C and simplify.

$$\begin{aligned} p_f F(C, p_f, p_c) + p_c C &= I \\ p_f C\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} + p_c C &= I \\ \left(p_f^{\frac{1}{2}} p_c^{\frac{1}{2}}\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c\right) C &= I \end{aligned}$$

$$C = \frac{1}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}}\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c} I = C(p_f, p_c, I)$$

(g) *(6 points)* Find the demand for F using two different methods and verify that your answer is correct.

Solution 12 *This is not a simple exercise because the two answers seem drastically different at first.*

$$\begin{aligned} p_f F + p_c C\left(p_f, p_c, I\right) &= I \\ p_f F + p_c \frac{1}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c} I &= I \end{aligned}$$

$$\begin{aligned} p_f F &= I - p_c \frac{1}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c} I = \left(1 - p_c \frac{1}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c}\right) I \\ F &= \left(1 - p_c \frac{1}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c}\right) \frac{I}{p_f} \\ &\left(\frac{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c - p_c}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c}\right) \frac{1}{p_f} I \\ &\frac{p_f^{-\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c} I \end{aligned}$$

$$\begin{aligned} C\left(p_f, p_c, I\right) \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} \left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} &= F \\ I \frac{\sqrt{\frac{p_c}{p_f}} \sqrt{\frac{\alpha}{\beta}}}{p_c + \sqrt{p_c}\sqrt{p_f}\sqrt{\frac{\alpha}{\beta}}} &= F \end{aligned}$$

The easiest way to show these two formulations are the same is to set them equal to each other:

$$\begin{aligned} I \frac{\sqrt{\frac{p_c}{p_f}}}{p_c + \sqrt{p_c}\sqrt{p_f}\sqrt{\frac{\alpha}{\beta}}} \sqrt{\frac{\alpha}{\beta}} &= \frac{p_f^{-\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c} I \\ \frac{p_c + \sqrt{p_c}\sqrt{p_f}\sqrt{\frac{\alpha}{\beta}}}{I} \left(I \frac{\sqrt{\frac{p_c}{p_f}}}{p_c + \sqrt{p_c}\sqrt{p_f}\sqrt{\frac{\alpha}{\beta}}} \sqrt{\frac{\alpha}{\beta}}\right) &= \left(\frac{p_f^{-\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} + p_c} I\right) \frac{p_c + \sqrt{p_c}\sqrt{p_f}\sqrt{\frac{\alpha}{\beta}}}{I} \end{aligned}$$

$$\begin{aligned}
\sqrt{\frac{p_c}{p_f}}\sqrt{\frac{\alpha}{\beta}} &= p_f^{-\frac{1}{2}} p_c^{\frac{1}{2}} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}} \\
\sqrt{\frac{p_c}{p_f}}\sqrt{\frac{\alpha}{\beta}} &= \frac{\sqrt{p_c}}{\sqrt{p_f}}\sqrt{\frac{\alpha}{\beta}}
\end{aligned}$$

so they are the same.