ECON 203
Final

Be sure to show your work for all answers, even if the work is simple.

This exam will approximately begin at:

9:10 in A125, 9:15 in A127, 9:20 in A229, and 9:25 in A329

Students in the wrong room will loose at least 5 minutes from the time they have to complete the exam.

1. *(4 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person. I will also not use a calculator or other electronic aid for calculation.

   Name and Surname: _

   Student ID: _

   Signature:

   _

2. *(16 points total)* Robinson Crusoe is a stupid fellow, as we all know. Despite their being a large community on the next island over he produces everything himself and does not trade with them. Fortunately for him Friday (real name unknown because Robinson couldn't pronounce it) decides to hang out with him, and being absolutely brilliant (he has a Ph.D. in economics from a nearby university)[1] he learns English and tries to convince Robinson he would be better off trading with the locals. Robinson has a series of arguments against trading, explain how each one is wrong using economics.

   (a) *(4 points)* Robinson Crusoe is a European and thus vastly more capable at producing everything than some uncivilized locals.

   Solution 1 *Even if it was true that Robinson Crusoe had an **absolute advantage** over the local population, he almost certainly does not have a **relative advantage.***
   Friday needs to explain to him that he would be better off producing what is easiest for him to produce and trading for the other goods. Specialization will make his life easier and allow him to get all he needs with less work.

   Remark 2 *If you started your answer with "he is wrong" you need to secure a job where you don't have to convince anyone of anything. Of course he's wrong, but his point is absolutely irrelevant. Especially as Turks you should know that you need to show respect in order to convince someone.*

[1] A little know fact in the history of Economic Theory is that modern exchange economy economics was first perfected in the Carribean. I am not saying Smith and Ricardo copied off of them, but their discoveries were really re-discoveries.

(b) *(6 points)* Even if he was to trade with them, he has no idea how valuable each of the goods he produces is. (You may assume he produces only food and clothing for the purposes of your argument, however you must ask him questions he can answer.)

Solution 3 *Let $h(F, C) \leq X$ be his production possibilities set, then Friday knows the mathematics behind what he needs to figure out is:*

$$\frac{MU_F}{MU_C} = \frac{p_f}{p_c} = \frac{\frac{\partial h}{\partial F}}{\frac{\partial h}{\partial C}}$$

or the ratio of the prices is equal to both the marginal rate of substitution and the marginal rate of transformation.

Now... how does he elicit this information from that blockhead? He has to ask questions like: "How much more or less do you value the ***last*** *unit of clothing over the* ***last*** *unit of food?" "At the end of the day, let's say we have one unit of each to produce, I agree to produce one of them, which one would you have me produce?" "If I offered you a free unit of either of the goods, how much more or less would you value a unit of clothing over a unit of food?"*

After a long series of questions like this he should be able to roughly figure out the answer. The key parts of each question is it always has to be about the last *unit and* relative *to the other good(s).*

The experimental standard would be to bring several piles of goods, offer him several different trades and then enact one at random. I.e. "will you give me three items of clothing for these 50 units of food?" etc. etc. After several weeks of doing this you should have his internal price vector pinned down.

Remark 4 *Several of you said "it doesn't matter, he can just start trading for what he wants" but he asked the question and you need to know how to answer it.*

(c) *(6 points)* Finally, why would there be a benefit of trading anyway? How could it possibly make him better off than producing everything himself?

Solution 5 *The basic answer is that by exporting (selling) some of his output and buying others he can consume something that was simply infeasible relying on his own production alone.*

The idea is that he can consume more of both (if he wants to) because the cost of importing some goods is lower than the cost of producing it locally.

θ	ε	ω	η	β	μ	$\frac{p_c}{p_f}$	F_a^*	C_a^*	F_b^*	C_b^*
$\frac{1}{2}$	6	3	$\frac{1}{3}$	7	7	2	6	3	7	7
$\frac{1}{3}$	2	8	$\frac{1}{2}$	6	12	$\frac{1}{2}$	2	8	6	12
$\frac{1}{3}$	2	4	$\frac{2}{3}$	5	16	$\frac{1}{4}$	1	8	6	12
$\frac{1}{2}$	2	7	$\frac{1}{5}$	8	1	2	8	4	2	4

3. *(16 points total)* Consider an exchange economy, person a has the preferences $U_a(F_a, C_a) = F_a^\theta C_a^{1-\theta}$ and an initial endowment of food and clothing of $\left(F_a^0, C_a^0\right) = (\varepsilon, \omega)$. Person b has preferences $U_b(F_b, C_b) = F_b^\eta C_b^{1-\eta}$ and an initial endowment of $\left(F_b^0, C_b^0\right) = (\beta, \mu)$.

   (a) *(6 points)* If a person has the preferences $u(x, y) = x^\alpha y^{1-\alpha}$ for $0 < \alpha < 1$ show that their demand for x given the budget set $px + qy \leq I$ is $x(p, q, I) = \alpha \frac{I}{p}$. You may use this below even if you cannot show it.

$$\begin{aligned} \frac{MU_x}{p} &= \frac{MU_y}{q} \\ \frac{\alpha \frac{u}{x}}{p} &= \frac{(1-\alpha)\frac{u}{y}}{q} \\ y &= \frac{1-\alpha}{\alpha}\frac{p}{q}x \end{aligned}$$

$$\begin{aligned} px + qy &= I \\ px + q\left(\frac{1-\alpha}{\alpha}\frac{p}{q}x\right) &= I \\ \alpha\left(px + q\left(\frac{1-\alpha}{\alpha}\frac{p}{q}x\right)\right) &= \alpha I \\ px &= \alpha I \\ x &= \alpha\frac{I}{p} \end{aligned}$$

   (b) *(10 points)* Find the equilibrium prices of food and clothing (denoted p_f and p_c) and the final consumptions of food and clothing for both person a and person b. To be technical, find $(p_f, p_c, F_a^*, C_a^*, F_b^*, C_b^*)$ where Z_i^* is the equilibrium consumption of Z by person i.

Solution 6 *Since the income in an exchange economy is the prices times endowments we know that:*

$$\begin{aligned} F_a &= \theta\frac{1}{p_f}(p_f\varepsilon + p_c\omega) \\ F_b &= \eta\frac{1}{p_f}(p_f\beta + p_c\mu) \end{aligned}$$

And since supply has to equal to demand we know that:

$$F_a(p_f, p_c) + F_b(p_f, p_c) = F_a^0 + F_b^0$$

$$\theta\frac{1}{p_f}(p_f\varepsilon + p_c\omega) + \eta\frac{1}{p_f}(p_f\beta + p_c\mu) = \varepsilon + \beta$$

By walrus's law we know we can normalize one price to one, so we let $p_f = 1$.

$$\theta(\varepsilon + p_c\omega) + \eta(\beta + p_c\mu) = \varepsilon + \beta$$

$$p_c = \frac{1}{\theta\omega + \mu\eta}(\beta + \varepsilon - \theta\varepsilon - \beta\eta)$$

or

$$\frac{p_c}{p_f} = \frac{1}{\theta\omega + \mu\eta}(\beta + \varepsilon - \theta\varepsilon - \beta\eta)$$

is a more accurate answer. Then:

$$\begin{aligned}
F_a^* &= \theta\left(\varepsilon + \left(\frac{1}{\theta\omega + \mu\eta}(\beta + \varepsilon - \theta\varepsilon - \beta\eta)\right)\omega\right) \\
&= \frac{\theta}{\theta\omega + \mu\eta}(\beta\omega + \varepsilon\omega + \mu\varepsilon\eta - \beta\eta\omega) \\
F_b^* &= \eta\left(\beta + \frac{1}{\theta\omega + \mu\eta}(\beta + \varepsilon - \theta\varepsilon - \beta\eta)\mu\right) \\
&= \frac{\eta}{\theta\omega + \mu\eta}(\beta\mu + \mu\varepsilon - \theta\mu\varepsilon + \theta\beta\omega)
\end{aligned}$$

there are many ways to find (C_a^*, C_b^*), I am just going to plug in the prices into the demand curves for a and b.

$$\begin{aligned}
C_a^* &= (1-\theta)\frac{1}{\frac{1}{\theta\omega+\mu\eta}(\beta + \varepsilon - \theta\varepsilon - \beta\eta)}\left(\varepsilon + \left(\frac{1}{\theta\omega + \mu\eta}(\beta + \varepsilon - \theta\varepsilon - \beta\eta)\right)\omega\right) \\
&= -\frac{\theta - 1}{\beta + \varepsilon - \theta\varepsilon - \beta\eta}(\beta\omega + \varepsilon\omega + \mu\varepsilon\eta - \beta\eta\omega) \\
C_b^* &= (1-\eta)\frac{1}{\frac{1}{\theta\omega+\mu\eta}(\beta + \varepsilon - \theta\varepsilon - \beta\eta)}\left(\beta + \frac{1}{\theta\omega + \mu\eta}(\beta + \varepsilon - \theta\varepsilon - \beta\eta)\mu\right) \\
&= -\frac{\eta - 1}{\beta + \varepsilon - \theta\varepsilon - \beta\eta}(\beta\mu + \mu\varepsilon - \theta\mu\varepsilon + \theta\beta\omega)
\end{aligned}$$

and of course at this point this is just random gobbledygook, but the answers do coincide with the above.

4. *(23 points total)* In a given industry there are two competing technologies. Type a firms have a cost function of $c_a(q) = \mu q + \tau q^2 + \phi$ with a fixed sunk cost of $F_{su}^a = \psi$, there are currently n_a of them. Type b firms have a cost function of $c_b(q) = \chi q + \kappa q^2 + \iota$, with a fixed start up cost of $F_{st}^b = \nu$ and there are n_b of them.

μ	τ	ϕ	ψ	n_a	χ	κ	ι	ν	n_b	P_{sd}^a	P_{in}^a	P_{sd}^b	P_{in}^b	P^{sr}
8	1	16	7	9	2	1	25	9	6	2	16	8	12	7
4	4	4	3	6	2	1	49	1	13	8	12	4	16	7
10	1	16	15	12	4	1	25	16	14	8	18	12	14	11
4	1	9	8	5	1	$\frac{1}{4}$	121	1	8	6	10	2	12	5

(a) *(5 points)* For type a firms find the marginal cost, the average cost, the average variable cost, the price at which they will shut down (P^a_{sd}) and the price at which new firms will enter with this technology (P^a_{in}).

$$\begin{aligned} MC &= \mu + 2\tau q \\ AVC &= \frac{\mu q + \tau q^2 + \phi - \psi}{q} \\ AC &= \frac{\mu q + \tau q^2 + \phi}{q} \end{aligned}$$

$$\begin{aligned} MC(q) &= AVC(q) \\ \mu + 2\tau q &= \frac{\mu q + \tau q^2 + \phi - \psi}{q} \\ q_{sd} &= \frac{1}{\tau}\sqrt{\tau\phi - \tau\psi} \\ P^a_{sd} &= \mu + 2\tau\left(\frac{1}{\tau}\sqrt{\tau\phi - \tau\psi}\right) \\ &= \mu + 2\sqrt{\tau\phi - \tau\psi} \end{aligned}$$

$$\begin{aligned} MC(q) &= AC(q) \\ \mu + 2\tau q &= \frac{\mu q + \tau q^2 + \phi}{q} \\ q_{in} &= \frac{1}{\tau}\sqrt{\tau\phi} \\ P^a_{in} &= \mu + 2\tau\left(\frac{1}{\tau}\sqrt{\tau\phi}\right) \\ &= \mu + 2\sqrt{\tau\phi} \end{aligned}$$

(b) *(5 points)* For type b firms find the marginal cost, the average cost, the average variable cost, the price at which they will shut down (P^b_{sd}) and the price at which new firms will enter with this technology (P^b_{in}).

$$\begin{aligned} MC &= \chi + 2\kappa q \\ AVC &= \frac{\chi q + \kappa q^2 + \nu}{q} \\ AC &= \frac{\chi q + \kappa q^2 + \iota}{q} \end{aligned}$$

$$\begin{aligned} MC(q) &= AVC(q) \\ \chi + 2\kappa q &= \frac{\chi q + \kappa q^2 + \nu}{q} \\ q &= \frac{1}{\kappa}\sqrt{\kappa\nu} \\ P_{sd}^b &= \chi + 2\kappa\left(\frac{1}{\kappa}\sqrt{\kappa\nu}\right) \\ &= \chi + 2\sqrt{\kappa\nu} \end{aligned}$$

$$\begin{aligned} MC(q) &= AC(q) \\ \chi + 2\kappa q &= \frac{\chi q + \kappa q^2 + \iota}{q} \\ q_{in} &= \frac{1}{\kappa}\sqrt{\iota\kappa} \\ P_{in}^b &= \chi + 2\sqrt{\iota\kappa} \end{aligned}$$

(c) *(2 points)* Assume that right now $P = x$, how many firms of each type will produce?

Solution 7 *Firms for whom $x > P_{sd}^i$ will shut down. Depending on the exam this was either one group or both groups.*

(d) *(2 points)* If some firms enter, which type will they be and why?

Solution 8 *By construction the firms that will enter depend on P_{in}^a and P_{in}^b. If $P_{in}^a < P_{in}^b$ then type a firms will enter. If the reverse is true then type b firms will enter.*

I can not tell you how disappointed I was in people who said "well if price rises high enough..." The point of entry is that it stops the price from rising that high. Only the firms with a lower P_{in} will enter.

Remark 9 *I was extremely disappointed in students who said "well if the price is x neither type would like to enter." Why did you not ask? I understand you were confused, but obviously I did not mean your interpretation. If the price is so low some firms are shutting down then obviously no one will enter.*

If there was any doubt in your mind you should have raised your hand and asked. No one did.

(e) *(3 points)* In the long run what will be the **unique** equilibrium price? How many of each type of firm will there be?

Solution 10 *In the long run $P = \min\left(P_{in}^a, P_{in}^b\right)$, thus only the firms that would enter in the last question will exist. We can not be sure how many of this type of firm there will be, but the other type will not exist.*

Remark 11 *You know, I can not say how disappointed I am in those who said "well for type a firms it would be* P_{in}^{a} *and for type b it would be* P_{in}^{b}*." Or even better, "it will be the maximum of* $\left(P_{in}^{a}, P_{in}^{b}\right)$*." I started the question by saying these technologies were competing. Is it common in competitions to choose the slowest runner and say anyone who did better than them has won?*

I even bolded the word "unique" in the question.

(f) *(4 points)* Assume the government charges a per-unit tax of t, in the long run what will be the tax burden of the firms? Explain.

Solution 12 *If* $P < \min\left(P_{in}^{a}, P_{in}^{b}\right)$ *then firms will shut down because they can't cover their average (variable) cost. Thus firms have their backs to the wall and can not assume any of the tax burden. All of the tax burden will be born by the consumers.*

Solution 13 *Alternatively, in the long run the supply curve is flat. It is fairly easy to show graphically (which some did) that this means all the tax burden will be born by the consumer.*

5. *(11 points total)* Let $c(q)$ be a cost function, $AVC(q) = \frac{c(q)-F_{su}}{q}$, and $MC(q) = \frac{dc(q)}{dq} > 0$. You may assume that $\frac{dMC(q)}{dq} \geq 0$.

(a) *(4 points)* Show that if $\frac{dAVC(q)}{dq} \geq 0$ then $MC(q) \geq AVC(q)$.

$$AVC = \frac{c(q) - F_{su}}{q}$$

$$\begin{aligned} \frac{dAVC(q)}{dq} &= \frac{c'(q)}{q} - \frac{c(q) - F_{su}}{q^2} \\ &= \frac{1}{q}\left(\frac{dc(q)}{dq} - \frac{c(q) - F_{su}}{q}\right) \\ &= \frac{1}{q}(MC - AVC) \end{aligned}$$

$$\begin{aligned} \frac{dAVC(q)}{dq} &\geq 0 \\ \frac{1}{q}(MC - AVC) &\geq 0 \\ (MC - AVC) &\geq 0 \\ MC &\geq AVC \end{aligned}$$

Remark 14 *Many of you lost points by saying that:*

$$\frac{dAVC(q)}{dq} = \frac{c'(q)\,q - (c(q) - F_{su})}{q^2}$$

without explanation, while that is true you are obviously combining at least two steps and were marked down a point for it.

(b) *(3 points)* Further show that if q^{sd} minimizes average cost, then $MC\left(q^{sd}\right) = AVC\left(q^{sd}\right)$.

Solution 15 *At any extrema we know that:*

$$\frac{dAVC(q)}{dq} = 0 = \frac{1}{q}(MC - AVC)$$

thus we must have $MC = AVC$. *If you are worried about second order conditions, don't be. Since* $\frac{d^2c}{dq^2} \geq 0$ *there will be at most one unique minimum.*

(c) *(4 points)* We assume that if $P \geq MC\left(q^{sd}\right)$ then the firm will supply $q(P)$ which is defined as $P = MC(q(P))$, use what you showed above to show that the firms variable profit $(\pi + F_{su})$ will be greater than zero.

Proof. Since $P \geq P_{sd}$ and $MC(q)$ is an increasing function we know that $q \geq q_{sd}$ thus we know that

$$\frac{dAVC}{dq} = \frac{1}{q}(MC - AVC) \geq 0$$

since $MC(q) = P$ and $AVC = \frac{c(q)-F_{su}}{q}$ this is equivalent to:

$$P - \frac{c(q) - F_{su}}{q} \geq 0$$

since $q > 0$ we know that this is the same as:

$$Pq - c(q) + F_{su} \geq 0$$

which is:

$$\pi + F_{su} \geq 0$$

■

6. *(14 points total)* Assume we have a cost function $c(w, r, p_m, q)$ where $w > 0$ is the cost of a unit of labor, $r > 0$ is the opportunity cost of a unit of capital, $p_m > 0$ is the cost of a unit of materials and $q > 0$ is the amount of output produced.

(a) *(8 points)* Prove that this function is non-decreasing in input prices, i.e. that $\frac{\partial c}{\partial w} \geq 0$, $\frac{\partial c}{\partial r} \geq 0$, and $\frac{\partial c}{\partial p_m} \geq 0$. You may use a graphic proof, but you will get at most a third of the credit unless you can explain how your proof generalizes.

Solution 16 *Assume that* $\tilde{w} \geq w$, $\tilde{r} \geq r$, *and* $\tilde{p}_m \geq p_m$ *and let* $\left(\tilde{L}, \tilde{K}, \tilde{M}\right)$ *be cost minimizing at* $(\tilde{w}, \tilde{r}, \tilde{p}_m)$ *then:*

$$\begin{aligned} c\left(\tilde{w}, \tilde{r}, \tilde{p}_m, q\right) &= \tilde{w}\tilde{L} + \tilde{r}\tilde{K} + \tilde{p}_m\tilde{M} \\ &\geq w\tilde{L} + r\tilde{K} + p_m\tilde{M} \end{aligned}$$

simply because $\tilde{w} \geq w$, $\tilde{r} \geq r$, *and* $\tilde{p}_m \geq p_m$ *and* $\tilde{L} \geq 0$, $\tilde{K} \geq 0$, *and* $\tilde{M} \geq 0$. *But by the definition of the cost function:*

$$c\left(w, r, p_m, q\right) \leq w\tilde{L} + r\tilde{K} + p_m\tilde{M}$$

resulting in if $\tilde{w} \geq w$, $\tilde{r} \geq r$, *and* $\tilde{p}_m \geq p_m$ *then* $c\left(\tilde{w}, \tilde{r}, \tilde{p}_m, q\right) \geq c\left(w, r, p_m, q\right)$ *or the desired conclusion.*

Solution 17 *Another method to prove this is the Envelope Theorem, but for this proof I really require that you prove the result, at least for one variable. This argument is that for the objective function:*

$$c\left(w, r, p_m, q\right) = \min_{L,K,M} \max_{\mu} wL + rK + p_mM - \mu\left(f\left(K, L, M\right) - q\right)$$

$$\frac{\partial c}{\partial w} = L + \frac{\partial L}{\partial w}\left(w - \mu\frac{\partial f}{\partial L}\right) + \frac{\partial K}{\partial w}\left(r - \mu\frac{\partial f}{\partial K}\right) + \frac{\partial M}{\partial w}\left(p_m - \mu\frac{\partial f}{\partial M}\right) - \frac{\partial \mu}{\partial w}\left(f\left(K, L, M\right) - q\right)$$

by the first order conditions of this objective function:

$$\left(w - \mu\frac{\partial f}{\partial L}\right) = \left(r - \mu\frac{\partial f}{\partial K}\right) = \left(p_m - \mu\frac{\partial f}{\partial M}\right) = \left(f\left(K, L, M\right) - q\right) = 0$$

and thus

$$\begin{aligned} \frac{\partial c}{\partial w} &= L \\ \frac{\partial c}{\partial r} &= K \\ \frac{\partial c}{\partial p_m} &= M \end{aligned}$$

and we know that $\min\left(L, K, M\right) \geq 0$, *this the conclusion is proven.*

(b) *(3 points)* How do we know that $\frac{\partial c}{\partial w} = L\left(w, r, p_m, q\right)$ or the input demand for labor? Explain.

Solution 18 *By the envelope theorem we know that the derivative of an optimized function like the cost function is only the direct effect, or the derivative of the function it is based on previously to optimization. Let*

$$\mathcal{L}\left(L, K, \mu\right) = wL + rK - \mu\left(f\left(L, K\right) - q\right)$$

our cost function is derived by optimizing this over (L, K, μ) *thus*

$$\frac{\partial c}{\partial w} = \frac{\partial \mathcal{L}}{\partial w} = L$$

where L *is at its optimal value, or* $L\left(w, r, p_m, q\right)$.

(c) *(3 points)* How do we know that $\frac{\partial L(w,r,p_m,q)}{\partial w} \leq 0$ or an input demand is decreasing in its own price? Explain.

Solution 19 *One of the properties of a cost function is that it is concave, a necessary condition for a function to be concave is that the second derivative with regards to any variable is negative. Since:*

$$\begin{aligned} L(w, r, p_m, q) &= \frac{\partial c}{\partial w} \\ \frac{\partial L}{\partial w} &= \frac{\partial^2 c}{\partial w^2} \leq 0 \end{aligned}$$

as desired.

Algebraically the proof is fairly simple too. Let $\left(\tilde{L}, \tilde{K}, \tilde{M}\right)$ *be cost minimizing at* $(\tilde{w}, r, p_m)$ *and* (L, K, M) *be cost minimizing at* (w, r, p_m) *then:*

$$\begin{aligned} \tilde{w}\tilde{L} + r\tilde{K} + p_m\tilde{M} &\leq \tilde{w}L + rL + p_m M \\ w\tilde{L} + r\tilde{K} + p_m\tilde{M} &\geq wL + rL + p_m M \end{aligned}$$

taking the negative of the second equation and adding them we get:

$$\begin{aligned} \tilde{w}\tilde{L} - w\tilde{L} &\leq \tilde{w}L - wL \\ (\tilde{w} - w)\tilde{L} &\leq (\tilde{w} - w)L \\ (\tilde{w} - w)\left(\tilde{L} - L\right) &\leq 0 \end{aligned}$$

which is the same statement, with a little more generality.

Remark 20 *I was surprised how many of you got this. I expected this to flummox most of you. One person even proved it using the algebraic proof.*