ECON 203
Midterm on Consumer Theory

Be sure to show your work for all answers, even if the work is simple.

This exam will last 100 minutes.

1. *(4 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person. I will also neither help others nor use a calculator or other electronic aid for calculation.

   Name and Surname: _

   Student ID: _

   Signature:

   _

   Remark 1 *If you write down abstract coefficients (like α, β, σ) in your answer you will get no credit. If you do not show your work you will get no credit.*

   Finally, if you give multiple answers then only the first one will be graded.

2. *(27 points total)* A consumer has the utility funciton $U(F, C) = \frac{1}{3}\ln F + \frac{1}{2}\ln C$, and they want to maximize their utility so that $p_f F + p_c C \leq I$, where $p_f > 0$, $p_c > 0$, and $I > 0$.

   (a) *(4 points)* Prove that this utility function is strictly montonic, what does it tell us about their optimal consumption?

   Solution 2 $\frac{\partial U}{\partial F} = \frac{1}{3}\frac{1}{F} > 0$ *for* $F > 0$ *and* $\frac{\partial U}{\partial C} = \frac{1}{2}\frac{1}{C} > 0$ *if* $C > 0$, *thus the utility function is strictly monotonic.*

   This tells us that they spend all their income, or $p_f F + p_c C = I$.

   (b) *(2 points)* Set up the Lagrangian objective function they will use.

$$\mathcal{L}(F, C, \lambda) = \frac{1}{3}\ln F + \frac{1}{2}\ln C - \lambda (p_f F + p_c C - I)$$

   (c) *(4 points)* Find the first order conditions of this objective function.

$$\frac{\partial \mathcal{L}}{\partial F} = \frac{1}{3F} - \lambda p_f = 0$$

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{1}{2C} - \lambda p_c = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -(p_f F + p_c C - I) = 0$$

(d) *(4 points)* Find a function for the quantity of C in terms of prices and the quantity of F they consume, it should be linear in F.

$$\begin{aligned}
\frac{1}{3F} - \lambda p_f &= 0 \\
\frac{1}{3Fp_f} &= \lambda \\
\frac{1}{2C} - \lambda p_c &= 0 \\
\frac{1}{2Cp_c} &= \lambda
\end{aligned}$$

$$\begin{aligned}
\frac{1}{3Fp_f} &= \frac{1}{2Cp_c} \\
C &= \frac{3}{2}\frac{p_f}{p_c}F
\end{aligned}$$

(e) *(4 points)* Find the demand curve for F.

$$\begin{aligned}
-\left(p_f F + p_c C - I\right) &= 0 \\
-\left(p_f F + p_c\left(\frac{3}{2}\frac{p_f}{p_c}F\right) - I\right) &= 0 \\
-\left(p_f F + \frac{3}{2}Fp_f - I\right) &= 0 \\
-\left(\frac{5}{2}Fp_f - I\right) &= 0
\end{aligned}$$

$$F = \frac{2}{5}\frac{I}{p_f}$$

(f) *(4 points)* Find the demand curve for C two different ways and verify they are the same.

$$\begin{aligned}
-\left(p_f F + p_c C - I\right) &= 0 \\
-\left(p_f\left(\frac{2}{5}\frac{I}{p_f}\right) + p_c C - I\right) &= 0 \\
-\left(\frac{2}{5}I + p_c C - I\right) &= 0 \\
-\left(p_c C - \frac{3}{5}I\right) &= 0
\end{aligned}$$

$$C = \frac{3}{5}\frac{I}{p_c}$$

$$\begin{aligned}
\frac{1}{3Fp_f} &= \frac{1}{2Cp_c} \\
\frac{1}{3\left(\frac{2}{5}\frac{I}{p_f}\right)p_f} &= \frac{1}{2Cp_c} \\
\frac{5}{6I} &= \frac{1}{2Cp_c} \\
2Cp_c &= \frac{6}{5}I \\
C &= \frac{3}{5}\frac{I}{p_c}
\end{aligned}$$

(g) *(5 points)* Let $F(p_f, p_c, I)$ be the demand curve for F, find $\frac{\partial F}{\partial p_c}$. What does this tell us about whether they are *substitutes* or *compliments*? Is this a *gross* or *net* measure of substitues versus compliments?

$$\begin{aligned}
F &= \frac{2}{5}\frac{I}{p_f} \\
\frac{\partial F}{\partial p_c} &= 0
\end{aligned}$$

This tells us that these are neither gross substitutes nor gross compliments.

(h) *(4 points)* Find $s_F = \frac{p_f F}{I}$ and $e_F(I)$, what does this tell us about how this conumer's behavior will change as they get richer (income increases)?

$$s_F = \frac{p_f F}{I} = \frac{p_f\left(\frac{2}{5}\frac{I}{p_f}\right)}{I}\frac{2}{5}$$

$$e_F(I) = \frac{\partial F}{\partial I}\frac{I}{F} = \left(\frac{2}{5}\frac{1}{p_f}\right)\frac{I}{\frac{2}{5}\frac{I}{p_f}} = 1$$

Notice that the first always implies the second. This tells us that when these consumers get richer they do not change the mixture of how much they buy, they just buy more of every good in the exact same proportions.

3. *(4 points)* Why is it important that a model is "wrong"?

Solution 3 *A model needs to capture the* key aspects *of an interaction. By definition this means they will overlook many small aspects of an interaction. Thus it will not be "correct" in the sense that it will perfectly predict reality, but on the other hand it will capture the important aspects of the interaction.*

4. *(17 points total)* About rationality:

(a) *(9 points)* Write down the three preference axioms that are required to give us the normative definition of rationality. For each one either explain what it implies or give a counter-example.

Solution 4 *The axioms are:*

i. Reflexivity: $A \succsim A$. This requires that our preferences are over outcomes, not process. This is contradicted in cases like the "second largest slice of cake" *where the only thing you care about is the signal you are sending through your consumption.*

ii. Transitivity: $A \succsim B$ and $B \succsim C$ implies that $A \succsim C$. This rules out decision cycles.

iii. Completeness: For all A and B, either $A \succsim B$ or $B \succsim A$. This guarentees that we can compare everything, and is obviously falsified by life changing decisions (like professor versus buddhist monk) or situations where we do not have enough information (like living on Mars or Venus).

(b) *(5 points)* What is the motivation for a social scientist like an economist to assume that people are rational?

Solution 5 *It is the extreme arrogance of a scientist assuming they are smarter than the subjects under their analysis, especially when both parties are from the same species this is ridiculous. There are many examples where populations do the rational thing (clearly satisfy a precise objective) but figuring this out took social scientists a very long time. If they had simply assumed their subjects were stupid they would have never been smart enough to investigate the question.*

(c) *(3 points)* Explain how the motivation for us assuming rationality could result in social scientists assuming people are infinitely smart.

Solution 6 *If my subjects are* as *smart as me... well that is hard to be sure of. And as smart as Einstein or as smart as me? Clearly since we cannot allow less than, the only viable option is more than: i.e. our subjects are smarter than any scientist investigating them. Now they are super human, how hard is it to take the next step and assume they are infitely smart? Not hard at all.*

5. *(18 points total)* On the income effect:

(a) *(8 points)* Write down the slutsky equation in elasticity form, defining each term in isolation and indicating which terms are the substitution effect and which are the income effect.

$$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x$$

$e_x(p_x)$—the elasticity of the normal or Marshallian demand curve.

$e_{h_x}(p_x)$—the elasticity of the income compensated or Hicksian demand curve.

$e_x(I)$—the income elasticity of the normal or Marshallian demand curve.

s_x—the share of income spent on x.

$e_{h_x}(p_x)$—the substitution effect, this is always negative.

$-e_x(I)\,s_x$—the income effect, its sign is positive for inferior goods and negative for normal or luxury goods.

(b) *(4 points)* Define a *Giffen good*

Definition 7 *A Giffen good is one that when the price increases the amount bought also increases. In other words* $\frac{\partial X}{\partial p_x} > 0$ *or* $e_x(p_x) > 0$

(c) *(3 points)* Using the slutsky equation, explain how Giffen goods could exist.

Solution 8 *We know that:*

$$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\,s_x$$

and we are interested in a case where $e_x(p_x) > 0$. *Thus we need* $e_{h_x}(p_x) - e_x(I)\,s_x > 0$ *when we know that* $e_{h_x}(p_x) \leq 0$. *This requires that* $e_x(I) < 0$ *or that the good is inferior.*

(d) *(3 points)* Given how rare they are, why do we care about Giffen goods at all?

Solution 9 *While they are exceedingly rare, they are important because they contradict the standard intuition sometimes called the "law of demand" (which is* $e_x(p_x) \leq 0$*). This illustrates how—in some cases—the extreme power of the income effect.*

6. *(30 points total)* A consumer wants to maximize their utility, $U(x,y) = \frac{1}{4}x + 2y$ such that $px + qy \leq I$, where $p > 0$, $q > 0$, and $I > 0$.

(a) *(3 points)* Prove that this utility function is strictly montonic, what does it tell us about their optimal consumption?

Solution 10 $\frac{\partial U}{\partial x} = \frac{1}{4} > 0$ *and* $\frac{\partial U}{\partial y} = 2 > 0$ *thus the utility function is strictly monotonic.*

This tells us that they spend all their income, or $px + qy = I$.

(b) *(1 point)* Set up the Lagrangian objective function they will use.

$$\mathcal{L}(x,y,\lambda) = \frac{1}{4}x + 2y - \lambda(px + qy - I)$$

(c) *(3 points)* Find the first derivatives of this objective function.

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x} &= \frac{1}{4} - \lambda p \\
\frac{\partial \mathcal{L}}{\partial y} &= 2 - \lambda q \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= -(px + qy - I)
\end{aligned}$$

(d) *(2 points)* Which of these first derivatives will always equal zero? How can you tell?

Solution 11 *Since the utility function is strictly montonic we know they will spend all their income or* $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$.

(e) *(3 points)* Explain why what you've done in parts *b* and *c* is basically useless.

Solution 12 *With this utility function we will almost always be in a corner solution. When that happens we have to carefully utilize the mathematics of corner solutions to figure out what the value for* λ *is.*

It is much simpler and faster simply to utilize Bang for the Bucks to figure out what to do.

(f) *(3 points)* Define the Bang for the Buck (BfB).

$$BfB_x = \frac{MU_x}{p_x}$$

(g) *(3 points)* If $BfB_x > BfB_y$ and you have convex preferences how should you change your consumption?

Solution 13 *You should increase your consumption of* x *and decrease your consumption of* y.

(h) *(6 points)* Define a *corner solution*. If a consumer does not buy a good does it mean its marginal utility is negative? What does it mean?

Solution 14 *A* corner solution *is when a consumer buys nothing of one or more goods. It means that* $BfB_x > BfB_y$ *when* $y = 0$. *In this case we* can *have* $MU_y \leq 0$, *but usually this is not true—it is simply that the marginal utility per dollar/lira is not high enough.*

(i) *(6 points)* Find the BfB_x and BfB_y for the consumer in this problem. Using these find a condition under which they buy no y, and another condition under which they buy no x.

Solution 15

$$
\begin{aligned}
BfB_x &= \frac{\frac{1}{4}}{p} = \frac{1}{4p} \\
BfB_y &= \frac{2}{q}
\end{aligned}
$$

They will buy no y if:

$$
\begin{aligned}
BfB_x &> BfB_y \\
\frac{1}{4p} &> \frac{2}{q} \\
q &> 8p
\end{aligned}
$$

They wil buy no x if:

$$
\begin{aligned}
BfB_x &< BfB_y \\
\frac{1}{4p} &< \frac{2}{q} \\
q &< 8p
\end{aligned}
$$