ECON 203
Final

Be sure to show your work for all answers, even if the work is simple.

This exam will begin around 12:30 in V04, 12:35 in V02

1. *(5 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person. I will also not use a calculator or other electronic aid for calculation.

   Name and Surname: _

   Student ID: _

   Signature:

   _

2. *(8 points)* Tariffs have been on my mind a lot lately. Using firm based analysis (*NOT the elasticity of demand and supply*) explain the maximum amount that a firm can absorb of a new tax like Trump's tariffs. How will it change as we move towards the long run? In the end, who will pay Trump's tariffs? Explain how—in theory—the tariffs could be a good idea in the long run.

   Solution 1 *Let $P_s(t)$ be the price they receive after they pay the tariff, then in order for them to produce we need:*

$$\begin{aligned} P_s(t)\,q - c(q) &\geq -F_{su} \\ P_s(t)\,q &\geq c(q) - F_{su} \\ P_s(t) &\geq \frac{c(q) - F_{su}}{q} = AVC(q) \end{aligned}$$

   thus the maximum they can absorb is the difference between the current price and their average variable costs. The rest either must be born by the consumers or the firm will shut down.

   As we move towards the long run fixed sunk costs will become fixed start up costs, which will increase average variable cost, and thus decrease the amount of the tax the firm can absorb.

   In the long run entry and exit will occur until all firms are making zero profit, and thus can not pay any of the tariff out of their (non-existent) profit. Thus customers must pay it all.

   In theory a tariff can be a good idea if it actually increases investment in the country. It is an "import substitution" strategy that has sometimes worked in some industries when the tariff is well designed and temporary (lasting only ten to twenty years). However this has never been shown to *work in the case of a post-industrial economy trying to get back low wage manufacturing jobs.*

In general the experience when this sort of tax is applied on all *goods is that it just results in that nation's economy crashing, and local firms becoming non-competitive on the global stage. It also is unclear if a blanket import tariff is realistic in the modern economy. Supply chains encircle the world these days. The large potential of comparative advantage is being exploited, resulting in a product (which may be assembled in India or the United States) having parts from countries all around the world. For example I watched a piece just today explaining that quality jigsaw puzzles (yapboz) usually use European cardstock, and the best dies to cut the cardboard are made in China.*

3. *(8 points)* Since Ankara is in a valley surrounded by mountains, the air quality downtown sometimes becomes very bad. This is part of the reason Bilkent is located up in the hills, here the air is (fairly) fresh at all times. It has been proposed to put a tax on fuel in the county of Ankara (Buyuksehir) to curb this problem. Explain how this would *not* be Pareto improving. Be careful to discuss who would benefit and who would not from this tax.

   Solution 2 *As I have clearly indicated in the question, the ones damaged the most by air quality are those that live downtown—who often do not have cars or at least use them very little.*

   In contrast, those of us who live on Bilkent campus or in villages a distance away from downtown pay very little personal cost from the pollution. At the same time we more often depend on our cars, and thus would pay a lot of taxes.

   Thus we would be made worse off and pay most of the tax, while those downtown would be better off and pay less.

   To be clear, I am fully in support of such a tax. Indeed I am strongly in favor of the more general "carbon tax" to help combat global warming. However I recognize that usually applying such taxes is not a Pareto improvement.

4. *(6 points)* In general equilibrium, what important fact does the *decentralization theorem* tell us about command economies like Robinson Crusoe's? Explain. (**NOTE**: A *command economy* is defined as one that works without a market. Instead the government tells everyone what to produce and consume. Robinson Crusoe's economy is a command economy because he is both the sole producer and the only consumer.)

   Solution 3 *In answering this question, I had a revelation. A revelation much bigger than the decentralization theorem. Indeed I finally get the* *second welfare theorem. One can read it as saying "any economy, of any structure, can be replicated by a market economy." I.e. under the listed conditions*[1]*, any economy can be replicated as a competitive market economy.*

In this case the decentralization theorem says that for the appropriate prices (price vector) that Robinson Crusoe can fulfill two roles to achieve the same outcome.

First they need to maximize their revenue given the production possibilities set, next they need to maximize their utility given their income is the revenue from the previous step.

One can go further and break the problem up more, in essence he can act as a bunch of competitive firms and still achieve the same outcome. Might sound boring to you, but remember Robinson Crusoe is too stupid to try to learn the local language. He must be very *bored.*

a	b	c	d	τ	P^*	Q^*	$P_s(t)$	$P_d(t)$	$Q(t)$	DWL
30	$\frac{1}{2}$	10	2	5	16	22	15	20	20	5
36	2	6	1	6	14	8	10	16	4	12
28	$\frac{1}{2}$	2	1	6	20	18	18	24	16	6
44	2	8	2	4	13	18	11	15	14	8
35	$\frac{1}{2}$	25	2	10	24	23	22	32	19	20

5. *(18 points)* In a given industry the Demand curve is $Q_d = a - bP_d$ and the supply curve is $Q_s = -c + dP_s$.

   (a) *(2 points)* What conditions determine an equilibrium in a market like this?

   Solution 4 *We must have $P_d = P_s = P$ and then we must have $Q_d = Q_s$ at the equilibrium price.*

   (b) *(4 points)* Find the equilibrium price and quantity.

$$\begin{aligned} a - bP &= -c + dP \\ P &= \frac{a+c}{b+d} \\ Q &= a - b\left(\frac{a+c}{b+d}\right) = \frac{ad - bc}{b+d} \end{aligned}$$

   (c) *(4 points)* The government now imposes a per-unit tax of $t = \tau$. Find the equilibrium price sellers receive, the price demanders pay, and quantity in this market.

$$P_d = P_s + \tau$$

[1] Which, I must note, include there being no externalities or public goods. They are quite unrealistic, so in point of fact there is a large roll for government in any economy. (Not to mention establishing a legal code, a basis for the complicated contracts often necessary.)

$$a - b\left(P_s + \tau\right) = -c + dP_s$$

$$\begin{aligned} P_s &= \frac{1}{b+d}\left(a + c - b\tau\right) \\ P_d &= \frac{1}{b+d}\left(a + c - b\tau\right) + \tau = \frac{1}{b+d}\left(a + c + d\tau\right) \end{aligned}$$

$$\begin{aligned} Q &= a - bP_d = a - b\left(\frac{1}{b+d}\left(a + c - b\tau\right) + \tau\right) \\ Q &= -\frac{1}{b+d}\left(bc - ad + bd\tau\right) \end{aligned}$$

(d) *(4 points)* Find the deadweight loss in this market with the tax, explaining how you derive it.

Solution 5 *Since both supply and demand are linear it is a triangle. The height of this triangle is $P_d - P_s = \tau$, the base of this triangle is Q^* from part b minus $Q(t)$ from part c. Thus:*

$$\begin{aligned} DWL &= \frac{1}{2}\tau\left(Q^* - Q(t)\right) \\ &= \frac{1}{2}\tau\left(\frac{ad - bc}{b+d} - \left(-\frac{1}{b+d}\left(bc - ad + bd\tau\right)\right)\right) \\ &= \frac{1}{2}bd\frac{\tau^2}{b+d} \end{aligned}$$

the exact values are in the table above.

(e) *(4 points)* Since there is deadweight loss in this market, would it be Pareto improving to remove the tax? Explain your reasoning.

Solution 6 *Absolutely not, Dead Weight Loss indicates their is a* cost *to raising revenue in this market. We must show their is a* benefit *outside of this market.*

That is usually easy enough, taxes are necessary for all sorts of purposes. Specifically they need to provide public goods, one of which is a legal system without which markets would be greatly hampered.

α	β	γ	λ	χ	μ	κ	F_{nt}	C_{nt}	F_t	C_t
3	2	1	1	85	3	2	$\frac{2}{13}\sqrt{13}\sqrt{85}$	$\frac{3}{13}\sqrt{13}\sqrt{85}$	$\frac{3}{13}\sqrt{13}\sqrt{85}$	$\frac{2}{13}\sqrt{13}\sqrt{85}$
4	1	1	1	145	2	$\frac{1}{2}$	$\frac{1}{17}\sqrt{17}\sqrt{145}$	$\frac{4}{17}\sqrt{17}\sqrt{145}$	$\frac{4}{17}\sqrt{17}\sqrt{145}$	$\frac{1}{17}\sqrt{17}\sqrt{145}$
4	3	1	1	73	4	3	$\frac{3}{5}\sqrt{73}$	$\frac{4}{5}\sqrt{73}$	$\frac{4}{5}\sqrt{73}$	$\frac{3}{5}\sqrt{73}$
3	1	1	1	82	3	1	$\frac{1}{5}\sqrt{5}\sqrt{41}$	$\frac{3}{5}\sqrt{5}\sqrt{41}$	$\frac{3}{5}\sqrt{5}\sqrt{41}$	$\frac{1}{5}\sqrt{5}\sqrt{41}$
5	1	1	1	101	5	$\frac{1}{2}$	$\frac{1}{26}\sqrt{26}\sqrt{101}$	$\frac{5}{26}\sqrt{26}\sqrt{101}$	10	1

6. *(19 points total)* Robinson Crusoe's preferences are $U(F, C) = \min(\alpha F, \beta C)$ and they have the production possibilities set of $\gamma F^2 + \lambda C^2 \leq \chi$.

(a) *(4 points)* Explain how we know that $C = F\frac{\alpha}{\beta}$ in any Pareto efficient allocation in this economy. **Note:** You may use this even if you can not explain it.

Solution 7 *With Leontief preferences you always will consume at the point where $\alpha F = \beta C$, after all you get no benefit from consuming more C (or F) than is given by that perfect balance. To be a little more mathematical.*

$$MU_c = \begin{cases} \beta & \text{if } \beta C < \alpha F \\ \text{undefined} & \text{if } \beta C = \alpha F \\ 0 & \text{if } \beta C > \alpha F \end{cases}$$

$$MU_f = \begin{cases} \alpha & \text{if } \beta C > \alpha F \\ \text{undefined} & \text{if } \beta C = \alpha F \\ 0 & \text{if } \beta C < \alpha F \end{cases}$$

thus the only logical choice is to have $\beta C = \alpha F$.

(b) *(2 points)* Explain why we can be sure that he will consume on the production possibilities frontier. **Note:** You may use this even if you can not explain it.

Solution 8 *We notice that for any $\varepsilon > 0$ $U(F, C) < U(F + \varepsilon, C + \varepsilon)$ or preferences are monotonic, with monotonic preferences you will always spend all your income or, in this case, consume on the outer boundary of the production possibilities set.*

(c) *(4 points)* Find the optimal amount of F and C to produce.

$$\begin{aligned} C &= F\frac{\alpha}{\beta} \\ \gamma F^2 + \lambda C^2 &= \chi \end{aligned}$$

$$\begin{aligned} \gamma F^2 + \lambda \left(F\frac{\alpha}{\beta}\right)^2 &= \chi \\ \frac{F^2}{\beta^2}\left(\lambda\alpha^2 + \gamma\beta^2\right) &= \chi \end{aligned}$$

$$\begin{aligned} F_{nt} &= \sqrt{\frac{\beta^2\chi}{\lambda\alpha^2 + \gamma\beta^2}} \\ C_{nt} &= \frac{\alpha}{\beta}\sqrt{\frac{\beta^2\chi}{\lambda\alpha^2 + \gamma\beta^2}} \end{aligned}$$

(d) *(6 points)* Now he has discovered he can trade with the (very advanced) inhabitants of the next island. In their economy $p_f = \mu$ and $p_c = \kappa$. Given this find out how much food and clothing he should produce.

$$\max_{F,C} \mu F + \kappa C - \omega \left(\gamma F^2 + \lambda C^2 - \chi\right)$$

$$\begin{aligned}
\frac{\partial L}{\partial K} &= \mu - \omega 2\gamma F = 0 \\
\frac{\partial L}{\partial C} &= \kappa - \omega 2\lambda C = 0 \\
\frac{\partial L}{\partial \omega} &= -\left(\gamma F^2 + \lambda C^2 - \chi\right) = 0
\end{aligned}$$

$$\omega = \frac{1}{2F\gamma}\mu = \frac{1}{2C}\frac{\kappa}{\lambda}$$

$$C = F\frac{\kappa}{\lambda}\frac{\gamma}{\mu}$$

$$\begin{aligned}
\gamma F^2 + \left(F\frac{\kappa}{\lambda}\frac{\gamma}{\mu}\right)^2 &= \chi \\
\frac{F^2}{\lambda}\frac{\gamma}{\mu^2}\left(\gamma\kappa^2 + \lambda\mu^2\right) &= \chi
\end{aligned}$$

$$\begin{aligned}
F_t &= \sqrt{\frac{\lambda\mu^2\chi}{\gamma\left(\gamma\kappa^2 + \lambda\mu^2\right)}} \\
C_t &= \sqrt{\frac{\lambda\mu^2\chi}{\gamma\left(\gamma\kappa^2 + \lambda\mu^2\right)}}\frac{\kappa}{\lambda}\frac{\gamma}{\mu}
\end{aligned}$$

(e) *(3 points) Without any further analysis* explain how we can be certain that Robinson Crusoe is better off if he trades with the next island.

Solution 9 *Since $(F_{nt}, C_{nt}) \neq (F_t, C_t)$ one can easily establish that $\mu F_t + \kappa C_t > \mu F_{nt} + \kappa C_{nt}$, thus he can consume (F_{nt}, C_{nt}) and also for small $\varepsilon > 0$ $(F_{nt} + \varepsilon, C_{nt} + \varepsilon)$. Since his preferences are monotonic he must be strictly better off.*

7. *(28 points total)* The cost function is defined as the minimum over (L, K) of $wL + rK$ such that $f(L, K) \geq q$. Note that $L \geq 0$, $K \geq 0$, $w > 0$, $r > 0$ and $q > 0$.

(a) *(8 points)* Prove the envelope theorem. Let L^* and K^* be the optimal quantities of labor and capital, and assume that $L^* > 0$ and $K^* > 0$. You may also assume that $f(L, K)$ is twice continuously differentiable.

Solution 10 *Let:*

$$\mathcal{L}(L,K,\mu,w,r,q) = wL + rK - \mu(f(L,K) - q)$$

then clearly:

$$c(w,r,q) = \min_{L,K}\max_{\mu}\mathcal{L}(L,K,\mu,w,r,q)$$

and at an interior solution the first order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial L} &= w - \mu MP_L = 0 \\
\frac{\partial \mathcal{L}}{\partial K} &= r - \lambda f_k = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu} &= -(f(L,K) - q) = 0
\end{aligned}$$

The optimal values of (L,K,μ) for all (w,r,q) should be written as: $(L(w,r,q), K(w,r,q), \mu(w,r,q))$ thus:

$$\begin{aligned}
c(w,r,q) &= \mathcal{L}(L(w,r,q), K(w,r,q), \mu(w,r,q), w, r, q) \\
&= wL(w,r,q) + rK(w,r,q) - \mu(w,r,q)(f(L(w,r,q), K(w,r,q)) - q)
\end{aligned}$$

Thus using the chain rule:

$$\frac{\partial c}{\partial w} = L(w,r,q) + \frac{\partial \mathcal{L}}{\partial L}\frac{\partial L}{\partial w} + \frac{\partial \mathcal{L}}{\partial K}\frac{\partial K}{\partial w} + \frac{\partial \mathcal{L}}{\partial \mu}\frac{\partial \mu}{\partial w}$$

but since $\frac{\partial \mathcal{L}}{\partial L} = \frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \mathcal{L}}{\partial \mu} = 0$ we derive the result, $\frac{\partial c}{\partial w} = L(w,r,q)$.
Of course if you hate yourself you could do it the long way, namely:

$$\begin{aligned}
\frac{\partial c}{\partial w} &= L(w,r,q) + w\frac{\partial L}{\partial w} + r\frac{\partial K}{\partial w} + (-(f(L,K) - q))\frac{\partial \mu}{\partial w} - \mu f_L\frac{\partial L}{\partial w} - \mu MP_K\frac{\partial K}{\partial w} \\
&= L(w,r,q) + \left(w - \mu\frac{\partial f}{\partial L}\right)\frac{\partial L}{\partial w} + (r - \mu f_k)\frac{\partial K}{\partial w} + (-(f(L,K) - q))\frac{\partial \mu}{\partial w}
\end{aligned}$$

and then we notice from the first order conditions above:

$$(w - \mu MP_L) = \left(r - \mu\frac{\partial f}{\partial K}\right) = -(f(L,K) - q) = 0$$

and the result is immediate.[2]

(b) *(8 points)* Prove that this function is non-decreasing in input prices using only algebra and the definition of the cost function.

[2] The confusion in my answer between MP_x, $\frac{\partial f}{\partial x}$, and f_x is intentional. You need to be comfortable that they all represent the same thing.
Different professors will use different notation and you should not be confused.

Proof. Let $w \geq \tilde{w}$ and $r \geq \tilde{r}$ then let (L^*, K^*) be cost minimizing at (w, r, q), then by definition:

$$c(w, r, q) = wL^* + rK^*$$

and since (L^*, K^*) is *feasible* at $(\tilde{w}, \tilde{r}, q)$ we know that:

$$\tilde{w}L^* + \tilde{r}K^* \geq c(\tilde{w}, \tilde{r}, q)$$

thus completing the proof. ■

(c) *(4 points)* Prove that this function is *strictly* increasing in (w, r) using the envelope theorem when $L^* > 0$ and $K^* > 0$.

Proof. From the envelope theorem we know that $\frac{\partial c}{\partial w} = L^*$ and $\frac{\partial c}{\partial r} = K^*$, thus if both of these terms are strictly positive we have the result. *(Only almost everywhere, when the cost function is differentiable, but we won't worry about that here.)* ■

(d) *(8 points)* Prove that this function is non-decreasing in output using only algebra and the definition of the cost function. What does this tell us about marginal cost?

Proof. Let $q \geq \tilde{q}$ and (L^*, K^*) be cost minimizing at (w, r, q), then like before:

$$c(w, r, q) = wL^* + rK^*$$

and like before we notice that (L^*, K^*) are *feasible* at $(w, r, \tilde{q})$ thus:

$$wL^* + rK^* \geq c(w, r, \tilde{q}) \ .$$

To unpack the "feasible" statement a little more. The important constraint is that $f(L, K) \geq q$, but since $q \geq \tilde{q}$ this obviously implies $f(L, K) \geq \tilde{q}$. ■

Solution 11 *This verifies that marginal cost can never be strictly negative. It must be at least zero.*