ECON 203
Midterm on Consumer Theory

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 12:05 in AZ25, 12:10 in AZ27 and 12:15 in A125.

It will end 100 minutes later.

1. *(12 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person. I will also neither help others nor use a calculator or other electronic aid for calculation.

   Name and Surname: _
   Student ID: _
   Signature: _

2. *(35 points total)* In a YouTube video Drew Gooden explained how he first waited until it was almost too late to watch a series on Netflix, and then bought the Blu-ray collection for $150 to finish the series. He then realized his CD player would not play blu-ray discs, and he bought a blu-ray player for $120. Finally he realized he didn't have software for watching Blu-rays, and spent 85$ on that. He stated this was an example of the *sunk cost fallacy*—which I must admit is a much cooler way of describing the insight than "sunk costs are sunk costs."

   (a) *(3 points)* Define a *sunk cost.* I simply want a definition, not examples.

   Definition 1 *A* sunk cost *is a non-recoverable cost. It is a cost that must be paid regardless of what happens in the future.*

   (b) *(4 points)* What is the "sunk costs fallacy" or, as I put it, "sunk costs are sunk costs"? I am looking for a simple explanation without any examples.

   Solution 2 *The* sunk cost fallacy *is paying attention to sunk costs when you make decisions.*

   (c) *(4 points)* Is he correct? You should assume he cannot resell anything in your answer.

   Solution 3 *Simply put, his decision making is rational. First he is willing to pay $150 for the blu-rays. This reveals his value of finishing the show is at least $150.*

He then has the shock of realizing that he will not be able to watch the show unless he buys a blu-ray player. Since \$120 < \$150 ≤ his value he obviously shoud do this.

Then he has the shock of realizing he can't watch the show unless he buys the correct software, which is another \$85, but since \$85 < \$150 ≤ his value he obviously shoud do this.

This does result in him paying \$355 to watch the show, but heh, life's like that some time. He followed an optimal decision making process.

(d) *(6 points)* If he did consider reselling (which he did not) how would that change your answer? What new fundamental economic concept would become relevant?

Solution 4 *this would bring into his though process* opportunity cost*, or the value of resale. He probably could sell the Blu-rays for \$100 (Maybe? I don't know, he did open the box to verify his CD player wouldn't work), so after buying the Blu-rays his sunk cost was probably now around \$50. Likewise he could sell the player, but here he would face the problems of adverse selection and he could use it to watch other blu-ray discs... so maybe he could sell it for \$60. Then when it comes to the software he cannot resell that.*

The main change is that now the highest sunk cost in his transaction is the last one. Each time he made a purchase decision he would have to make a harder one. I am sure he would do the same thing, but each choice would be harder than the one before.

(e) *(6 points)* Write down the three axioms that are required for normative rationality, and define each one.

i. Reflexivity—$A \succcurlyeq A$—it is hard to write this in words. If you also wrote down $A \succcurlyeq B$, well that was meaningless garbage and probably resulted in you losing points.
ii. Transitivity—If $A \succcurlyeq B$ and $B \succcurlyeq C$ then $A \succcurlyeq C$. In words you do not fall into decision cycles.
iii. Completeness—for all A and B either $A \succcurlyeq B$ or $B \succcurlyeq A$, you can compare all options.

(f) *(2 points)* What implicit assumption do we make about our subject's intelligence do we make when we assume they are rational.

Solution 5 *That they are infinitely intelligent—more intelligent than us or any feasible computer.*

(g) *(6 points)* If we include the implicit assumption, is Drew rational? If we do not, is he rational now? Explain your answer.

Solution 6 *Well now, Drew, why didn't you check your player and software first? Huh? If you were infinitely intelligent then you would* *definitely have done this and known it was going to cost you $355 to watch the show. In which case you might have made a different decison.*

Without that caveat, yes, he is clearly rational. Perhaps he should have had better information, but rationality is always dependent on what information you have. Perhaps ex-post *it seems silly, but he made the right decision at each step.*

(h) *(4 points)* Why is it important to assume our subjects are rational even when we are faced with such silly behavior? What motivates this assumption?

Solution 7 *What silly behavior? Sheesh, your professor... It is important because their is nothing more arrogant than assuming us—living in our lovely ivory tower—are more knowledgable about decisions people have to make than the people making those decisions. In fact this justifies our assuming they are infinitely intelligent—even though that might not be a realistic assumption.*

I mean, they definitely are more intelligent than me, *so...*

I thought those who said "it makes the math easier" or "we need it for our theories" were rather naive. There are entire social science disciplines that write tons of papers without assuming rationality. A sociology textbook I read started out by saying that individuals did not matter. Likewise recently there has been a fascination in Economics about "bounded rationality," and they publish tons *of papers (many of which are fairly simple) about the impact of agents who are not* fully *rational.*

Economics would be fine without rationality, but it should not go there.

α	σ	$\frac{\partial U}{\partial F}$	$F(p_f, p_c)$	$C(p_f, p_c, I)$
49	2	$\frac{49}{F^2}$	$7\sqrt{\frac{p_c}{p_f}}$	$\frac{1}{p_c}\left(I - 7p_f\sqrt{\frac{p_c}{p_f}}\right)$
36	2	$\frac{36}{F^2}$	$6\sqrt{\frac{p_c}{p_f}}$	$\frac{1}{p_c}\left(I - 6p_f\sqrt{\frac{p_c}{p_f}}\right)$
64	3	$\frac{64}{F^3}$	$4\sqrt[3]{\frac{p_c}{p_f}}$	$\frac{1}{p_c}\left(I - 4p_f\sqrt[3]{\frac{p_c}{p_f}}\right)$
27	3	$\frac{27}{F^3}$	$3\sqrt[3]{\frac{p_c}{p_f}}$	$\frac{1}{p_c}\left(I - 3p_f\sqrt[3]{\frac{p_c}{p_f}}\right)$
16	4	$\frac{16}{F^4}$	$2\sqrt[4]{\frac{p_c}{p_f}}$	$\frac{1}{p_c}\left(I - 2p_f\sqrt[4]{\frac{p_c}{p_f}}\right)$
32	5	$\frac{32}{F^5}$	$2\sqrt[5]{\frac{p_c}{p_f}}$	$\frac{1}{p_c}\left(I - 2p_f\sqrt[5]{\frac{p_c}{p_f}}\right)$

3. *(28 points total)* Heddy has the utility function $U(F, C) = \frac{\alpha}{1-\sigma}F^{1-\sigma} + C$ where $F \geq 0$ and $C \geq 0$.

(a) *(4 points)* Show that this utility function is strictly monotonic for $F \geq 0$ and $C \geq 0$. You may assume this below even if you cannot prove it.

Solution 8 $\frac{\partial U}{\partial F} = (1-\sigma)\frac{\alpha}{1-\sigma}F^{1-\sigma-1} = \frac{1}{F^{\sigma}}\alpha > 0$ *for* $F > 0$ *and diverges to positive infinity as* $F \to 0$.
$\frac{\partial U}{\partial C} = 1 > 0$.
Thus this utility function is strictly monotonic.

(b) *(2 points)* Heddy wants to maximize his utility over the budget set $p_f F + p_c C \leq I$ where $p_f > 0$, $p_c > 0$ and $I > 0$. Set up the Lagrangian objective function we want to maximize.

$$L(F, C, \lambda) = \frac{\alpha}{1-\sigma}F^{1-\sigma} + C - \lambda(p_f F + p_c C - I)$$

(c) *(4 points)* Find the first derivatives of this objective function.

$$\begin{aligned} \frac{\partial L}{\partial F} &= \frac{1}{F^{\sigma}}\alpha - \lambda p_f \\ \frac{\partial L}{\partial C} &= 1 - \lambda p_c \\ \frac{\partial L}{\partial \lambda} &= -(p_f F + p_c C - I) \end{aligned}$$

(d) *(4 points)* Which of these derivatives will be equal to zero in any optimum? Which might be strictly negative? Explain.

Solution 9 *Since the utility function is strictly monotonic,* $p_f F + p_c C = I$ *so* $\frac{\partial L}{\partial \lambda} = 0$. *And* $\lim_{F \to 0} \frac{1}{F^{\sigma}}\alpha \to \infty$ *so for any finite value of* λ *there is an* $F > 0$ *such that* $\frac{\partial L}{\partial F} = 0$.
On the other hand, $\frac{\partial L}{\partial C} = 1 - \lambda p_c$ *will only be zero if* $\lambda = \frac{1}{p_c}$. *This might or might not happen, so that one might be strictly negative.*

(e) *(4 points)* Assuming that all first derivatives are equal to zero, find the demand for F by equalizing bang for the bucks. **NOTE:** It will not be affected by Income.

Solution 10 *If we know that:*

$$\begin{aligned} \frac{\partial L}{\partial F} &= \frac{1}{F^{\sigma}}\alpha - \lambda p_f = 0 \\ \frac{\partial L}{\partial C} &= 1 - \lambda p_c = 0 \end{aligned}$$

then we can immeaditely derive that $\lambda = \frac{1}{p_c}$ *and thus*

$$\frac{1}{F^{\sigma}}\alpha - \left(\frac{1}{p_c}\right)p_f = 0$$

$$\begin{aligned} \frac{\alpha p_c}{p_f} &= F^{\sigma} \\ F &= \left(\frac{\alpha p_c}{p_f}\right)^{\frac{1}{\sigma}} \end{aligned}$$

(f) *(4 points)* Assuming that all first derivatives are equal to zero, find the demand for C.

Solution 11 *Now we use budget balancing to find the demand for C:*

$$\begin{aligned} p_f F + p_c C &= I \\ F &= \left(\frac{\alpha p_c}{p_f}\right)^{\frac{1}{\sigma}} \\ C &= \frac{1}{p_c}\left(I - p_f\left(\frac{\alpha p_c}{p_f}\right)^{\frac{1}{\sigma}}\right) \\ &= \frac{1}{p_c}\left(I - p_f\left(p_c\right)^{\frac{1}{\sigma}}\left(p_f\right)^{-\frac{1}{\sigma}}\left(\alpha\right)^{\frac{1}{\sigma}}\right) \\ &= \frac{I}{p_c} - \alpha^{\frac{1}{\sigma}}\left(\frac{p_c}{p_f}\right)^{\frac{1}{\sigma}-1} \end{aligned}$$

(g) *(6 points)* When and which of the first derivatives might be strictly negative? Explain.

Solution 12 *We find that* $C^* = \frac{I}{p_c} - \alpha^{\frac{1}{\sigma}}\left(\frac{p_c}{p_f}\right)^{\frac{1}{\sigma}-1}$*, but if the left hand side is strictly negative then* $C^* = 0$ *because we can not have negative demand. This will happen when:*

$$\begin{aligned} \frac{I}{p_c} - \alpha^{\frac{1}{\sigma}}\left(\frac{p_c}{p_f}\right)^{\frac{1}{\sigma}-1} &< 0 \\ I &< p_c\alpha^{\frac{1}{\sigma}}\left(\frac{p_c}{p_f}\right)^{\frac{1}{\sigma}-1} \\ I &< \alpha^{\frac{1}{\sigma}} p_f\left(\frac{p_c}{p_f}\right)^{\frac{1}{\sigma}} \end{aligned}$$

or income is too low.

4. *(25 points total)* The duality theorem tells us that:

$$h_x\left(p_x, p_y, p_z, u\right) = X\left(p_x, p_y, p_z, I\left(p_x, p_y, p_z, u\right)\right)$$

and the envelope theorem tells us that $\frac{\partial I}{\partial p_x} = x$, $\frac{\partial I}{\partial p_y} = y$, $\frac{\partial I}{\partial p_z} = z$. (You may assume that all exogenous variables—(p_x, p_y, p_z, u)—are strictly positive.)

(a) *(9 points)* Derive the Slutsky equation in elasticity form, defining each term in the final equation as you derive them.

$$\begin{aligned}
\frac{\partial h_x}{\partial p_x} &= \frac{\partial x}{\partial p_x} + \frac{\partial x}{\partial I}\frac{\partial I}{\partial p_x} \\
\frac{\partial h_x}{\partial p_x} &= \frac{\partial x}{\partial p_x} + \frac{\partial x}{\partial I}x \\
\frac{\partial h_x}{\partial p_x}\frac{p_x}{x} &= \frac{\partial x}{\partial p_x}\frac{p_x}{x} + \frac{\partial x}{\partial I}x\frac{p_x}{x} \\
e_{h_x}(p_x) &= e_x(p_x) + \frac{\partial x}{\partial I}x\frac{p_x}{x}\frac{I}{I}
\end{aligned}$$

where $e_{h_x}(p_x)$ is the own price elasticity of t Hicksian or Income compensated demand curve and $e_x(p_x)$ is the same for the Marshallian or normal demand curve.

$$e_{h_x}(p_x) = e_x(p_x) + e_x(I)\, s_x$$

where $e_x(I) = \frac{\partial x}{\partial I}\frac{I}{x}$ is the income elasticicty of x, and $s_x = \frac{p_x x}{I}$ is the share of income you spend on x. In final form the Slutsky equation is:

$$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x$$

(b) *(4 points)* Which term(s) represent the substitution effect, which represent the income effect? Explain

Solution 13 $e_{h_x}(p_x)$ *is the* substitution effect*, it is always negative.* $-e_x(I)\, s_x$ *is the* income effect*. The* -1 *is because when a price rises your (real) income or your budget set shrinks. The term* $e_x(I)$ *tells us how you will react to this change, and* s_x *tells us the amount this will affect your decisions.*

(c) *(3 points)* What is a *Giffen good*? Why are they strange? Why do we care about such a rarely observed type of good?

Solution 14 *A* Giffen good *is a good with an upward sloping demand curve. When the price increases so does the amount demanded.*

They are strange because they contradict the law of Demand *which is that when a price rises you buy less.*

We care about them because they are the strongest evidence of how important the income effect can be. While it is often a small effect, if it can turn a downward sloping demand curve into an upward sloping one this is an amazing impact.

(d) *(3 points)* What is the only empirically verified Giffen good?

Solution 15 *Rice in the Hunan province of China—for some consumers.*

(e) *(6 points)* Using the Slutsky equation, explain why the only empirically verified Giffen good is an excellent candidate for being a Giffen good.

Solution 16 *We need* $e_x(p_x) \geq 0$ *so we need:*

$$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x \geq 0$$

This means that we must have:

i. $e_x(I) < 0$*, and the larger it is the better. Rice is very inferior in China, to the extent that some rich people are developing heart problems because they don't eat enough rice.*

ii. $e_{h_x}(p_x)$ *"small" or their are few substitutes of this good. Again a staple like rice seems obviously to qualify.*

iii. s_x *"large" to be specific* $s_x \geq \frac{e_{h_x}(p_x)}{e_x(I)}$*. Again this is only possible for a good like rice, which is probably most of their expenditure on food.*