Answers to:
Practice Questions—Chapters 1 to 6.
Introduction and Consumer Theory
ECON 203
Kevin Hasker

These questions are to help you prepare for the exams only. Do not turn them in. Note that not all questions can be completely answered using the material in the chapter in which they are asked. These are all old exam questions and often the answers will require material from more than one chapter. Questions with lower numbers were asked in more recent years.

1 Chapter 1 and 2—Economic Models and The Mathematics of Optimization.

1. Why is every model wrong by construction?

   A model is intended to find the key variables that affect some given outcome. By focusing on key variables one must, necessarily, ignore other factors that may affect an interaction but probably will not have a large impact.

2. What are the three insights from Nezarettin Hoca and the Ants? Which of these is most important? Show that the other two can be derived from this one.

   (a) At any maximum $f'(x^*) = 0$

   (b) At any maximum $f''(x^*) \leq 0$

   (c) If $f'(x) > 0$ then increase x, if $f'(x) < 0$ then decrease x.
   The most important insight is the last one. If you follow this rule then anyplace you stop must have $f'(x^*) = 0$. Furthermore for $x < x^*$ it must be that $f'(x) > 0$ thus it must be that $f''(x^*) \leq 0$.

3. List three of the four great insights of rationality. For each one explain what the insight is and for two of them give an example of how you can use this insight.

   For the answer to this question please see the introductory handout posted on the class web page.

4. On rationality:

   (a) What is the motivation for economists to assume rationality, or what is the motivational definition of rationality?
   Do not assume that people are stupid.

   (b) Give the Positive definition of Rationality.
   People do not choose inferior options.

(c) Give the Normative definition of Rationality.

People always choose the best option.

(d) Write down the three preference axioms we need for the Normative definition of Rationality, defining each and giving an intuitive (not graphical) counter-example to one of them.

i. *Reflexivity*—$A \succapprox A$—*an option is at least as good as itself.*

ii. *Transitivity*—$A \succapprox B, B \succapprox C \Rightarrow A \succapprox C$—*no decision cycles A counter example would be any example of a decision cycle, where because you are comparing multi-faceted goods you are unable to make up your mind.*

iii. *Complete*—*either* $A \succapprox B$ *or* $B \succapprox A$—*everything can be compared. A counter example is a life changing decision, like for example I am unable to compare my current life with being a monk in Tibet. Since the options are so far apart, and I have never tried being a Monk, I can not tell you which I think is better.*

2 Chapter 3—Preferences and Utility

1. We usually assume that people have *monotonic* preferences. A person has *monotonic* preferences if when you give them more of every good they are always happier.

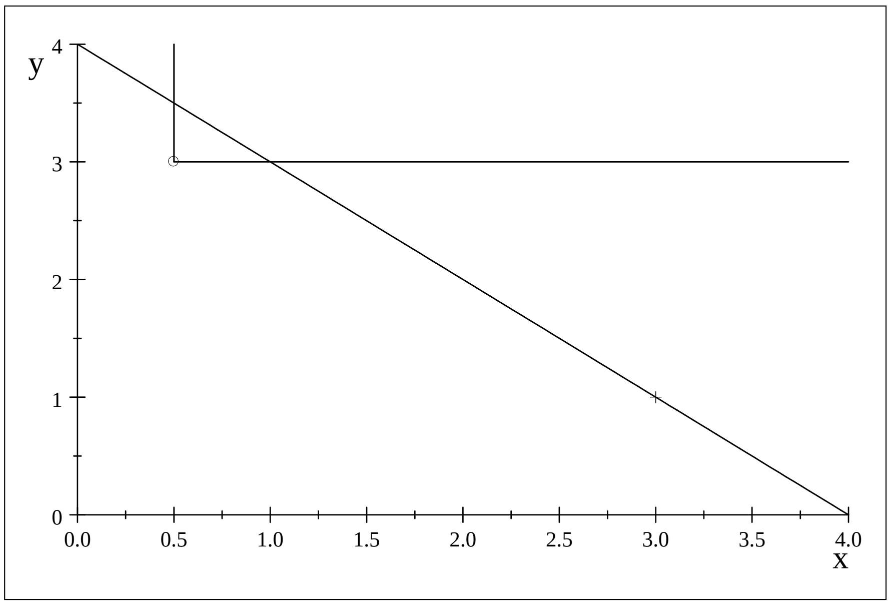
(a) Give an example that makes it clear that no one's preferences are monotonic.

The best example is the "infinite glasses of raki. If someone prefers one glass of raki to none and they have monotonic preferences then they prefer 1000 glasses of raki to 1, but 1000 glass of raki will kill you, so this is not true. This holds equally well for most foods, like water, too much is not a good thing, and so no one's preferences are monotonic.

(b) Prove that someone with monotonic preferences will spend all of their income. You may either use a graph or algebra, and you should start your proof by assuming that the person does not spend all of her income.

First of all, I will give the proof in words, which is easier. By contradiction assume that (F_A, C_A) *is optimal but that* $p_f F_A + p_c C_A < I$, *then clearly there is some* $\varepsilon > 0$ such that $p_f (F_A + \varepsilon) + p_c (C_A + \varepsilon) \leq I$. However since $F_A + \varepsilon > F_A$ and $C_A + \varepsilon > C_A$ $(F_A + \varepsilon, C_A + \varepsilon)$ is strictly better, a contradiction.

Graphically, Consider the circle in the graph below (at $\left(\frac{1}{2}, 3\right)$*), call that circle A. Now by montonicity everything to the North East of that point is strictly better, this is the area inside the wedge that* *starts at that point. Thus A can not be optimal. On the other hand at some point like the cross (at* $(3,1)$*), if we drew such a wedge from that point everything that would be strictly preferred would be not affordable, thus it could be an optimum.*



2. We generally assume that people have convex preferences. Define this assumption either using words or mathematics, show an indifference curve that is ruled out by this assumption and why it is ruled out, and give a example using only words of preferences that are not convex. Why do we make this assumption? (*HINTS:* your answer to the last question will be related to utility maximization, and writing down a precise definitions using only words is very difficult.)

   For consumption vectors, A, B, C *if* $A \succeq C$ *and* $B \succeq C$ *then for* $0 \leq \lambda \leq 1$ $\lambda A + (1-\lambda) B \succeq C$.

   Consider the indifference curves generated by $\max(x_1, x_2)$ *then* $(20, 80)$ *and* $(80, 20)$ *are on the same indifference curve but* $\frac{1}{2}(20, 80) + \frac{1}{2}(80, 20) = (50, 50)$ *is on a strictly lower indifference curve since it gives a utility of* 50. *A more general answer would show an indifference curve that is bowed away from the origin, so that a line between any two points on the indifference curve is strictly below the indifference curve.*

   'An example of preferences that are not convex is preferences over housing location. Say that you are indifferent between a job in New York City and Paris. Would spending odd weeks in NYC and even weeks in Paris be better than both of these options? I think not, the amount of travelling

you would have to do would make either location strictly preferred to a convex combination of the two locations.

We make this assumption because it guarentees that when the indifference curve is tangent to the budget constraint this is a utility maximizing bundle.

3. On the preference axiom of *monotonicity.*

   (a) Define this axiom. Your answer does not have to have mathematical terms in it.

   If one consumption bundle has at least as much of every good as another, and at least one good is strictly more, then it is prefered to that other good.

   If A has more of every good than B (and strictly more of at least one good) then $A \succ B$.

   (b) What does this axiom tell us about indifference curves and marginal utilities?

   It tells us that indifference curves are downward sloping and marginal utilities are positive.

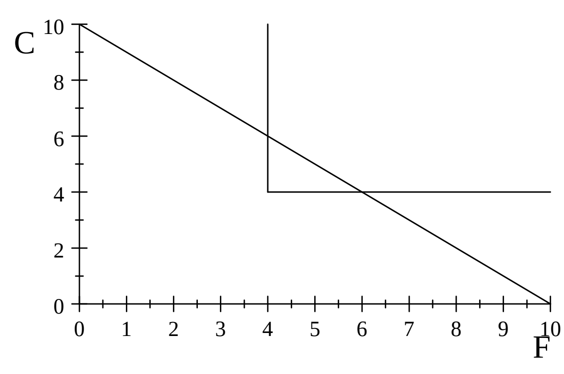
   (c) Why is this axiom completely unreasonable as an assumption about human behavior? Your answer should include an example.

   My favorite example is the "infinite glasses of raki." According to this axiom if you think that one glass of raki is better than none, then 300 glasses is better than 3. Unfortunately that much raki would kill you.

   (d) Prove either using a graph or math that this means that at your optimal consumption ($\{F^*, C^*\}$) you will spend all your income ($p_f F^* + p_c C^* = I$).

   I am going to use both techniques, but math first. Assume that at $\{F^*, C^*\}$ $p_f F^* + p_c C^* < I$. *But clearly there is an* $\varepsilon > 0$ *so that* $p_f (F^* + \varepsilon) + p_c (C^* + \varepsilon) \leq I$ *and since* $\{F^* + \varepsilon, C^* + \varepsilon\}$ *has strictly more of every good than* $\{F^*, C^*\}$ $\{F^* + \varepsilon, C^* + \varepsilon\} \succ \{F^*, C^*\}$ *contradicting that* $\{F^*, C^*\}$ *was optimal.*

   Graphically consider the budget set below:



at any point on the interior of the budget set (like the corner of the wedge above) we can draw two lines, one parallel to the F axis and one to the C axis. Any points inside this wedge are preferred to the point on the corner, so the original point can not be optimal.

4. About *rationality.*

   (a) Write down the three axioms that give us the normative definition of rationality. The best answers will use both words and mathematical symbols, but you may use only one for partial credit.

      i. *Reflexivity, A is at least as good as A, or* $A \succsim A$

      ii. *Transitivity, A is better than another option, that one is better than a third, then A is better than the third, or there are no decision cycles, or* $A \succsim B, B \succsim C \Rightarrow A \succsim C$

      iii. *Completeness, everything can be compared to everything else. Either* $A \succsim B$ or $B \succsim A$.

   (b) For each one write down a counter example using words.

      i. *Reflexivity, to satisfy this you only can use two items to determine which is better. A counter example is the second largest slice of cake example. If you are offered the choices* $\left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$ *you will choose* $\frac{1}{4}$. *If you are offered* $\left\{\frac{1}{4}, \frac{1}{5}, \frac{1}{6}\right\}$ *you will choose* $\frac{1}{5}$. *This implies (if we tried to represent this as a reflexive ordering) that* $\frac{1}{4} \succ \frac{1}{5} \succ \frac{1}{4}$, *a clear violation of reflexivity.*

      ii. *Transitivity, counter examples are decision cycles. I.e. situations where you can not make a choice because there are too many characteristics to choose among. Say, for example, you are deciding what car to buy. You are thinking about an economy car because you don't use your car that much. But then a Jeep (SUV) gives you so much more room so the Jeep is better than the economy car, but then a sports car might not give you more room but it will give you so much more power, so the sports car is better than the Jeep. But then the price of the economy car is so much lower than the sports car so the economy car is better than the sports car.*

      iii. *Completeness, any counter example will have life-changing choices that you don't have enough basis to compare. For example being a professor or a Tibetan monk.*

   (c) For two of them draw indifference curves that do not satisfy these axioms.

      i. *Reflexivity can't be done.*

      ii. *Transitivity will have crossing indifference curves.*

      iii. *Completeness will have non-nested indifference curves. There will be one family of nested indifference curves on the right and another on the left.*

(d) Write down the positive definition of rationality, and explain why it does not need to satisfy the three axioms you wrote down in part 4a.

The positive definition of rationality is that you don't choose inferior options. Since you don't have to be able to choose to satisfy this definition your preferences can be incomplete. Since you can not choose anything at all you can have intransitive preferences. And if we don't need these two axioms then we don't have to worry about Reflexivity.

5. On the normative definition of rationality:

   a What three axioms of choice make up this definition? Define each one using words or symbols.

      i. Reflexivity: $A \gtrapprox A$

      ii. Transitivity: $A \gtrapprox B, B \gtrapprox C \Rightarrow A \gtrapprox C$. In words there can't be any "decision cycles."

      iii. Completeness: Either $A \gtrapprox B$ or $B \gtrapprox A$ for every A and B. In words everything is comparable.

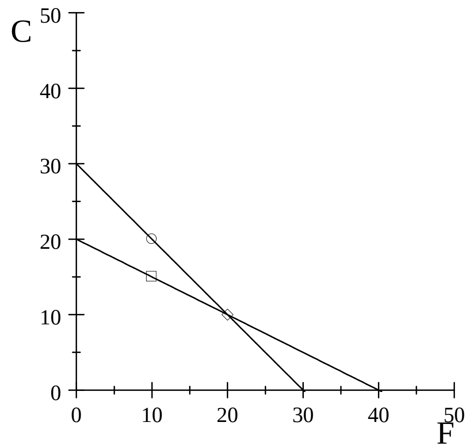
   b For two of these axioms give a real life example of how these preferences might fail. (You can not use indifference curves in this answer.)

      i. *Transitivity: A "decision cycle" is a counter-example. A decision cycle is any time you have multiple options you are choosing among and you can't figure out which is best. For example say you want to buy a new car, you are considering a Hyundai Getz because it's small, compact, and cheap. But then you think, hey, I'd like a Hyundai Tuscon—it's so much more powerful and not that much more expensive. But then you say, but I'd really like a Toyota Rav4. Then you look at the price sticker, and realize that you could buy three Hyundai Getz for that price. What are you thinking? You probably go through cycles like this with every major purchase, transitivity says you can resolve them and make a purchase.*

      ii. *Completeness: Everything is not comparable. When faced with major life-style changes you don't know enough to make a choice. Like for example giving up your studies to become a Tibetan Buddhist Monk. How do you know if you'd like it better?*

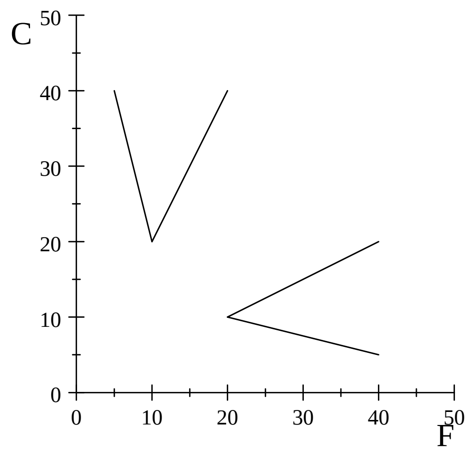
   c Show what two of these axioms rule out in indifference curves.

      i. Axiom 1: *Transitivity rules out crossing indifference curves.*



In the graph the diamond at the crossing of the two indifference curves is indifferent to both the circle (on the higher indifference curve) and the box (on the lower indifference curve). Transitivity would imply that you are indifferent between the circle and the box. However by direct comparison the circle is strictly better than the box, so we have a contradiction.

ii. Axiom 2: *Completeness rules out non-nesting indifference curves. These are awfully hard to draw using my computer, but in the graph below you can kind of get the idea.*



Anything that is inside of these indifference curves is clearly better than anything on either indifference curve, but we can not compare things on one indifference curve to things on the other since one is not inside of the other.

6. Show that a consumer with the utility function $u(x, y) = xy^2$ has the

same demand curves as the consumer with the utility function $u(x,y) = \frac{1}{1000}\ln x + \frac{1}{500}\ln y$. Why is this?

ANSWER:

Let $g(z) = \frac{1}{1000}\ln z$. *Then,* $g'(z) = \frac{1}{1000z} > 0$ *for every* $z > 0$. *Hence* $u_1(x,y) = g(u(x,y)) = \frac{1}{1000}\ln x + \frac{1}{500}\ln y$ *is a positive monotonic transformation of* $u(x,y)$. *Since utility functions are defined up to a positive monotonic transformation we are done. To see the reason for this property of utility functions, first note that we say* $u(\cdot)$ *represents preferences of consumer I, if, A is preferred to C implies* $u(A) \geq u(C)$ *and* $u(A) \geq u(C)$ *implies A is preferred to C. Now, since any positive monotonic transformation of* $u(\cdot)$ *would preserve the same property, it represents preferences of consumer I as well.*

Another way to see the similarity is to notice that the slope of indifference curves does not change

$$MRS = \frac{\frac{\partial u}{\partial F}}{\frac{\partial u}{\partial C}} = \frac{g'(u)\frac{\partial u}{\partial F}}{g'(u)\frac{\partial u}{\partial C}}$$

as long as $g'(u) > 0$, *since this will be true for any monotonic* $g(\cdot)$ *this means that* $U(F,C)$ *and* $g(U(F,C))$ *have the same indifference curves, all that changes is the "number" we associate with each indifference curve.*

3 Chapter 4—Utility Maximization and Choice.

1. Consider the utility function $U(F,C) = C - \frac{1}{F}$, let p_f be the price of F, p_c be the price of C, and I be the income of this consumer. Assume that the demand for F and C is strictly positive throughout this question unless the question specifically says this is not true.

   (a) Find both of the marginal utilities and verify that they are strictly positive for $C > 0$ and $F > 0$. What does this tell us about this utility function? What does it tell us about the relationship between expenditure $(p_f F + p_c C)$ and income (I)?

   $$\begin{aligned} MU_C &= 1 > 0 \\ MU_F &= \left(-\frac{1}{F^2}\right)(-1) = \frac{1}{F^2} > 0 \end{aligned}$$

   this tells us the utility function is **monotonic** *and that* $p_f F + p_c C = I$.

   (b) Write down the Lagrangian function he will maximize.

   $$\max_{F,C}\min_{\lambda} C - \frac{1}{F} - \lambda(p_f F + p_c C - I)$$

(c) Find the first order conditions of his maximization problem.

$$\begin{aligned} 1 - \lambda p_c &= 0 \\ \frac{1}{F^2} - \lambda p_f &= 0 \\ -(p_f F + p_c C - I) &= 0 \end{aligned}$$

(d) Solve for the bang for the buck of F and C. Using these find the demand for F.

$$\lambda = \frac{1}{p_c} = \frac{1}{p_f F^2}$$

$$F = \left(\frac{p_c}{p_f}\right)^{\frac{1}{2}}$$

(e) Find the demand for C.

$$\begin{aligned} p_f F + p_c C &= I \\ p_f \left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} + p_c C &= I \\ C &= \frac{I - p_f \left(\frac{p_c}{p_f}\right)^{\frac{1}{2}}}{p_c} \\ &= \frac{I}{p_c} - \left(\frac{p_f}{p_c}\right)^{\frac{1}{2}} \end{aligned}$$

(f) What is a *corner solution*? What mathematical condition tells us that a consumer will be in a corner solution? With this utility function is the consumer ever in a corner solution? If so, when?

A **corner solution** *is a situation where a consumer buys none of one or more goods. The mathematical condition is*

$$\frac{MU_X}{p_x} > \frac{MU_Y}{p_y}$$

for every $Y > 0$. *With this utility function this is completely possible.*

$$\frac{1}{p_f F^2} > \frac{1}{p_c}$$

and for this to be true we have to have $F = \frac{I}{p_f}$, *the maximum possible value.*

$$\begin{aligned} \frac{1}{p_f \left(\frac{I}{p_f}\right)^2} &> \frac{1}{p_c} \\ \frac{p_c}{p_f} &> \left(\frac{I}{p_f}\right)^2 \end{aligned}$$

and this condition is fine enough for an answer.

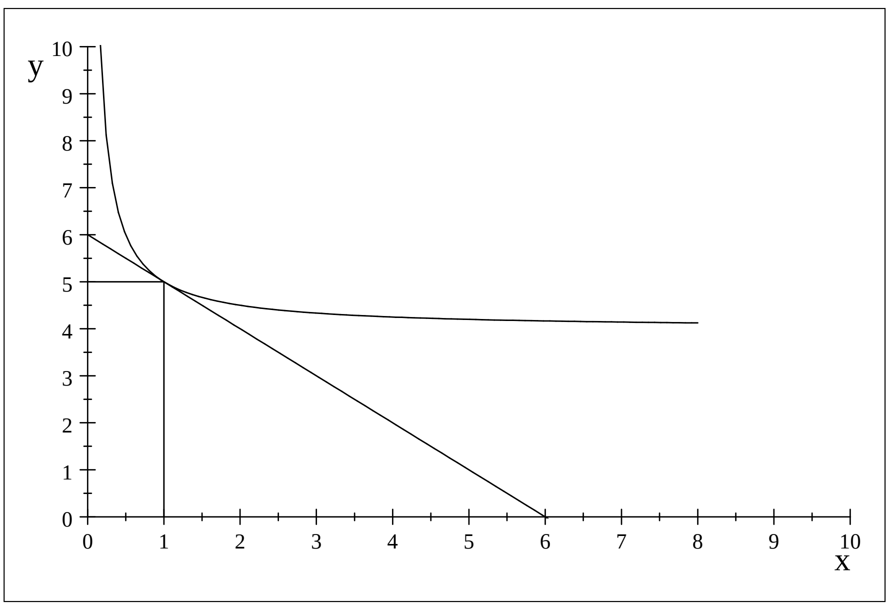
(g) In the table below find four values for F and C so that $U(F,C) = 4$.
To solve this question you should first find the formula for an indifference curve:

$$\begin{aligned} U &= C - \frac{1}{F} \\ C &= U + \frac{1}{F} \end{aligned}$$

and then it is easy to plug in values for F and fill out the table. From there we can go to the graph.

C	F
6	$\frac{1}{2}$
5	1
$4\frac{1}{2}$	2

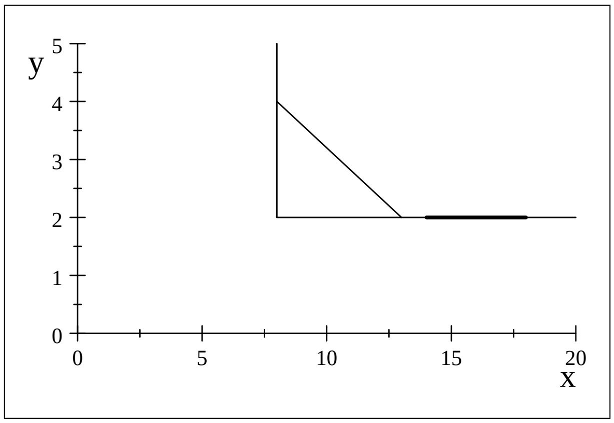
and then in the graph below graph this indifference curve.
For the graph I will assume that $4 = C - \frac{1}{F}$, and that $p_c = 3$, $p_f = 3$ and $I = 18$ using the demand curves I find that $F = 1$, $C = \frac{18}{3} - \left(\frac{3}{3}\right)^{\frac{1}{2}} = 5$.



2. Roderick is a most unlucky lad. He was born with 5 legs, so he has 1 left feet and 4 right feet. Since he has special needs, his parents let him buy his own shoes, and give him an allowance of I to buy shoes with. His utility function is $U(L,R) = \min\left\{L, \frac{R}{4}\right\}$ where L is the quantity of left shoes he buys and R is the quantity of right shoes he buys.

(a) In the graph provided, draw the indifference curve for Roderick where $U(L,R) = 2$. Warning: this graph must be precise.

This answer is given for $U(L,R) = \min\left\{L, \frac{R}{4}\right\}$ *the other answers can be found by generalizing this one. Given the graph provided they should all look the same.*



(b) Define what we mean when we say that preferences are *convex*. Are these preferences convex? Explain why or why not using the graph above.

Preferences are convex if when $A \succeq C$ *and* $B \succeq C$ *then for all* $\lambda \in [0,1]$ $\lambda A + (1-\lambda) B \succeq C$ *if you let* $C \in \{A, B\}$ *then you can see that this means that a line between any two points on the indifference curve must be weakly above and to the right of that indifference curve. On the graph two examples are given, one where* A *and* B *are on different sections of the indifference curve and one (in a heavy black line) where they are on the same indifference curve.*

(c) Define what we mean when we say that preferences are monotonic. Are these preferences monotonic? *Warning: Your definition and your answer must agree, be careful that they do.*

There are two possible definitions of monotonic. Preferences are **strongly monotonic** *if whenever* $A_l \geq B_l$ *for all* l *and* $A_l > B_l$ *for at least one* l *then* $A \succ B$. *By this definition these preferences are not monotonic.* $U(2,3) = U(2,5) = 2$. *Preferences are* **weakly monotonic** *if whenever* $A_l > B_l$ *for all* l *then* $A \succ B$. *This utility function is weakly monotonic, because it is always strictly increasing in the lesser of* L *and* $\frac{R}{4}$.

(d) By analyzing the graph find the ratio of L to R that Roderick will always buy, regardless of prices. Explain your reasoning.

Looking at the graph, obviously it doesn't make sense to consume anywhere except at the corner where $\min\left\{L, \frac{R}{4}\right\} = L = \frac{R}{4}$. *Consuming* *anywhere else would bring no utility but would increase expenditure.*

$$\begin{aligned} L &= \frac{R}{4} \\ R &= L4 \end{aligned}$$

(e) Find the demand for L.

$$\begin{aligned} p_l L + p_r R &= I \\ p_l L + p_r (L4) &= I \\ L &= \frac{I}{p_l + 4p_r} \end{aligned}$$

(f) Find the demand for R.

$$R = L4 = 4\frac{I}{p_l + 4p_r}$$

$$\begin{aligned} p_l L + p_r R &= I \\ p_l \left(\frac{1}{4}R\right) + p_r R &= I \\ R &= \frac{I}{p_l \frac{1}{4} + p_r} \end{aligned}$$

(g) Assume that the price of right shoes, p_r, increases. What will be the substitution effect from this price change? Explain.

To find the substitution effect we change the prices **while staying on the same indifference curve**. *As stated in part d the consumption point will always be the same in this case, So the substitution effect is always zero.*

3. Tolga Han is a simple fellow, since his parents pay for his food and clothes all he spends money on is footballs (F) and candy (C). His utility function is $U(F, C) = FC^4$.

(a) Is this utility function monotonic? (You should only consider $F > 0$ and $C > 0$).

$$\begin{aligned} \frac{\partial U}{\partial F} &= C^4 = \frac{U}{F} > 0 \\ \frac{\partial U}{\partial C} &= 4FC^{4-1} = 4\frac{U}{C} > 0 \end{aligned}$$

since both first derivatives are strictly positive yes, it is monotonic.

(b) Write down the Lagrangian function he will maximize.

$$\max_{F,C} \min_{\lambda} FC^4 - \lambda (p_f F + p_c C - I)$$

(c) Find the first order conditions of his maximization problem.

$$\begin{aligned} \frac{U}{F} - \lambda p_f &= 0 \\ 4\frac{U}{C} - \lambda p_c &= 0 \\ p_f F + p_c C - I &= 0 \end{aligned}$$

(d) Solve for the bang for the buck of Footballs and Candy. Equalize the bangs for the bucks and find a formula for C in terms of F and prices.

$$\begin{aligned} \frac{1}{F}\frac{U}{p_f} &= \lambda = \frac{1}{C}U\frac{4}{p_c} \\ C &= F\frac{4}{p_c}p_f \end{aligned}$$

(e) Find the demand for Footballs.

$$\begin{aligned} p_f F + p_c\left(F\frac{4}{p_c}p_f\right) - I &= 0 \\ F &= \frac{I}{p_f}\frac{1}{5} \end{aligned}$$

(f) Find the demand for Candy.

$$\begin{aligned} \frac{1}{\left(\frac{I}{p_f+4p_f}\right)}\frac{U}{p_f} &= \frac{1}{C}U\frac{4}{p_c} \\ C &= \frac{I}{p_c}\frac{4}{5} \end{aligned}$$

4. For the utility function $U(F,C) = F^{\frac{3}{5}}C^{\frac{1}{3}}$.

(a) Is this utility function monotonic? (You should only consider $F > 0$ and $C > 0$).

$$\begin{aligned} MU_f &= \frac{3}{5}\frac{U}{F} > 0 \\ MU_c &= \frac{1}{3}\frac{U}{C} > 0 \end{aligned}$$

(b) Find another utility function that is ordinally equivalent to this one. You may use this utility function throughout the rest of the question. Be sure to show that the two utility functions are equivalent.

$$\begin{aligned} \ln(U) &= \frac{3}{5}\ln F + \frac{1}{3}\ln C \\ U^{\frac{1}{\frac{3}{5}+\frac{1}{3}}} &= F^{\frac{9}{14}} C^{\frac{5}{14}} \end{aligned}$$

both of these are positive monotonic transformations thus they represent the same preferences. To proove this we can merely point out that $\frac{d\ln(U)}{dU} = \frac{1}{U} > 0$ and $\frac{dU^{\frac{1}{\frac{3}{5}+\frac{1}{3}}}}{dU} = \frac{1}{\frac{3}{5}+\frac{1}{3}} U^{\frac{1}{\frac{3}{5}+\frac{1}{3}}-1} > 0$ *or we can compare the MRS's*

$$\begin{aligned} MRS_{original} &= \frac{\frac{3}{5}\frac{U}{F}}{\frac{1}{3}\frac{U}{C}} = \frac{9}{5}\frac{C}{F} \\ MRS_{\ln(U)} &= \frac{\frac{\frac{3}{5}}{F}}{\frac{\frac{1}{3}}{C}} = \frac{9}{5}\frac{C}{F} \\ MRS_{U^{\frac{1}{\frac{3}{5}+\frac{1}{3}}}} &= \frac{\frac{9}{14}\frac{U}{F}}{\frac{5}{14}\frac{U}{C}} = \frac{9}{5}\frac{C}{F} \end{aligned}$$

:

(c) Write down the Lagrangian function he will maximize.

$$\max_{F,C}\min_{\lambda} F^{\frac{3}{5}}C^{\frac{1}{3}} - \lambda\left(p_f F + p_c C - I\right)$$

(d) Find the first order conditions of his maximization problem.

$$\begin{aligned} \frac{3}{5}\frac{U}{F} - \lambda p_f &= 0 \\ \frac{1}{3}\frac{U}{C} - \lambda p_c &= 0 \\ -\left(p_f F + p_c C - I\right) &= 0 \end{aligned}$$

(e) Solve for the bang for the buck of Footballs and Candy. Equalize the bangs for the bucks and find a formula for C in terms of F and prices.

$$\begin{aligned}
\frac{1}{F}U\frac{\frac{3}{5}}{p_f} &= \lambda \\
\frac{1}{C}U\frac{\frac{1}{3}}{p_c} &= \lambda \\
\frac{1}{F}U\frac{\frac{3}{5}}{p_f} &= \frac{1}{C}U\frac{\frac{1}{3}}{p_c} \\
C &= \frac{5}{9}\frac{F}{p_c}p_f
\end{aligned}$$

:

(f) Find the demand for F.

$$\begin{aligned}
-\left(p_f F + p_c C - I\right) &= 0 \\
-\left(p_f F + p_c\left(\frac{F}{\frac{3}{5}}\frac{\frac{1}{3}}{p_c}p_f\right) - I\right) &= 0 \\
I - Fp_f - \frac{F}{\frac{3}{5}}\frac{1}{3}p_f &= 0 \\
F &= \frac{9}{14}\frac{I}{p_f}
\end{aligned}$$

(g) Find the demand for C.

$$\begin{aligned}
C &= \frac{F}{\frac{3}{5}}\frac{\frac{1}{3}}{p_c}p_f \\
&= \frac{\left(\frac{9}{14}\frac{I}{p_f}\right)}{\frac{3}{5}}\frac{\frac{1}{3}}{p_c}p_f \\
&= \frac{5}{14}\frac{I}{p_c}
\end{aligned}$$

$$\begin{aligned}
-\left(p_f F + p_c C - I\right) &= 0 \\
-\left(p_f\left(\frac{9}{14}\frac{I}{p_f}\right) + p_c C - I\right) &= 0 \\
\frac{5}{14}I - Cp_c &= 0 \\
C &= \frac{5}{14}\frac{I}{p_c}
\end{aligned}$$

:

5. Mark Aydin is a simple lad, all he really wants in life is Popcorn (C) and Olives (O). His parents (who do ensure that he eats a healthy diet)

have decided to give Mark Aydin an allowance of A to spend on his two favorite goods. The price of olives is p_o, the price of popcorn is p_c and Mark Aydin's utility function is $U(O,C) = 2C + 12\sqrt{O}$.

(a) Write down the Lagrangian function he will maximize.

$$L = 2C + 12\sqrt{O} - \lambda (p_o O + p_c C - A)$$

(b) Find the first order conditions of his maximization problem.

$$\begin{aligned} 0 &= \frac{6}{\sqrt{O}} - \lambda p_o \\ 0 &= 2 - \lambda p_c \\ 0 &= -(p_o O + p_c C - A) \end{aligned}$$

:

(c) Solve for the bang for the buck of Olvies and Popcorn.

$$\begin{aligned} \frac{1}{O^{\frac{1}{2}}} \frac{6}{p_o} &= \lambda \\ \frac{2}{p_c} &= \lambda \end{aligned}$$

(d) By equalizing the bang for the bucks find his demand for olives. *Note: You may assume that he consumes a strictly positive amount of both goods.*

$$\begin{aligned} \frac{1}{O^{\frac{1}{2}}} \frac{6}{p_o} &= \frac{2}{p_c} \\ \frac{1}{\frac{2}{p_c}} \frac{6}{p_o} &= O^{\frac{1}{2}} \\ 9\frac{p_c^2}{p_o^2} &= O \end{aligned}$$

: **Some of you wrote $9\frac{p_c^2}{p_o^2} = O$ this cost you two points here, but many people who made this mistake then got four points for the next part.**

(e) Find the demand for popcorn.

$$\begin{aligned} 0 &= -\left(p_o 9\frac{p_c^2}{p_o^2} + p_c C - A \right) \\ C &= \frac{A}{p_c} - 9\frac{p_c}{p_o} \end{aligned}$$

:

(f) Find conditions on p_o, p_c, and A such that Mark Aydin will choose not to consume any popcorn.

The next two parts, f and g, have been asked many times in the past. However they are not primary questions, so if you got the points good, if you did not it is just a sign you are not an A student.

formally this will occur whebn the $BfB_C \leq BFB_o$ *for any positive* C, *or when* $O = \frac{A}{p_o}$

$$\begin{aligned}
\frac{1}{O^{\frac{1}{2}}}\frac{6}{p_o} &\geq \frac{2}{p_c} \\
\frac{6}{2}\frac{p_c}{p_o} &\geq O^{\frac{1}{2}} \\
9\frac{p_c^2}{p_o^2} &\geq \frac{A}{p_o} \\
9p_c^2 p_o^{-2} &\geq \frac{A}{p_o} \\
9\frac{p_c^2}{p_o} &\geq A
\end{aligned}$$

You can also find this critical combination by solving for when the demand curve for C *is weakly negative.*

$$\begin{aligned}
\frac{A}{p_c} - 9\frac{p_c}{p_o} &\leq 0 \\
\frac{A}{p_c} &\leq 9\frac{p_c}{p_o} \\
A &\leq 9\frac{p_c^2}{p_o}
\end{aligned}$$

(g) What kind of solution to his maximization problem is it when he consumes none of one (or more) goods? Is this type of solution common or weird for real consumers? Explain.

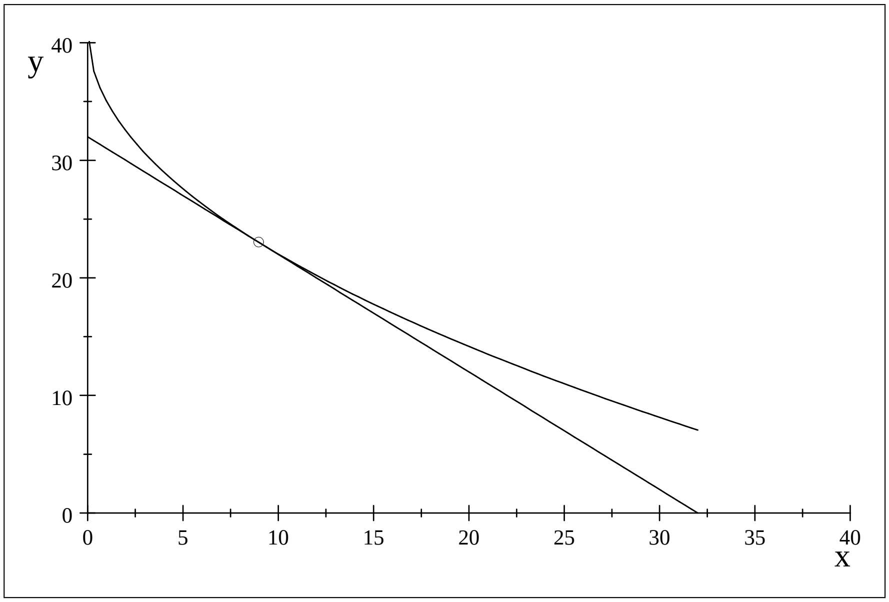
This is a **corner solution**, *this type of solution is perfectly normal for most real consumers. For example I am currently at a corner solution with regards to apartments in San Francisco. I want one, but I don't even have a fraction of one. There are simply too many goods for me to consume them all.*

(h) If $p_o = 3$ and $p_c = 3$ and $A = 96$, graph and clearly label his budget set, his optimal consumption, and an indifference curve through that consumption bundle below.

You should not have lost points on this part. You must be able to graphically draw utility maximization, and you should be able to clearly label it. It is possible that you

might be due more points on this part, but in order to be able to get them you really must have labeled your graph.

From the demand curves, $O = 9\frac{(3)^2}{(3)^2} = 9$, $C = \frac{96}{3} - 9\left(\frac{3}{3}\right) = 23$, $U = 2(23) + 12\sqrt{9} = 82$. *The indifference curve is* $C = \frac{82}{2} - 6\sqrt{O}$. *This results in the graph below.*



the circle is the optimal consumption, the budget set is the triangle $(32, 0), (0, 0)\,(0, 32)$ *and the indifference curve is the curved line drawn above the budget set.*

6. If you have Constant Elasticity of Substitution utility function with 3 goods (X, Y, and Z) the demand curves will have the form $X = \theta_x(p_x, p_y, p_z)\,I$, $Y = \theta_y(p_x, p_y, p_z)\,I$, and $Z = \theta_z(p_x, p_y, p_z)\,I$.

   (a) Find a general formula for the share of your income that you spend on the good X. Find a constraint on $\theta_x(p_x, p_y, p_z)$ using this formula.

$$\begin{aligned} s_X &= \frac{p_x X}{I} = \frac{p_x \theta_x I}{I} = \theta_x p_x \\ 0 &\leq \theta_x p_x \leq 1 \\ \theta_x &\leq \frac{1}{p_x} \end{aligned}$$

   (b) Find the income elasticity of X, is this good normal, inferior, or a luxury good? Explain your answer, if it can be more than one list all of the categories it fits into.

$$\begin{aligned}\frac{\partial X}{\partial I} &= \theta_x \\ e_x(I) &= \frac{\partial X}{\partial I}\frac{I}{X} = \theta_x \frac{I}{\theta_x I} = 1\end{aligned}$$

Thus it is either a normal or a luxury good since the elasticity is positive it is normal, since it is weakly greater than one it can be a luxury as well.

(c) Using the budget constraint find a constraint on $\theta_x(p_x, p_y, p_z)$, $\theta_y(p_x, p_y, p_z)$ and $\theta_z(p_x, p_y, p_z)$.

$$\begin{aligned}p_x X + p_y Y + p_z Z &= I \\ p_x \theta_x I + p_y \theta_y I + p_z \theta_z I &= I \\ p_x \theta_x + p_y \theta_y + p_z \theta_z &= 1\end{aligned}$$

(d) What is the most unrealistic aspect of Constant Elasticity of Substitution demand curves (compared to real demand curves)?

There are many ways to write down this answer. One excellent way is to point out the share of income that you spend on all goods is constant. The only thing that affects your demand is relative prices.

Another way to point it out is to point out that all goods can be considered luxuries, or that the income elasticity of all goods is one. This flies in the face of standard goods like public transport or bread which in general is an inferior good, or the income elasticity is negative. Even food in general is known to be normal but not a luxury good.

(e) Write down three common utility functions that are Constant Elasticity of Substitution Utility functions. You may answer either using verbal descriptions or by writing down sample Utility functions.

The three utility functions are Perfect Substitutes, Perfect Compliments (or Leontief), and the Cobb-Douglass. The general functional forms are;

$$\begin{aligned}U(F,C) &= \alpha F + \beta C \text{—Perfect Substitutes} \\ U(F,C) &= \min\{\alpha F, \beta C\} \text{—Perfect Compliments} \\ U(F,C) &= F^{\alpha} C^{\beta} \text{—Cobb-Douglass}\end{aligned}$$

7. Assume that your utility function is:

$$U(F,C) = -\frac{1}{4F} - \frac{1}{C}$$

where F is food, C is clothing, p_f and p_c are the prices of food and clothing respectively, and I is your income.

(a) Is this utility function monotonic? If it is what can you assume about the budget set? (The Budget Set is the $\{F, C\}$ such that $p_f F + p_c C \leq I$.)

$$\begin{aligned}\frac{\partial U}{\partial F} &= \frac{1}{4F^2} > 0 \\ \frac{\partial U}{\partial C} &= \frac{1}{C^2} > 0\end{aligned}$$

thus the utility function is monotonic, and this means that you will always consume on the budget constraint, or where $p_f F + p_c C = I$.

(b) Write down the Lagrangian function you will maximize.

$$\max_{F,C} \min_{\lambda} \frac{-1}{4F} + \frac{1}{C} - \lambda \left(p_f F + p_c C - I\right)$$

(c) Find the first order conditions of her maximization problem.

$$\begin{aligned}\frac{1}{4F^2} - \lambda p_f &= 0 \\ \frac{1}{C^2} - \lambda p_c &= 0 \\ p_f F + p_c C - I &= 0\end{aligned}$$

(d) Solve for the bang for the buck for Clothing and Food.

$$\begin{aligned}\lambda &= \frac{1}{C^2 p_c} \\ \lambda &= \frac{1}{4F^2 p_f}\end{aligned}$$

(e) Solve for the quantity of Food demanded in terms of Clothing and prices.

$$\begin{aligned}\frac{1}{4F^2 p_f} &= \frac{1}{C^2 p_c} \\ F &= \frac{1}{2}\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} C\end{aligned}$$

(f) Solve for the demand curve for Clothing.

$$\begin{aligned}p_f F + p_c C - I &= 0 \\ p_f \left(\frac{1}{2}\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} C\right) + p_c C &= I \\ p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} C + 2 p_c C &= I \\ \left(p_f^{\frac{1}{2}} + p_c^{\frac{1}{2}}\right) p_c^{\frac{1}{2}} C &= I \\ C &= \frac{p_c^{\frac{-1}{2}}}{p_f^{\frac{1}{2}} + 2 p_c^{\frac{1}{2}}} I\end{aligned}$$

(g) Solve for the demand curve for Food.

$$\begin{aligned} F &= \frac{1}{2}\left(\frac{p_c}{p_f}\right)^{\frac{1}{1-\rho}} C \\ &= \frac{1}{2}\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} \left(\frac{p_c^{\frac{-1}{2}}}{p_f^{\frac{1}{2}} + 2p_c^{\frac{1}{2}}} I\right) \\ &= \frac{p_f^{\frac{-1}{2}}}{p_f^{\frac{1}{2}} + 2p_c^{\frac{1}{2}}} \frac{I}{2} \end{aligned}$$

(h) What is the income elasticity of these goods? Are they normal, inferior, or luxury goods? (Hint: Guesses will get no credit but the answer should be a constant like 2 or $\frac{1}{3}$.)

Our demand function has the form:

$$X = \theta\left(p_x, p_y\right) I$$

where

$$\theta\left(p_x, p_y\right) \in \left\{ \frac{2p_c^{\frac{1}{-2}}}{p_f^{\frac{1}{2}} + 2p_c^{\frac{1}{2}}}, \frac{p_f^{\frac{1}{-2}}}{p_f^{\frac{1}{2}} + 2p_c^{\frac{1}{2}}} \right\}$$

and therefore

$$\frac{\partial X}{\partial I} = \frac{X}{I}$$

and

$$e_x\left(I\right) = \frac{X}{I}\frac{I}{X} = 1$$

thus these goods are either normal or luxury goods.

8. Assume that the utility function is: $U\left(F, C\right) = 2\sqrt{F} + 2C$ where F is food, C is clothing. Let p_f and p_c be the prices of food and clothing respectively, and I be the income.

   a Is this utility function monotonic? If it is what can you assume about the budget set? (The Budget Set is the $\{F, C\}$ such that $p_f F + p_c C \leq I$.)
   $\frac{\partial U}{\partial F} = \frac{1}{\sqrt{F}} > 0$, $\frac{\partial U}{\partial C} = 2 > 0$ so this utility function is monotonic and we can assume $p_f F + p_c C = I$.

   b Find the bang for the buck of food and clothing. Under what conditions will this person consume only food? What type of solution is this to her utility maximization problem?

$$
\begin{aligned}
BFB_f &= \frac{MU_f}{p_f} = \frac{1}{F^{\frac{1}{2}} p_f} \\
BFB_c &= \frac{MU_c}{p_c} = \frac{2}{p_c}
\end{aligned}
$$

this person will spend all of their income on food if

$$\frac{MU_c}{p_c} \leq \frac{MU_f}{p_f}$$

when $F = \frac{I}{p_f}$

$$
\begin{aligned}
\frac{2}{p_c} &< \frac{1}{\left(\frac{I}{p_f}\right)^{\frac{1}{2}} p_f} \\
\frac{2}{p_c} &< I^{-1+\frac{1}{2}} p_f^{-\frac{1}{2}} \\
p_f &\leq (4I)^{-1} p_c^2
\end{aligned}
$$

For the rest of the question you can assume that this consumer consumes a positive amount of both food and clothing.

c Write down the Lagrangian function she will maximize.

$$\max_{F,C} \min_{\lambda} 2F^{\frac{1}{2}} + 2C - \lambda \left(p_f F + p_c C - I\right)$$

d Find the first order conditions of her maximization problem.

$$
\begin{aligned}
F^{\frac{-1}{2}} - \lambda p_f &= 0 \\
2 - \lambda p_c &= 0 \\
p_f F + p_c C - I &= 0
\end{aligned}
$$

e Solve for the bang for the buck of Clothing and Food.

$$
\begin{aligned}
\lambda &= \frac{1}{p_f F^{\frac{1}{2}}} \\
\lambda &= \frac{2}{p_c}
\end{aligned}
$$

f By equalizing the bang for the bucks solve for the demand for Food.

$$
\begin{aligned}
\frac{1}{p_f F^{\frac{1}{2}}} &= \frac{2}{p_c} \\
F^{\frac{1}{2}} &= \frac{p_c}{2p_f} \\
F &= \left(\frac{p_c}{2p_f}\right)^2
\end{aligned}
$$

g Using the budget constraint demand for Clothing.

$$\begin{aligned} p_f F + p_c C - I &= 0 \\ p_f p_f^{-2}\left(\frac{p_c}{2p_f}\right)^2 + p_c C - I &= 0 \\ p_f^{-1}\left(\frac{p_c}{2}\right)^2 + p_c C - I &= 0 \\ C &= \frac{I - p_f^{-1}\left(\frac{p_c}{2}\right)^2}{p_c} \\ &= \frac{1}{p_c}I - \left(\frac{p_c}{p_f}\right)\left(\frac{1}{4}\right) \end{aligned}$$

h What is the derivative of these goods with respect to income? Are they normal or inferior?

$$\frac{\partial F}{\partial I} = 0$$

so food is either normal or inferior.

$$\frac{\partial C}{\partial I} = \frac{1}{p_c}$$

so it is normal.

9. Lale is a simple child, all she wants is Candy Bars (C) and French Fries (F). Her mom (who *does* make sure she eats a healthy diet) gives her I lira a week to buy whatever she wants. Assume her utility function is:

$$U(C, F) = -C^{-1} - F^{-1}$$

(a) Is Lale's utility function monotonic? Prove your answer.
Yes, $\frac{\partial U}{\partial C} = (-1)\left(-C^{-(2)}\right) = C^{-2} > 0$, $\frac{\partial U}{\partial F} = (-1)\left(-F^{-(2)}\right) = 1F^{-2} > 0$ *when* C *and* F *are strictly positive.*

(b) Write down the Lagrangian function she will maximize.

$$\max_{C,F}\min_{\lambda} -C^{-1} - F^{-1} + \lambda\left(I - p_f F - p_c C\right)$$

(c) Find the first order conditions of his maximization problem.

$$\begin{aligned} C^{-2} - \lambda p_c &= 0 \\ F^{-2} - \lambda p_f &= 0 \\ I - p_f F - p_c C &= 0 \end{aligned}$$

(d) Solve for the bang for the buck for candy bars and french fries, and equalize.

$$\begin{aligned} \lambda &= \frac{C^{-2}}{p_c} \\ \lambda &= \frac{F^{-2}}{p_f} \end{aligned}$$

$$\frac{F^{-2}}{p_f} = \frac{C^{-2}}{p_c}$$

(e) Solve for the quantity of French fries demanded in terms of Candy bars and prices.

$$F = C\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}}$$

(f) Solve for the demand curve for Candy bars by substituting the above term into the budget constraint.

$$\begin{aligned}
I - p_f F - p_c C &= 0 \\
I - p_f C\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} - p_c C &= 0 \\
C &= \frac{I}{p_f\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} + p_c} \\
&= \frac{I}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} + p_c}
\end{aligned}$$

(g) Solve for the demand curve for French fries.

$$\begin{aligned}
F &= C\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} \\
&= \frac{I}{p_f^{\frac{1}{2}} p_c^{\frac{1}{2}} + p_c}\left(\frac{p_c}{p_f}\right)^{\frac{1}{2}} \\
&= \frac{I}{p_f + p_c\left(\frac{p_f}{p_c}\right)^{\frac{1}{2}}} \\
&= \frac{I}{p_f + p_c^{\frac{1}{2}} p_f^{\frac{1}{2}}}
\end{aligned}$$

10. Assume that Ata Demirer (from Avrupa Yakasi) has decided to start an Extreme Aiken diet, where he eats nothing but kofte (K) and sis kebap (S). He doesn't really care about which he eats, just the total quantity so we can write his utility function as: $U(K, S) = \frac{3}{4}K + S$ where we multiply K by $\frac{3}{4}$ because each serving of kofte (K) is smaller than a serving of sis kebap (S).

    (a) Find the bang for the buck of K and S.

$$\frac{MU_k}{p_k} = \frac{\frac{3}{4}}{p_k}, \quad \frac{MU_s}{p_s} = \frac{1}{p_s}$$

(b) When will he consume no S? When will he consume no K? What kind of solution to his utility maximization problem is it when he consumes no K or no S?

No S if $\frac{MU_k}{p_k} > \frac{MU_s}{p_s}$ i.e. $\frac{\frac{3}{4}}{p_k} > \frac{1}{p_s}$ i.e. $\frac{3}{4}p_s > p_k$.

No K if $\frac{MU_k}{p_k} < \frac{MU_s}{p_s}$ i.e. $\frac{\frac{3}{4}}{p_k} < \frac{1}{p_s}$ i.e. $\frac{3}{4}p_s < p_k$

It is a corner solution.

(c) Find his demand functions for K and S. (Notice that little algebra is called for, given your answer to b you should be able to write them down and explain why they have the form the do.)

$$S(p_k, p_s, I) = \begin{cases} 0 & \text{if } \frac{3}{4}p_s > p_k \\ \rho\frac{I}{p_s} & \text{if } \frac{3}{4}p_s = p_k \\ \frac{I}{p_s} & \text{if } \frac{3}{4}p_s < p_k \end{cases}, K(p_k, p_s, I) = \begin{cases} \frac{I}{p_k} & \text{if } \frac{3}{4}p_s > p_k \\ (1-\rho)\frac{I}{p_k} & \text{if } \frac{3}{4}p_s = p_k \\ 0 & \text{if } \frac{3}{4}p_s < p_k \end{cases}$$

for any $\rho \in [0,1]$. To understand how I derived these equations, from part b I realize that if $\frac{3}{4}p_s < p_k$ Ata buys no Kofte, thus

$$\begin{aligned} p_s S + p_k K &= I \\ p_s S + p_k 0 &= I \\ S &= \frac{I}{p_s} \end{aligned}$$

and I can easily invert the argument for when $\frac{3}{4}p_s > p_k$. In the knife edge case, when $\frac{3}{4}p_s = p_k$, the BfB's are equal no matter what Ata consumes, so as long as they spend all of their income they are equally happy, thus we can split the consumption between the goods in any way we want.

11. Lale is a simple child, all she wants is Candy Bars (C) and French Fries (F). Her mom (who *does* make sure she eats a healthy diet) gives her I lira a week to buy whatever she wants. Assume her utility function is:

$$U(F,C) = F^2C^4$$

(a) What is another utility function that is ordinally equivalent to this utility function?

$$\begin{aligned} \tilde{U}(F,C) &= F^{\gamma}C^{1-\gamma} = F^{\frac{1}{3}}C^{\frac{2}{3}} \text{ since } \gamma = \frac{2}{2+4} = \frac{1}{3} \\ \tilde{\tilde{U}}(F,C) &= \frac{1}{3}\ln F + \frac{2}{3}\ln C \end{aligned}$$

(b) Write down the Lagrangian function she will maximize.

$$\max_{F,C}\min_{\lambda} \frac{1}{3}\ln F + \frac{2}{3}\ln C - \lambda(p_f F + p_c C - I)$$

(c) Find the first order conditions of her maximization problem.

$$\begin{aligned} \frac{1}{3F} - \lambda p_f &= 0 \\ \frac{2}{3C} - \lambda p_c &= 0 \\ p_f F + p_c C - I &= 0 \end{aligned}$$

(d) Solve for the bang for the buck for candy bars and french fries.
$\lambda = \frac{1}{3p_f F}$, $\lambda = \frac{1}{3p_f F}$, $\lambda = \frac{2}{3p_c C}$

(e) Solve for the quantity of French fries demanded in terms of Candy bars and prices.

$$\begin{aligned} \frac{1}{3p_f F} &= \frac{2}{3p_c C} \\ F &= \frac{1}{2}\frac{p_c}{p_f}C \end{aligned}$$

(f) Solve for the demand curve for Candy bars.

$$\begin{aligned} p_f F + p_c C - I &= 0 \\ p_f \left(\frac{1}{2}\frac{p_c}{p_f}C\right) + p_c C - I &= 0 \\ C &= \frac{2}{3}\frac{I}{p_c} \end{aligned}$$

(g) Solve for the demand curve for French fries.
There are two ways to do this, first

$$\begin{aligned} F &= \frac{1}{2}\frac{p_c}{p_f}C = \frac{1}{2}\frac{p_c}{p_f}\left(\frac{2}{3}\frac{I}{p_c}\right) = \frac{1}{3p_f}I \\ p_f F + p_c C - I &= 0 \\ p_f F + p_c\left(\frac{2}{3}\frac{I}{p_c}\right) - I &= 0 \\ F &= \frac{1}{3p_f}I \end{aligned}$$

12. Sam Longshanks' utility function for food and clothing is $U(F, C) = CF^2 + 2F^2$. He's a simple fellow so all he spends his income (I) on is food (F—which has a price of p_f) and clothing (C—which has a price of p_c). In this question always assume that Sam spends some money on both food and clothing.

(a) If you substitute in $X = C+2$ what standard type of utility function is the new function $V(F, X)$? You can use this to simplify your further analysis.

Then $V(F, X) = F^2 X$, and this is a Cobb-Douglass function. Thus the rest of the problem is easy.

(b) Set up Lagrangian

$$\max_{F,X=C+2} F^2 X - \lambda (p_f F + p_c C - I)$$
$$\max_{F,C} F^3 (C+2) - \lambda (p_f F + p_c C - I)$$

(c) Find the first order conditions for an optimum.

$$\begin{aligned} 2\frac{U}{F} - \lambda p_f &= 0 \\ \frac{U}{X} - \lambda p_c &= 0 \\ p_f F + p_c (X - 2) - I &= 0 \end{aligned}$$

(d) Using the first order conditions find a function of X (or C) in terms of F and prices.

$$\begin{aligned} \frac{U}{p_c X} &= 2\frac{U}{F p_f} \\ X &= F\frac{p_f}{2p_c} \end{aligned}$$

(e) Find the demand curve for F from the budget constraint.

$$p_f F + p_c \left(F\frac{p_f}{2p_c} - 2 \right) - I = 0$$

$$F = \frac{2}{3}\frac{I + p_c 2}{p_f}$$

(f) Find the demand curve for C.

$$\begin{aligned} C + 2 &= \left(\frac{2}{3}\frac{I + p_c 2}{p_f} \right) \frac{p_f}{2p_c} \\ C &= \frac{1}{3}\frac{I}{p_c} - \frac{2}{3} \end{aligned}$$

13. Consider a student who only consumes CDs (C) and Fast Food (F). Let the price of CDs be p_c and the price of Fast Food be p_f. Assume that their utility function is:

$$U(F, C) = 8 \ln F + C$$

(a) Set up the Lagrangian and find the first order conditions, where the income of the student is I.

ANSWER:

$$\begin{aligned} L(F,C,\lambda) &= 8\ln F + C - \lambda(p_f F + p_c C - I) \\ \frac{\partial L}{\partial F} &= \frac{8}{F} - \lambda p_f = 0 \\ \frac{\partial L}{\partial C} &= 1 - \lambda p_c = 0 \\ \frac{\partial L}{\partial \lambda} &= p_f F + p_c C - I = 0 \end{aligned}$$

(b) Assuming that she consumes both Fast Food and CDs solve for her demand curves.

i. By equalizing the bang for the buck's solve for F in terms of prices.

ANSWER:

$$\begin{aligned} \frac{8}{p_f F} &= \frac{1}{p_c} \\ F &= 8\frac{p_c}{p_f} \end{aligned}$$

ii. Using the budget constraint solve for the demand curve for C. Notice that you've already solved for the demand curve for F.

ANSWER:

$$\begin{aligned} p_f F + p_c C - I &= 0 \\ p_f\left(8\frac{p_c}{p_f}\right) + p_c C - I &= 0 \\ C &= \frac{I}{p_c} - 8 \end{aligned}$$

(c) Solve for the "Bang for the buck" for Fast Food and CDs. If $F = 1$, $C = 3$, $p_f = 1$, $p_c = 16$ what should she increase her consumption of? Why?

ANSWER:

$$\begin{aligned} \frac{8}{p_f F} &= \frac{8}{1*1} = 8 \\ \frac{1}{p_c} &= \frac{1}{16} \\ \frac{MU_f}{p_f} &> \frac{MU_c}{p_c} \\ 8 &> \frac{1}{16} \end{aligned}$$

so this student should consume more food

(d) Assume that $p_f = 1$.

i. How high does the price of CD's have to be before she will spend all of her money on Fast Food (the answer will be in terms of income, I and the price of CD's, p_c)?
ANSWER:
If this student spends all of her money on food, then $F = \frac{I}{p_f}$, what we want to know is when this is optimal or:

$$BFB_f \geq BFB_C$$

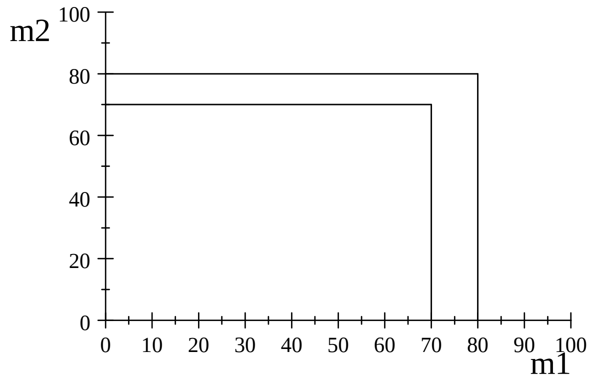
when $F = \frac{I}{p_f}$

$$\begin{aligned} \frac{8}{p_f F} &\geq \frac{1}{p_c} \\ \frac{8}{p_f \frac{I}{p_f}} &\geq \frac{1}{p_c} \\ \frac{8}{I} &\geq \frac{1}{p_c} \\ 8p_c &\geq I \end{aligned}$$

ii. Consuming all Fast food is what type of solution to her maximization problem?
ANSWER:
It is a corner solution

14. Professor Samaritan always gives two midterm exams in his Finance course. He only uses the higher of the two scores that a student gets on the midterms when he calculates the course grade.

(a) Kim Noodler wants to maximize her grade in this course. Let m_1 be her score on the first midterm and m_2 be her score on the second midterm. In the graph below with m_2 on the vertical axis and m_1 on the horizontal axis plot an indifference curve that passes through $m_1 = 10$ and $m_2 = 80$, and another that passes through $m_1 = 70$ and $m_2 = 60$. Which of these combination of scores would Kim prefer?
ANSWER:



Ind curves

Since her final grades m is given by $m = \max\{m_1, m_2\}$. Kim would prefer $m_1 = 10$ and $m_2 = 80$.

(b) Are Kim's preferences convex? What would be the best way to allocate her time between the two midterms?

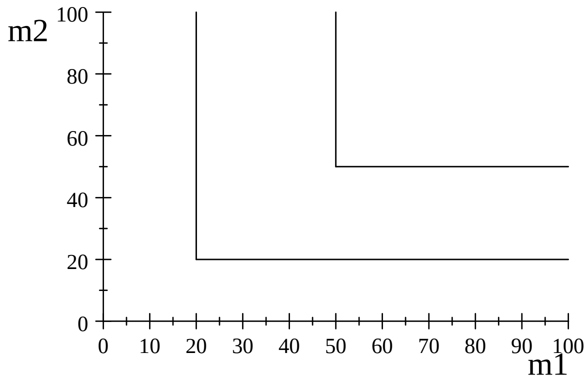
ANSWER:

Kim's preferences are not convex since the upper contour sets are not convex. She would spent all her time on only one of the exams and would not study for the other at all. Or $U(100,0) = U(0,100)$ *but* $U(50,50) = U\left(\frac{1}{2}100 + \frac{1}{2}0, \frac{1}{2}0 + \frac{1}{2}100\right) = 50 < U(100,0)$

It turns out that Kim is also taking an Economics of Law course from Professor Rasputin who also gives two midterms but—unlike Prof. Samaritan—she throws out the higher of the two grades when calculating the final score. Again, let m_1 be Kim's score on the first midterm and m_2 be her score on the second midterm.

(c) In the graph below plot an indifference curve for Kim that passes through m_1=20 and m_2=70 and another one that passes through m_1=60 and m_2=50. Which of these combinations of Economics scores would Kim prefer?

ANSWER:



Ind curves

Since her final grades m is given by $m = \min\{m_1, m_2\}$. Kim would prefer $m_1 = 60$ and $m_2 = 50$

(d) Are Kim's preferences convex?

ANSWER:

Yes as can be seen from the graph, but notice as well that $U(100, 0) = U(0, 100) = 0$ *but* $U(50, 50) = U\left(\frac{1}{2}100 + \frac{1}{2}0, \frac{1}{2}0 + \frac{1}{2}100\right) = 50 > U(100, 0)$

(e) I'm considering which of these two methods to use. Since all of the material in this class is important, I want to be sure that you guys understand everything to at least some degree. Which of these two methods would you recommend to me? Why?

ANSWER:

Choosing Rasputin's method would be more effective since it induces convex preferences, and this means that students study for both exams.

15. For this problem the consumers utility function is $U(F, C) = FC^2$, where income equals 6, and the price of food and clothing are both two.

(a) Find what the consumers will consume and graph her indifference curve through that consumption bundle.

ANSWER:

Let, $U_1(F, C) = \ln U(F, C) = \ln F + 2\ln C$.Note that since the objective function is strictly concave first order conditions are necessary and sufficient.

Then the problem is,

$$maxU_1(F, C) \qquad \text{s.t.} \qquad 2F + 2C = 6.$$

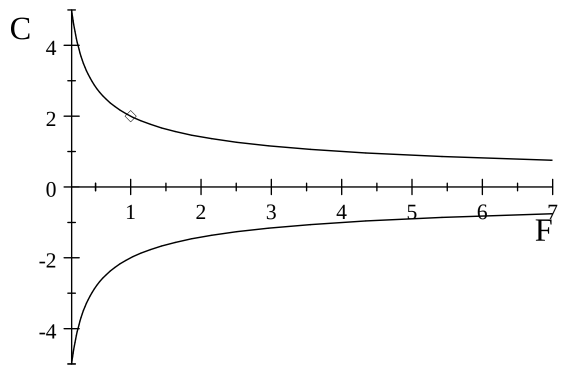
$L(F, C, \lambda) = \ln F + 2\ln C + \lambda(6 - 2F - 2C)$

FOC:

$$\begin{array}{rcl} \frac{\partial(L(F,C,\lambda))}{\partial F} & = & \frac{1}{F} - 2\lambda = 0 \\ \frac{\partial(L(F,C,\lambda))}{\partial C} & = & \frac{2}{C} - 2\lambda = 0 \\ \frac{\partial(L(F,C,\lambda))}{\partial \lambda} & = & 6 - 2F - 2C = 0 \end{array} ,$$

Solution is : $C = 2, F = 1$,

Now, since $U(1,2) = 4$, we should plot all the points on FxC plane such that $FC^2 = 4$. Which gives $F = \frac{4}{C^2}$;



(b) Now assume that the price of x falls to one, find her new consumption bundles and graph her new indifference curve.

ANSWER:

The new problem is,

i. $maxU_1(F, C)$ s.t. $F + 2C = 6$.

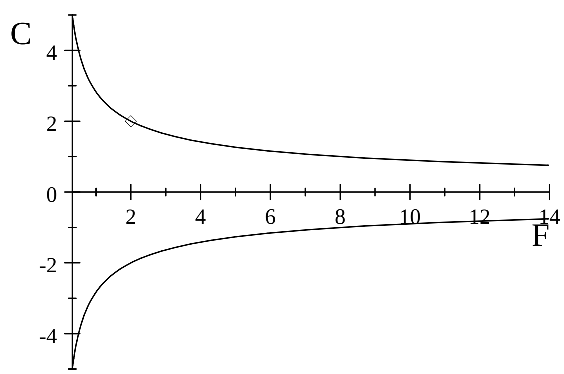
$L(F, C, \lambda) = \ln F + 2\ln C + \lambda(6 - F - 2C)$

FOC:

$$\begin{array}{rcl} \frac{\partial(L(F,C,\lambda))}{\partial F} & = & \frac{1}{F} - \lambda = 0 \\ \frac{\partial(L(F,C,\lambda))}{\partial C} & = & \frac{2}{C} - 2\lambda = 0 \\ \frac{\partial(L(F,C,\lambda))}{\partial \lambda} & = & 6 - F - 2C = 0 \end{array} ,$$

Solution is : $C = 2, F = 2$,

Now, since $U(2,2) = 8$, we should plot all the points on FxC plane such that $FC^2 = 8$. Which gives $F = \frac{8}{C^2}$;



ii. $maxU_1(F,C)$ s.t. $2F + C = 6.$

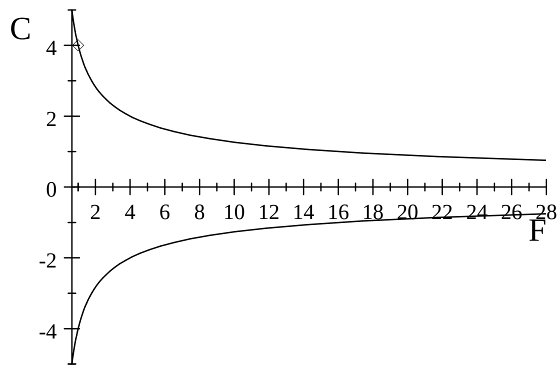
$L(F,C,\lambda) = \ln F + 2\ln C + \lambda(6 - 2F - C)$

FOC:

$$\begin{aligned}\frac{\partial(L(F,C,\lambda))}{\partial F} &= \frac{1}{F} - 2\lambda = 0\\ \frac{\partial(L(F,C,\lambda))}{\partial C} &= \frac{2}{C} - \lambda = 0\\ \frac{\partial(L(F,C,\lambda))}{\partial \lambda} &= 6 - 2F - C = 0\end{aligned}$$

Solution is : $C = 4, F = 1$

Now, since $U(1,4) = 16$, we should plot all the points on FxC plane such that $FC^2 = 16$. Which gives $F = \frac{16}{C^2}$;



(c) Find the point on the old indifference curve where the consumer would consume at the new prices.

ANSWER:

$$\begin{aligned}FC^2 &= 4\\ \frac{MU_f}{p_f} &= \frac{MU_c}{p_c}\end{aligned}$$

the second condition gives us the tangency.

$$\begin{aligned}MU_f &= \frac{U}{F}\\ MU_c &= 2\frac{U}{C}\end{aligned}$$

$$\begin{aligned}\frac{MU_f}{p_f} &= \frac{MU_c}{p_c}\\ \frac{\frac{U}{F}}{p_f} &= \frac{2\frac{U}{C}}{p_c}\\ F &= \frac{1}{2}C\frac{p_c}{p_f}\end{aligned}$$

Now addressing the two parts separately.

i. In the first problem the new price of clothing is 2 and the price of food is 1, so this is:

$$\begin{aligned} F &= C \\ FC^2 &= 4 \\ (C)\, C^2 &= 4 \end{aligned}$$

And the solution is $C = \sqrt[3]{4}$, (and we still wonder who wrote up this problem :-)) $F = C = \sqrt[3]{4}$

ii. In the second problem the new price of clothing is 1 and the price of food is 2, so this is:

$$\begin{aligned} F &= \frac{1}{4}C \\ FC^2 &= 4 \\ \left(\frac{1}{4}C\right) C^2 &= 4 \end{aligned}$$

And the solution is $C = 2\sqrt[3]{2}$, (gosh who wrote up this stinking problem :-)) $F = \frac{1}{4}C = \frac{1}{2}\sqrt[3]{2}$

(d) Indicate the Income and Substitution effects on a graph.
ANSWER:

i. The Substitution effect on F in the first problem is:

$$\begin{aligned} \frac{\Delta F}{\Delta\left(\frac{p_f}{p_c}\right)} &= F^I - F^o \\ &= \sqrt[3]{4} - 1 \\ &= .587\,4 \end{aligned}$$

The income effect is what is left, or

$$\begin{aligned} \frac{\Delta F}{\Delta RI} &= F^n - F^I \\ &= 2 - \sqrt[3]{4} \\ &= .412\,6 \end{aligned}$$

so we find that more of the change is due to the substitution effect than the income effect.

ii. The Substitution effect on C in the first problem is:

$$\begin{aligned} \frac{\Delta C}{\Delta\left(\frac{p_f}{p_c}\right)} &= C^I - C^o \\ &= 2\sqrt[3]{2} - 2 \\ &= .519\,84 \end{aligned}$$

The income effect is what is left, or

$$\begin{aligned}\frac{\Delta C}{\Delta RI} &= C^n - C^I \\ &= 4 - 2\sqrt[3]{2} \\ &= 1.4802\end{aligned}$$

4 Chapter 5—Income and Substitution Effects.

1. Define *Consumer Surplus*, you may use a graph or words. Why is it not a good measure of Consumer Welfare? What characteristics of a good make Consumer Surplus an acceptable measure of Consumer Welfare.

   Consumer Surplus *is the area underneath the Marshallian or Normal demand curve and above the price. It is not a good measure of consumer welfare because of the income effect. A good measure of consumer welfare is compensating variation, which is based on the area beneath the Hicksian or Income Compensated demand curve and above the price, and the two concepts are not the same.*

   Consumer surplus is an acceptable measure of consumer welfare if either the income elasticity is effectively zero or the share of your income spent on that good is nearly zero. For example I would be fine with consumer surplus when analyzing the market for gum (chicklet) but I would have serious problems with it in analyzing the market for housing.

2. Define a *normal* and a *luxury* good and give examples of both kinds of goods. Be sure that your definition makes it clear how to tell the two apart.

   A **normal** *good is a good for which* $0 \leq e_x(I) \leq 1$, *an example of a normal good is food, clothing, housing, most goods fall into this class.*

   A **luxury** *good is a good for which* $1 \leq e_x(I)$, *these are goods which you do not need to consume but you do as you get more wealthy. Examples are foreign vacations, diamonds, sports cars, etceteras.*

3. About the impact of income and prices.

   (a) Write down the Slutsky equation in elasticity form. Define all terms used.

   $$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x$$

   where $e_x(p_x)$ *is the elasticity of Marshallian demand with regards to price,* $e_{h_x}(p_x)$ *is the Hicksian elasticity with regards to price,* $e_x(I)$ *is the income elasticity of the good, and* s_x *is the share of income spent on that good.*

   (b) When a price changes what two effects does it have on demand? Define each effect and relate it to a term in the Slutsky equation.

   The two effects are:

i. *the substitution effect, the reaction of demand to the change in relative prices, this is* $e_{h_x}(p_x)$

ii. *the income effect, the reaction of demand to the change in real income caused by the change in price, this is* $-e_x(I)\, s_x$

(c) If we have a normal good, which will cause a bigger increase in the demand for that good?

i. A 1% increase in the income.

ii. A 1% decrease in the price.

Explain your answer. Note: Clearly a precise answer will depend on the elasticities, I am looking for a general guideline. A wrong answer carefully argued can get almost full credit.

With normal goods the income and substitution effect work together, so a 1% decrease in price is going to have a larger impact. Of course a precise answer depends on the elasticities, with the 1% decrease in price the effect is $-e_{h_x}(p_x) + e_x(I)\, s_x$ *with a 1% increase in income the effect is* $e_x(I)$ *so the question is which is bigger.*

$$
\begin{aligned}
-e_{h_x}(p_x) + e_x(I)\, s_x &> e_x(I) \\
-e_{h_x}(p_x) &> (1 - s_x)\, e_x(I)
\end{aligned}
$$

so under this condition the general insight will be correct. Notice I have written this condition so that the terms on both sides are positive.

(d) If we have an inferior good, which will cause a bigger increase in the demand for that good?

i. A 1% decrease in the income.

ii. A 1% decrease in the price.

Explain your answer. Note: Clearly a precise answer will depend on the elasticities, I am looking for a general guideline. A wrong answer carefully argued can get almost full credit.

With normal goods the income and substitution effect work against each other, so a 1% decrease in income will probably have a larger effect. Of course a precise answer depends on the elasticities, with the 1% decrease in price the effect is $-e_{h_x}(p_x) + e_x(I)\, s_x$ *with a 1% decrease in income the effect is* $-e_x(I)$ *so the question is which is bigger.*

$$
\begin{aligned}
-e_{h_x}(p_x) + e_x(I)\, s_x &< -e_x(I) \\
-e_{h_x}(p_x) &> -(1 + s_x)\, e_x(I)
\end{aligned}
$$

so under this condition the general insight will be correct. Notice I have written this condition so that the terms on both sides are positive.

4. A scientist in Moldavia has just invented *thneed*. A thneed's a fine something that everyone needs. It's a shirt. It's a sock. It's a glove. It's a hat. But it has other uses. Yes far beyond that. It can be used to build buildings. Made into a car. You can even eat it, with it you will go far.

   Unfortunately, for practically every use it's been put to (and it can do almost everything) it has always been worse than the original. As food it is rather nasty tasting, even though it's very good for you—you literally don't need to eat anything else. Houses built with thneed invariably leak, and the cars tend to need repair a lot. For clothing not only is it scratchy but it has a very ugly yellow-green color. Basically you have to be desperate to use thneed, on the other hand it is cheap—it sells for 10 kurus a kilo.

   (a) Based on the paragraph above, is thneed a normal good, a luxury good, or an inferior good? Explain your answer.

   It is described as a substitute—and an inferior one—for almost everything. One that you have to be desperate to use. Thus it must be an **inferior good**.

   (b) Write down the slutsky equation in elasticity form.

   $$e_T(p_t) = e_{h_T}(p_t) - e_T(I)\, s_T$$

   where $e_T(p_t)$ *is the price elasticity of the normal or marshallian demand curve,* $e_{h_T}(p_t)$ *is the price elasticity of the hicksian or income compensated demand curve,* $e_T(I)$ *is the elasticity of the good with regards to income, and* s_T *is the share of my income spent on thneeds,*

   (c) I argue that of any good ever described, thneed is the most likely to have an upward sloping demand curve. Using the slutsky equation explain my reasoning.

   From the description it is clear that we can have $s_T\tilde{}1$ *for thneeds, and also that* $e_T(I)$ *is large and negative. Thus* $-e_T(I)\, s_T$ *is as large and positive as possible for any good, and this gives us the best chance of* $e_T(p_t)$ *being large and positive.*

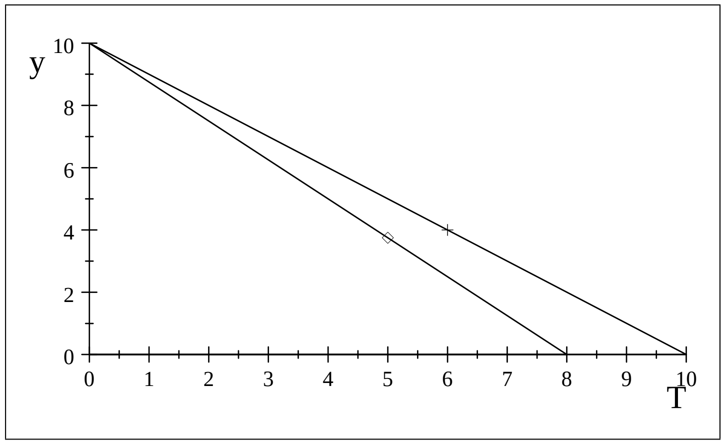
   (d) Define Consumer surplus, would consumer surplus be a good measure of consumer welfare for thneed? Why or why not.

   Consumer surplus is the area above the price and below the demand curve. No it would not be a good measure of consumer welfare because it only is if $e_T(I)\, s_T$ *is nearly zero, as argued above this will be very large so we should not use Consumer Surplus to measure welfare in the market for thneeds.*

   (e) *(9 points total)* The Turkish government wants to increase the consumption of thneed because of its health benefits. They are considering three different programs, for each program indicate which program will increase the consumption of thneed (if there is a choice) and using a graph show why you think this. You may assume that demand is downward sloping in this section.

i. A price subsidy or tax.

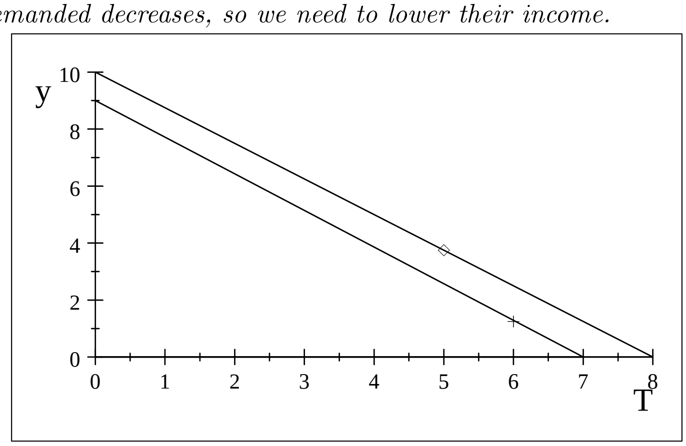
Here a price subsidy is called for. As illustrated in the graph this will increase the demand for thneeds since demand is downward sloping, indicating that the substitution effect outweighs the income effect.



in this graph consumption would change from some point like the diamond to the cross.

ii. An income subsidy or tax.

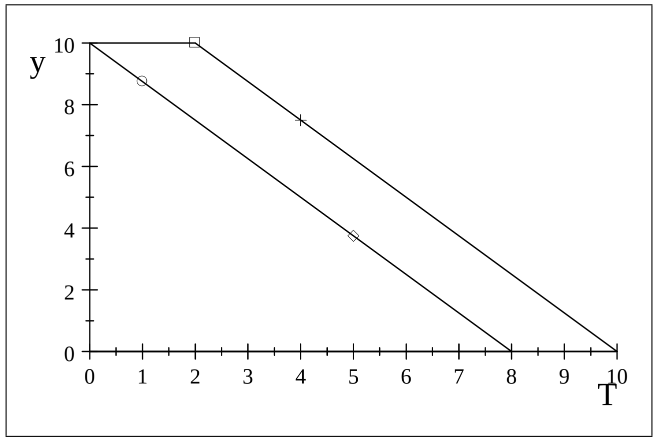
Since thneeds are an inferior good an Income tax is called for in this case. With an inferior good if income increases the quantity demanded decreases, so we need to lower their income.



in this graph consumption would change from some point like the diamond to the cross.

iii. Giving X units of thneed to each consumer. In this part indicate what size X should be to increase consumption of the average consumer.

In this case the reaction depends on whether the consumer was consuming more or less than X units of thneed beforehand. If they were consuming less—like the circle in the graph below—they *would start consuming exactly X units of thneed (the square). If they were consuming more (like the diamond) then the impact will be just like an increase in income, so they would consume less (the cross). If this is going to be effective X has to be more than the average consumption of thneed.*



(f) If the demand for thneed is upward sloping how would it change your answers to the last part of the question, if at all.

Then in part e.i. we would need a tax on thneed, this would increase the price and increase the consumption of thneed. Other than that all results would be the same.

(g) Which author in what book first invented thneed?

Just for your information Dr. Seuss invented thneed in "The Lorax." If you know an English speaking kid you should read it to him or her some time. His version of thneed is not as useful, quite, as mine.

5. About the Slutsky equation.

(a) Write down the Slutsky equation in elasticity or derivative form.

$$\begin{aligned} \frac{\partial X}{\partial p_x} &= \frac{\partial h_x}{\partial p_x} - \frac{\partial X}{\partial I} X \\ e_X(p_x) &= e_{h_X}(p_x) - e_X(I)\, s_X \end{aligned}$$

(b) Define $I(p_x, p_y, U_0) = \min_{X,Y} p_x X + p_y Y - \lambda (U(X,Y) - U_0)$, $h_x(p_x, p_y, U_0)$ as the Hicksian or Income Compensated demand for X, and $X(p_x, p_y, I)$ as the normal or Marshallian demand for X. Given the identity $X(p_x, p_y, I(p_x, p_y, U_0)) = h_x(p_x, p_y, U_0)$ derive the Slutsky equation in elasticity form.

$$\begin{aligned} X(p_x, p_y, I(p_x, p_y, U_0)) &= h_x(p_x, p_y, U_0) \\ \frac{\partial X}{\partial p_x} + \frac{\partial X}{\partial I}\frac{\partial I}{\partial p_x} &= \frac{\partial h_x}{\partial p_x} \end{aligned}$$

by the envelop theorem $\frac{\partial I}{\partial p_x} = X$.

$$\begin{aligned}
\frac{\partial X}{\partial p_x} &= \frac{\partial h_x}{\partial p_x} - \frac{\partial X}{\partial I} X \\
\frac{\partial X}{\partial p_x}\frac{p_x}{X} &= \frac{\partial h_x}{\partial p_x}\frac{p_x}{X} - \frac{\partial X}{\partial I} X \frac{p_x}{X} \\
e_X(p_x) &= e_{h_X}(p_x) - \frac{\partial X}{\partial I}\frac{1}{X} p_x X \frac{I}{I} \\
e_X(p_x) &= e_{h_X}(p_x) - \frac{\partial X}{\partial I}\frac{I}{X}\frac{p_x X}{I} \\
e_X(p_x) &= e_{h_X}(p_x) - e_X(I)\, s_X
\end{aligned}$$

(c) When a price changes what two effects does it have on demand? Define each effect and relate it to a term in the Slutsky equation.

i. The substitution effect, the change in demand in reaction to the change in the relative prices of X and the other goods. This is $e_{h_X}(p_x)$.

ii. The income effect, the change in demand in reaction to the change in the real income. This is $-e_X(I)\, s_X$.

(d) Illustrate the two effects with three graphs, one of the total effect and one of each effect in isolation. Which graph corresponds to which term in the Slutsky equation?

6. The price of gasoline has increased from $p_g^o = 1$ to $p_g^n = 1 + 1$. In order to lessen the impact on the citizens, the government is planning on giving out an income subsidy $S_g = (8)$. Assume that the income of all consumers is $I = (32)$ and that the price of all other goods (Y) is one. There are 3 consumers in this society, their consumption of gasoline (G) before and after the changes are listed below:

	G_0	G_n
Ahmet (A)	$\frac{3}{4}(8)$	$\frac{1}{2}(8)$
Baris (B)	$2(8)$	$\frac{3}{4}(8)$
Cansu (C)	$\frac{7}{4}(8)$	$\frac{3}{2}(8)$

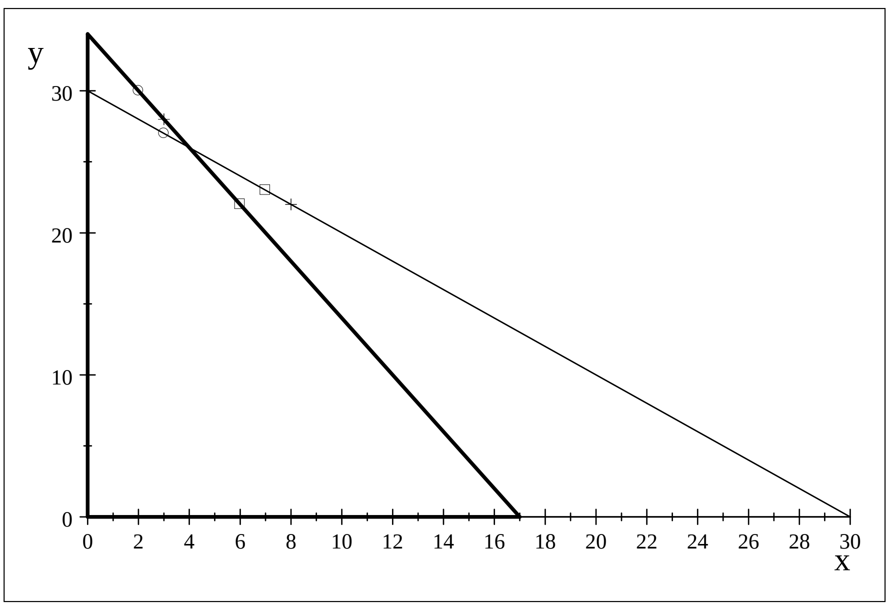
1	(32)	Σ	A_o	A_n	B_o	B_n	C_o	C_n	$\frac{I}{p_g^o}$	$\frac{I+S_g}{p_g^n}$
1	30	4	3	2	8	3	7	6	30	17
1	32	8	6	4	16	6	14	12	32	20
2	60	12	9	6	24	9	21	18	30	24
2	54	12	9	6	24	9	21	18	27	22
3	60	8	6	4	16	6	14	12	20	17
3	60	4	3	2	8	3	7	6	20	16
	$> 81\gamma$	4γ	3γ	2γ	8γ	3γ	7γ	6γ	$\frac{(32)}{\lambda}$	$\frac{(32)+4\gamma}{\lambda+1}$

assume that all of these consumers have monotonic preferences.

(a) In the graph below draw the old and new budget sets and the old and new consumption points of all consumers. Label all critical points. Label the consumption bundles by the first letter of the person's name and an o (old consumption) or n (new consumption) subscript. To be clear, for Cansu you should label the points C_o (her old consumption) and her new consumption C_n, Ahmet A_o and A_n and Baris B_o and B_n for his new consumption.

Your graph does not need to be too precise, but I expect it to look approximately correct.

To answer this using words: The old budget set is the triangle defined by the points:$(0, I)\,(0, 0)\,(I, 0)$ *the new budget set is the triangle defined by:* $(0, I + S_g)\,(0, 0)\left(\frac{I+S_g}{1+1}, 0\right)$ *for each of the consumers above:*$A_0 = \left(\frac{3}{4}(8), I - \frac{3}{4}(8)\right), A_n = \left(\frac{1}{2}(8), I + (8) - (1+1)\frac{1}{2}(8)\right)$, $B_0 = (2(8), I - 2(8)), B_n = \left(\frac{3}{4}(8), I + (8) - (1+1)\frac{3}{4}(8)\right)$, $C_0 = \left(\frac{7}{4}(8), I - \frac{7}{4}(8)\right), C_n = \left(\frac{3}{2}(8), I + (8) - (1+1)\frac{3}{2}(8)\right)$. *For the graph below I will assume that the various values are* $\lambda = 1$, $I = 30$, $S_g = 4$, *A's two consumption points are the circles to the upper left. B's consumption points are the crosses, one to the left and one to the right of the point where the budget constraints cross. C's consumption points are the boxes to the right of the point where the budget constraints cross.*



(b) How was the fact that preferences were monotonic critical to answering the last question correctly?

Because preferences are monotonic one knows that the consumer will spend all of their income, thus one is able to figure out what Y is only *from information about G. Otherwise this would not be possible.*

(c) Which of these consumers can you be sure will be better off after the change? Which will be worse off? Which can you not tell? In each case explain your reasoning, points will not be awarded for correct guesses.

Notice that I changed the labeling of the points on most of the exams, so you will not, for example, necessarily find that Ahmet is better off, you should find that one person is, and your reasoning should be as is given below (though you can make all of these arguments graphically)

They will be better off if after the change they can still afford the consumption bundle they consumed before:

$$\begin{aligned} A &: 12 + I - 6 = 6 + I < 8 + I \\ B &: 32 + I - 16 = 16 + I > 8 + I \\ C &: 28 + I - 14 = 14 + I > 8 + I \end{aligned}$$

so Ahmet will be better off.

They will be worse off if before the change they could buy the consumption bundle they consume after the change (reversing the reasoning above.)

$$\begin{aligned} A &: 4 + I + 8 - 8 = 4 + I > I \\ B &: 6 + I + 8 - 12 = 2 + I > I \\ C &: 12 + I + 8 - 24 = I - 4 < I \end{aligned}$$

so Cansu will be worse off. Notice that Cansu's old consumption also monitonically dominates her new consumption.

For Baris you can not tell whether she is better or worse off. Since neither consumption point is in the other budget set you are free to draw an indifference curve that makes the after consumption higher, and another that makes the before consumption higher.

You can actually answer more easily by looking at the graph. There it is clear that Cansu's consumption is monotonically dominated, and which person's consumption bundles are in the old or the new budget set.

(d) *(6 points total)* The government notices that more than half their citizens are worse off after the price increase and the subsidy.

i. Given the information above, they ask you to find the lowest subsidy that will guarentee at least half the citizens will be better off.

In order to do this we need to make sure that at least two of the three people can continue to consume their old consumption *bundle, so we want to find the minimum* Γ *such that:*

$$\begin{aligned} A &: \ 12 + I - 6 = 6 + I \leq \Gamma + I \\ B &: \ 32 + I - 16 = 16 + I \leq \Gamma + I \\ C &: \ 28 + I - 14 = 14 + I \leq \Gamma + I \end{aligned}$$

for at least two of the people, This clearly is $\Gamma = 14$

ii. What more information could you use to decrease the amount of the subsidy? How would you use it?

If we new their precise utility functions we could probably decrease the amount of the subsidy. For example Baris may already be happier after the change, and thus Γ *at most would need to be* (8). *If he is not then perhaps increase the subsidy by less than* $\frac{3}{4}(8)$ *would make either Baris or Cansu better off.*

7. About defining and using the slustky equation.

(a) Write down the Slutsky equation in elasticity or derivative form.

$$\begin{aligned} e_x(p_x) &= e_{h_x}(p_x) - e_x(I)\, s_x \\ \frac{\partial x}{\partial p_x} &= \frac{\partial h_x}{\partial p_x} - \frac{\partial x}{\partial I} x \end{aligned}$$

(b) Explain how this equation indicates the two effects of a change in price on the demand for a good. Describe each effect and explain what terms in the equation captures each effect.

The two effects are the income and the subsidy effect.

T*he subsidy effect is the impact of the change in relative price on demand, this is represented by* $e_{h_x}(p_x)$ *because this is the elasticity of hicksian demand. For hicksian or compensated demand your income is increased or decreased so that income does not have any effect on your demand, thus it has to represent a pure price effect.*

The income effect is $-e_x(I)\, s_x$*, this is the impact of the change in real income on your demand. It is negative because when a price increases real income falls,* $e_x(I)$ *represents how much that change in income will affect demand, and* s_x *indicates the magnitude of the change in price on the real income of the consumer.*

(c) Explain how this equation captures the difference between net and gross substitutes. Be sure to clearly explain the difference between the net and gross concepts.

Parts c and d are for better students, do not be totally upset if you didn't get these points. Points for part d were basically given for correct work towards an answer since so few students did well.

Net substitutes (or compliments) is about what you want irregardless of what you can afford, thus it is a function of hicksian demand, or $e_{h_x}(p_y)$.

Gross substitues (or compliments) is about what you want and can afford, so it is a function of marshallian demand elasticity, $e_x(p_y)$.

This equation show us that $e_x(p_y) = e_{h_x}(p_y) - e_x(I)\, s_y$, *thus telling us how the gross and net concepts are related.*

(d) Using this equation explain how X can be a gross substitute of Y while Y is a gross compliment of X.

We need $e_x(p_y) = e_{h_x}(p_y) - e_x(I)\, s_y > 0$ *and* $e_y(p_x) = e_{h_y}(p_x) - e_y(I)\, s_x < 0$ *the sign of* $e_{h_y}(p_x)$ *and* $e_{h_x}(p_y)$ *must be the same (they are either net compliments or substitutes) but to make* $e_x(p_y) > 0$ *we merely need* $e_x(I)\, s_y$ *to be sufficiently negative, likewise if* $e_y(p_x) < 0$ *then we need* $e_y(I)\, s_x$ *to be sufficiently positive.*

8. In this question I want you to illustrate graphically the two effects of an increase in price on the quantity demanded. Throughout this question assume $U(X,Y) = XY$. With this utility function the demand curves are $X = \frac{1}{2}\frac{I}{p_x}$ and $Y = \frac{1}{2}\frac{I}{p_y}$ and the bang for the buck of X is $\frac{U}{p_x X}$, and for Y is $\frac{U}{p_y Y}$.

(a) What are these two effects? Define each effect. What do we know about the sign of these effects? If we can't be sure about the sign of one of the effects when will it be positive and when will it be negative?

The two effects are the:

i. *Substitution effect—the pure impact of relative prices, if the price of a good increases you substitute to other goods. It is always negative.*

ii. *Income effect—the impact of income on consumption. When a price increases it decreases your real income, so this effect captures that. It is negative if the good is normal or a luxury, positive if the good is inferior.*

(b) In a graph draw a budget set when income (I) is 48, the price of X (p_x) is 2 and the price of Y (p_y) is 12. Also graph the budget set when p_x increases to (p_x^n) 8 (I and p_y are the same as before). Indicate the old and new consumptions of X in the graph, label the old consumption X_o and the new consumption X_n.

The corners of the graphs are $\left\{\frac{I}{p_x}, 0\right\}$ $\left\{0, \frac{I}{p_y}\right\}$, *where* p_x *increases from* p_x^o *to* p_x^n $X_0 = \frac{1}{2}\frac{I}{p_x^o}$, $X_n = \frac{1}{2}\frac{I}{p_x^n}$. *See the graph below.*

(c) What will utility be at the old consumption? Draw this indifference curve in a graph

$$
\begin{aligned}
U(X,Y) &= XY \\
&= \left(\frac{1}{2}\frac{I}{p_x}\right)\left(\frac{1}{2}\frac{I}{p_y}\right) \\
&= \frac{1}{4}\frac{I^2}{p_x p_y}
\end{aligned}
$$

the formula for the indifference curve is:

$$
\begin{aligned}
XY &= \frac{1}{4}\frac{I^2}{p_x p_y} \\
Y(X) &= \frac{1}{4}\frac{I^2}{p_y p_x}\frac{1}{X}
\end{aligned}
$$

(d) How much X and Y would this person consume if they were on the old indifference curve but faced the new prices (when $p_y = q = 12$, $p_x = p_x^n = 8$)? Indicate the level of X consumed on a graph, denote it X_i.

You can answer this question either mathematically or using the graph, but for full credit your answer must be exactly correct. The answer will be an integer.

I, of course, will use math. We have:

$$
\begin{aligned}
XY &= \frac{1}{2^2}\frac{I^2}{p_x p_y} \\
\frac{U}{p_x^n X} &= \frac{U}{p_y Y} \\
Y &= p_x^n \frac{X}{p_y} \\
X\left(q\frac{X}{p_y}\right) &= \frac{1}{4}\frac{I^2}{p_x p_y} \\
X_i &= \frac{1}{2}\frac{1}{(p_x^o)^{\frac{1}{2}}(p_x^n)^{\frac{1}{2}}} I
\end{aligned}
$$

and this also tells me that

$$
\begin{aligned}
Y_i &= p_x^n \frac{1}{p_y}\left(\frac{1}{2}\frac{1}{(p_x^o)^{\frac{1}{2}}(p_x^n)^{\frac{1}{2}}} I\right) \\
&= \frac{1}{2}\left(\frac{p_x^n}{p_x^o}\right)^{\frac{1}{2}}\frac{I}{p_y}
\end{aligned}
$$

so

$$\begin{aligned} I_i &= p_x^n\left(\frac{1}{2}\frac{1}{(p_x^o)^{\frac{1}{2}}(p_x^n)^{\frac{1}{2}}}I\right)+p_y\left(\frac{1}{2}\left(\frac{p_x^n}{p_x^o}\right)^{\frac{1}{2}}\frac{I}{p_y}\right) \\ & q\left(\frac{1}{2}\frac{1}{(p)^{\frac{1}{2}}(q)^{\frac{1}{2}}}I\right)+p_y\left(\frac{1}{2}\left(\frac{q}{p}\right)^{\frac{1}{2}}\frac{I}{p_y}\right) \\ &= \left(\frac{p_x^n}{p_x^o}\right)^{\frac{1}{2}} I \end{aligned}$$

(e) Is this good normal or inferior? You can answer this question either using math or the graph. If you answer it using math you must show how the graph is consistent with your answer.

$$\frac{\partial X}{\partial I}=\frac{1}{2}\frac{1}{p_x}>0$$

so this good is normal. You can see this by comparing X_i and X_n below. Since income has decreased and X is normal $X_i > X_n$.

(f) Using the three points $\{X_o, X_n, X_i\}$ identify the two effects of an increase in p_x on the demand for X. Explain why these differences represent the two effects.

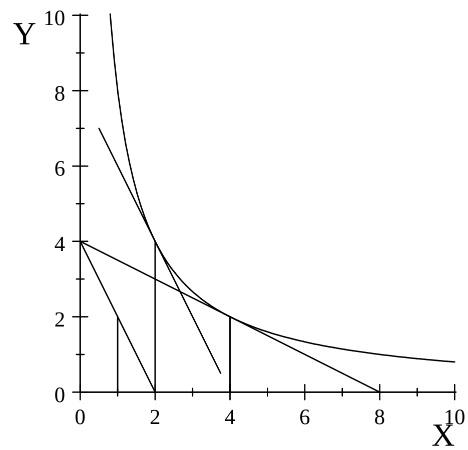
The total effect is $X_n - X_o$ the substitution effect is $X_i - X_o$ because these two points are on the same indifference curve, so the only impact is the reaction to prices. The income effect is $X_n - X_i$ because the only change that has been made between these two points is that income has fallen.

(g) Write down the Slutsky equation in either derivative or elasticity form—defining each term. Discuss how this captures the two effects of an increase in p_x on the demand for X, relating your answers to your answer in part f.

$$\begin{aligned} \frac{\partial X}{\partial p_x} &= \frac{\partial h_x}{\partial p_x}-\frac{\partial X}{\partial I}X \\ e_x(p_x) &= e_{h_x}(p_x)-e_x(I)\,s_x \end{aligned}$$

Where $\frac{\partial X}{\partial p_x}$ is the derivative of the marshallian demand curve with regards to price, or the total effect, and $e_x(p_x)=\frac{\partial X}{\partial p_x}\frac{p_x}{X}$. The second term is the substitution effect, the derivative of the hicksian demand curve with respect to price, $\frac{\partial h_x}{\partial p_x}$ and $e_{h_x}(p_x)=\frac{\partial h_x}{\partial p_x}\frac{p_x}{X}$. This is $X_i - X_o$ because hicksian demand curves are derived holding the indifference curve constant. The third term $-\frac{\partial X}{\partial I}X$ is the income effect, and $e_x(I)=\frac{\partial X}{\partial I}\frac{p_x}{X}$, $s_x=\frac{p_x X}{I}$, this is equivalent to $X_n - X_i$ because it is how the change in your income changes demand.

And here is the graph:



9. In this question I want you to graphically represent the income and the substitution effect. In this problem the price of X will increase from p_x^o to p_x^n, the price of Y will be unchanged at p_y and the income will be I. In the graph on page 48 the indifference curve is the level of utility this person gets at (p_x^o, p_y, I).

   (a) In the graph on page 48 draw the old budget constraint. Be sure to label the points where it touches the axes using (p_x^o, p_y, I). Also try to make it tangent to the indifference curve at $X = 4$ $Y = 8$. The old consumption of X, X_o, will be 4.

   I can't label the points on the graph, so I will do it here. The corners of the budget constraint are $(0, 0)$, $\left(0, \frac{I}{p_x^o}\right)$, and $\left(0, \frac{I}{p_y}\right)$

   (b) In the graph on page 48 draw the new budget constraint. Be sure to label the points where it touches the axes using (p_x^n, p_y, I).

   I can't label the points on the graph, so I will do it here. The corners of the budget constraint are $(0, 0)$, $\left(0, \frac{I}{p_x^n}\right)$, and $\left(0, \frac{I}{p_y}\right)$.

   (c) Now in the graph on page 48 draw an intermediate budget constraint where the prices are (p_x^n, p_y) but the income is high enough that the budget constraint just touches the indifference curve. Label the intermediate level of consumption of X you find X_i.

   This is the dashed budget constraint in the graph. With this graph $X_i = 3$.

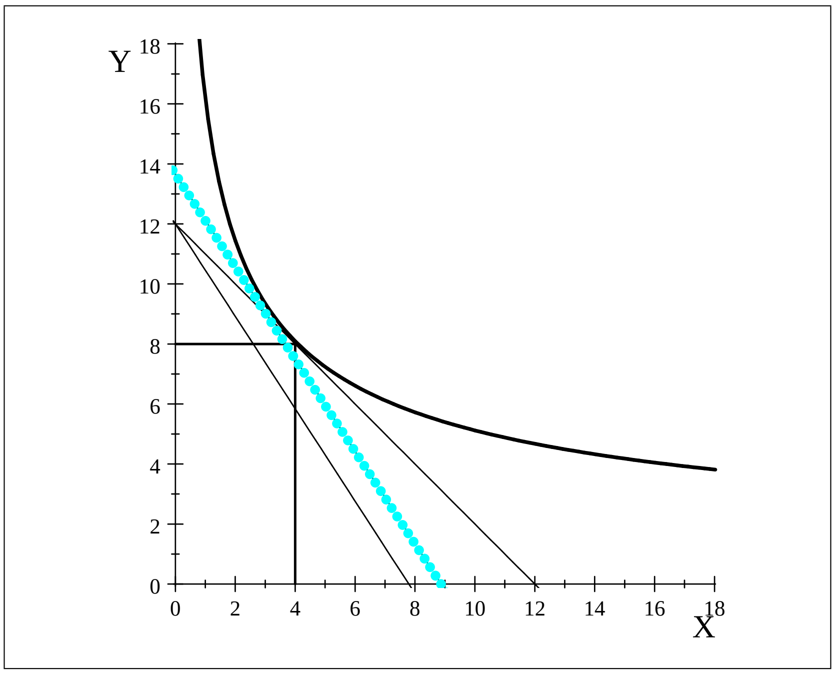
   (d) What is the relationship between X_o and X_i? Is $X_i \geq X_o$ or $X_i \leq X_o$? Is this just a coincidence or is this always true? Why? Is $X_i - X_o$ the income or the substitution effect?

   We know that $X_i \leq X_o$ *because this is the substitution effect, which is always negative.*

(e) If X is an inferior good what is the relationship between the final consumption (X_n) and X_i? Is $X_i \geq X_n$ or $X_i \leq X_n$? Is $X_n - X_i$ the income or the substitution effect?

If it is an inferior good $X_i \leq X_n$ because when we go from the intermediate budget constraint to the final one income falls, so the quantity demanded of inferior goods increases.

(f) If X is a normal or luxury good what is the relationship between the final consumption (X_n) and X_i? Is $X_i \geq X_n$ or $X_i \leq X_n$?*If it is a normal good $X_i \geq X_n$ because when we go from the intermediate budget constraint to the final one income falls, so the quantity demanded of normal goods decrease.*



10. In this question I want you to compare Consumer Surplus and Compensating Variation. Compensating Variation is the area between the Hicksian (or *compensated*) demand curve and the vertical axis between the old and new price.

(a) Define Compensating Variation in more intuitive terms. In other words explain the concept Compensating Variation captures that makes it a welfare concept.

Compensating variation is the amount of money we need to pay the consumers if they are going to be equally happy before and after the price change.

(b) At the old price what relationship will there be between Marshallian (or standard) demand for X and the Hicksian demand for X. Why is this true?

The Hicksian and Marshallian demand will be equal, since the Hicksian demand will be for the level of utility the consumer was getting before the price change.

(c) Write down the Slutsky equation in elasticity or derivative form, and explain how this captures the substitution and income effect.

$$e_x(p_x) = e_{h_x}(p_x) - e_x(I)\, s_x$$

$e_{h_x}(p_x)$*—a pure reaction to prices since the income is adjusted to keep this consumer equally happy. This is the substitution effect.*

$-e_x(I)\, s_x$*—a pure reaction to the change in income, which is the only variable that changes in this equation. This is the income effect. Notice that the minus sign in front is there because when the price of a good increases real income decreases.*

(d) Give an example of a good where the difference between Consumer Surplus and Compensating variation will be small and explain why the difference will be small using the Slutsky equation.

It will be small if the income elasticity is nearly zero or the share of your income you spend on the good is nearly zero. The latter is obviously an easier case to check, examples would be things like bubble gum, socks, and other things where the expenditure on those goods make up an insignificant part of someone's income.

(e) Explain how the Slutsky equation can be used to find the difference in the slope (or elasticity) of the Hicksian and Marshallian demand curves.

By definition this difference is $-e_x(I)\, s_x$ *unless this is zero one will be more steeply sloped than the other. Which one? Well that leads us to Marshall's mistake. Since he put the price on the vertical axis what we really graph is* $\frac{1}{e_x(p_x)}$ *so the relationship in terms of the graph is*

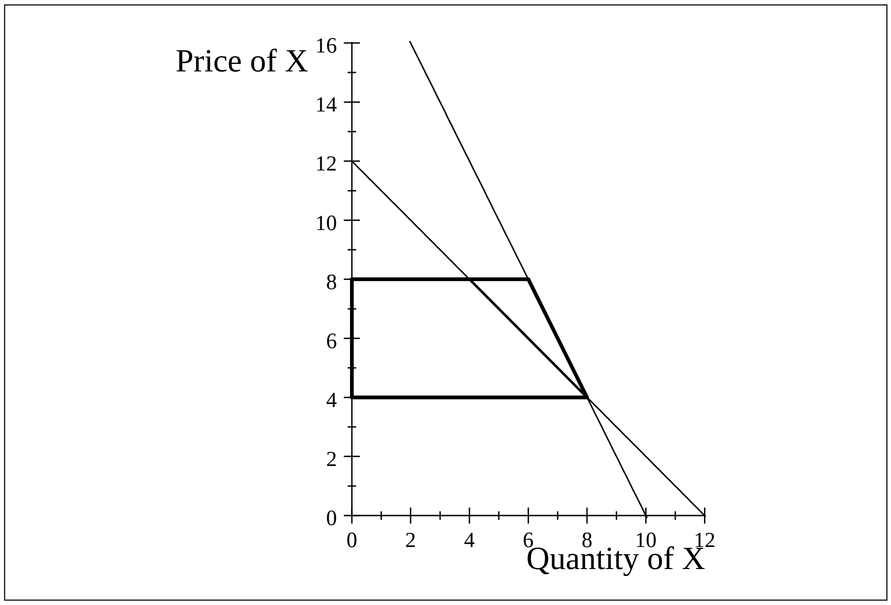
$$\frac{1}{e_x(p_x)} = \frac{1}{e_{h_x}(p_x) - e_x(I)\, s_x}$$

and if $-e_x(I)\, s_x < 0$ *the Marshallian demand curve will be steeper. Otherwise it will be flatter.*

(f) Using your answers to parts 10b and 10e in a graph draw a Marshallian demand curve and a Hicksian demand curve, and the change in Consumer Surplus and Compensating variation when the price of X changes from p_x^o to p_x^n. Be sure that the Marshallian demand is X_0 at the price p_x^o.

In my graph $X_0 = 8$ *and* $P_x^0 = 4$, $P_x^n = 6$. *The Hicksian demand curve has the steeper slope and passes through* $X_0 = 8$ *and* $P_x^0 = 4$.

This means that the change in Consumer Surplus is smaller than the Compensating Variation, and the change in Consumer surplus is the smaller shape surrounded by a dark line, the Compensating variation is the larger shape.



11. The Turkish government has decided they want to increase the consumption of bread and are considering three options.

    (a) An Income Subsidy (Increasing Income) so that people can afford more bread.

    (b) Giving out "Bread Tickets"—in this program the government gives each person a number of tickets that can only be used to purchase bread.

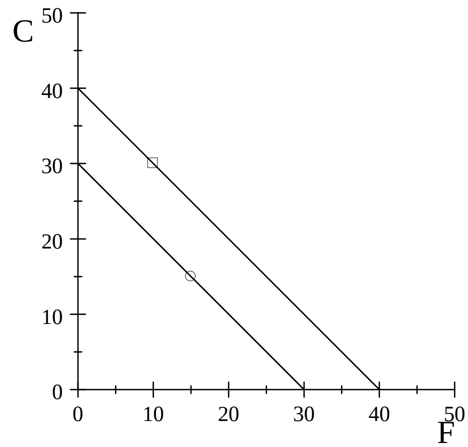
    (c) A price subsidy or price ceiling that will reduce the price of bread.

    Please answer the following questions about these three programs.

    (a) What type of good is bread in Turkey? Is it normal, inferior, or a luxury good?

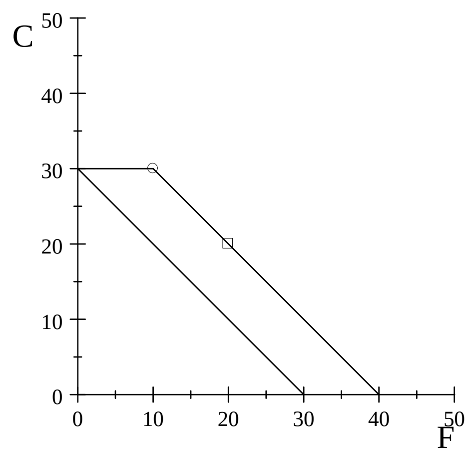
        Bread is an inferior good in Turkey.

    (b) In the graph below show the impact of an Income Subsidy. Who would increase their consumption of bread? Would anyone decrease their consumption of bread?



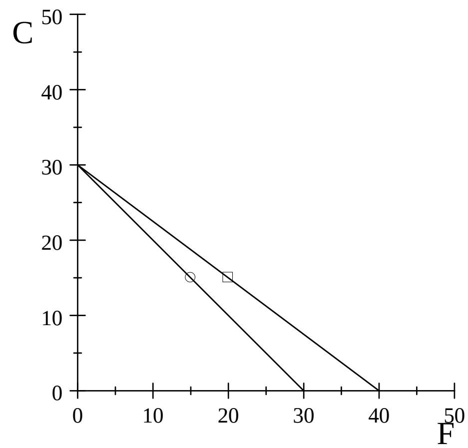
Since bread is an inferior good increasing income will decrease the consumption of bread for everyone. You would move from some consumption point like the circle on the lower budget constraint to the box on the higher.

(c) In the graph below show the impact of Bread Tickets. Who would increase their consumption of bread? Would anyone decrease their consumption of bread?



For most people this is the same as increasing income, so they will decrease their consumption of bread. These people would be consuming at some point like the box after the bread tickets are given out. For people who are consuming on the corner, like the circle above, they might increase their consumption of bread. But these people are rare.

(d) In the graph below show the impact of a price subsidy. Who would increase their consumption of bread? Would anyone decrease their consumption of bread?



Since the income effect is outweighed by the substitution effect, everyone will consume more bread. Moving from points like the circle on the lower budget constraint to the box on the higher one.

(e) Given that the Turkish government chose a price subsidy over bread tickets or an income subsidy what does this tell you about their goal? Do they want everyone to consume more bread or only some people? How can you tell?

They want to increase everyone's consumption of bread. We can tell this because if they just wanted to be sure that everyone consumed some bread they could do this by using bread tickets. Since they used a price subsidy they want to increase everyone's consumption—regardless of their current consumption.

12. Why can demand curves sometimes slope upwards? Why do we usually assume that demand curves are downwards sloping?

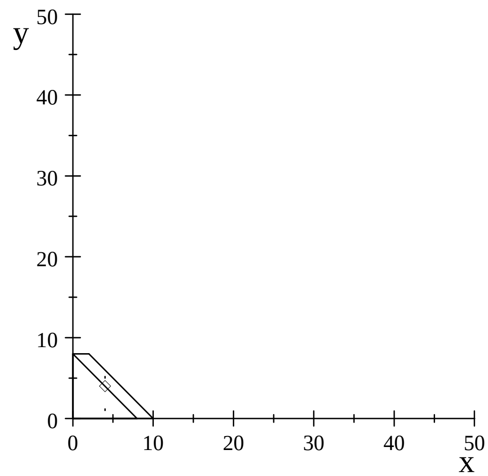
    Because of the Income Effect. Because the Substitution effect usually has a larger impact than the income effect.

13. The government is considering giving food tickets to people, these tickets entitle the holder to one unit of food per ticket. Assume that they are not planning on giving that many tickets to each person.

    (a) What type of good is food? What does this mean about the elasticity of food with respect to income?

    Food is a normal good and $0 \leq e_F(I) \leq 1$

    (b) Draw a representative consumer's budget set before and after the government institutes this policy.



In this graph the x axis is Food, and the diamond is where they are currently consuming.

(c) You think there are three possible reasons they might want to give food tickets to everyone.

i. To increase the amount they spend on food.
this will achieve this goal since food is a normal good.

ii. To decrease the amount they spend on food.
this will not achieve this goal since food is a normal good.

iii. To make sure that everyone spends at least a certain amount on food.
this will achieve this goal, the least Food anyone will consume will be at the corner of the new budget set. At that point everyone will consume the amount of Food the government hands out.

Evaluate whether the program achieves each of the three above objectives. Argue your case each time.

14. When there is a change in one price:

(a) Write down the Slutsky Equation in derivative form and then derive the equation in elasticity form using this equation.

$$\begin{aligned}
\frac{\partial X}{\partial p_x} &= \frac{\partial h_x}{\partial p_x} - \frac{\partial X}{\partial I} X \\
\frac{\partial X}{\partial p_x} \frac{p_x}{X} &= \frac{\partial h_x}{\partial p_x} \frac{p_x}{X} - \frac{\partial X}{\partial I} X \frac{p_x}{X} \frac{I}{I} \\
e_x(p_x) &= e_{h_x}(p_x) - \frac{\partial X}{\partial I} \frac{X}{I} \frac{p_x X}{I} \\
e_x(p_x) &= e_{h_x}(p_x) - e_x(I)\, s_x
\end{aligned}$$

where $s_x = \frac{p_x X}{I}$ *is the share of my income I spend on X.*

(b) Using words and symbols define the two effect captured by the Slutsky equation, and explain why each one is there.

$\frac{\partial h_x}{\partial p_x}$—the *substitution effect*, this is the effect of the change in relative price on the quantity demanded.

$-\frac{\partial X}{\partial I}X$—the *income effect*, this is the effect caused by the fact that real income $RI = \{X, Y | p_x X + p_y Y \leq I\}$ has changed, it is negative because RI always decreases when a price increases.

(c) For each of the following equations, state whether they can be demand functions, and if they can what share of income must be spent on X?

i. $X(p_x, p_y, I) = 10p_x^{\frac{3}{4}} p_y^{-2} I^{-3}$

$\frac{\partial X}{\partial p_x} = \frac{3}{4}\frac{X}{p_x}$ $e_X(p_x) = \frac{3}{4}\frac{X}{p_x}\frac{p_x}{X} = \frac{3}{4}$. $\frac{\partial X}{\partial I} = -3\frac{X}{I}$ $e_X(I) = -3\frac{X}{I}\frac{I}{X} = -3$. *According to the slutsky equation:*

$$
\begin{aligned}
e_{h_x}(p_x) &= e_x(p_x) + e_x(I)\, s_x \leq 0 \\
\frac{3}{4} - 3s_x &\leq 0 \\
\frac{1}{4} &\leq s_x
\end{aligned}
$$

so as long as the share of income spent on x is greater than $\frac{1}{4}$ *this is a well defined demand equation.*

ii. $X(p_x, p_y, I) = 12p_x^2 p_y^{-\frac{2}{3}} I^{-1}$

$\frac{\partial X}{\partial p_x} = 2\frac{X}{p_x}$ $e_X(p_x) = 2\frac{X}{p_x}\frac{p_x}{X} = 2$. $\frac{\partial X}{\partial I} = -\frac{X}{I}$ $e_X(I) = -\frac{X}{I}\frac{I}{X} = -1$. *According to the slutsky equation:*

$$
\begin{aligned}
e_{h_x}(p_x) &= e_x(p_x) + e_x(I)\, s_x \leq 0 \\
2.1 - 1.s_x &= 2 - s_x \leq 0 \\
2 &\leq s_x
\end{aligned}
$$

since this is impossible this can not be a demand equation.

15. The Turkish government is considering new methods to subsidize the consumption of bread, which is an inferior good. Their current method is a price ceiling, refusing to allow bread makers to charge more than a certain price.

(a) They are considering replacing this with a direct subsidy of bread, giving everyone a certain number of "bread tickets" that can only be used of purchasing bread. Explain why this policy will not increase the amount of bread consumed and might decrease it.

ANSWER:

Since bread is an inferior good people will consume less of it when their income increases.

Bread tickets increase real income, since the individual has to buy less bread he can buy more of everything with the same income.

Therefore this will decrease the amount of bread consumers eat.

(b) One evil politician suggests that instead of bread tickets the government should just raise the income tax. Explain how this policy is better than bread tickets.

ANSWER:

Since bread is an inferior good people will consume less of it when their income increases.

An income tax will reduce real income.

Therefore this will increase the amount of bread consumers eat.

(c) One politician listens carefully to your answer above and suggests that it might be best to tax bread. Is he completely crazy? Is it possible that this could increase the consumption of bread? Why? Is this as good as the evil politicians idea? Use the concept of income and substitution effects in your answer.

ANSWER:

No,Since this is an inferior good, it is theoretically possible for the income effect to be stronger than the substitution effect, and for an increase in the price of bread to actually increase the amount consumed.

This will be worse than the evil politician's idea since the substitution effect will be fighting against the income effect.

(An aside: I said in class that this type of good is called a "Giffen good" after the wacko who proved this was possible. (Notice he was not a complete wacko). But as several of you more elegantly expressed on your exams this is really "Giffen's paradox." While it is theoretically possible it doesn't really happen, and it is a paradox relative to our normal way of thinking.

16. The state of Kansas in the United States is concerned because the consumption of Margarine (made with corn oil–one of their chief products) is too low. Margarine is an inferior good, in this case discuss the effect of:

(a) Giving each Kansan ten pounds of Margarine per month.

ANSWER:

Assuming that Kansans can sell ten pounds of Margarine in the market, income of each consumer will increase and since the good is inferior it will have a negative effect on consumption. Assuming prices will remain the same they will consume less of margarine.

Assuming that they cannot sell ten pounds of Margarine in the market, we cannot draw a conclusion about whether the new level of consumption is higher or lower than the old one. All we can say is that since real income has increased with ability of consuming ten pounds, *the amount purchased from the market will decrease. But this amount plus ten pounds, which is ready for consumption may be higher than the old level.*

(b) Changing the Kansans' income through an income tax/subsidy. (Hint, why didn't I say "increasing" the Kansans' income?)

ANSWER:

If the state imposes income tax, since real income decreases and since Margarine is an inferior good, consumption will increase. Similarly if the state imposes income subsidy, since real income increases consumption will decrease.

(c) Changing the price of Margarine through a direct subsidy tax.

Compare the effect of each of these policies on the demand for margarine, and rank them where possible.

ANSWER:

In case of price subsidy, substitution effect would force consumption up. But at the same since price decrease implies an increase in real income, income effect will be negative. Hence we cannot say whether the new level of consumption will be higher or lower than the old level.

In case of a direct tax, substitution effect would force consumption down. But at the same since price increase implies a decrease in real income, income effect will be positive. Hence we cannot say whether the new level of consumption will be higher or lower than the old level.

If the state knows that substitution effect will be stronger, in order to increase consumption first policy should be implemented. If the converse is true, second policy should be implemented.

Compare the effect of each of these policies on the demand for margarine, and rank them where possible.

17. When the price of gasoline was \$1.00 per gallon, Joe consumed 1000 gallons of gasoline. The price increased to \$1.50, and the government told Joe that it would give him \$500 to offset the price increase. Will Joe be happier, the same, or less happy after the price increase and transfer? How much gasoline will he consume? Use the concepts of substitution and income effect in your answer. THERE IS A CLEAR ANSWER TO THIS, THINK IT THROUGH!

ANSWER:

He will consume less of gasoline. Since price of gasoline has increased, substitution effect will be negative, since its sign is always the opposite of the sign of price change. Hence it reduces consumption. Now, since the subsidy is just enough for the real income to remain unchanged (note that if Joe wanted to do so, he could buy the old bundle of goods), there will be no income effect and hence consumption of gasoline will decline. His utility must have increased, since although he could buy the old bundle, he did not so.

5 Chapter 6—Demand Relationships Among Goods.

1. Assume that a consumer has a Cobb-Douglass utility function for X and Y, and the demand functions $X = \frac{\alpha I}{p_x}$, $Y = \frac{\beta I}{p_y}$ for $\alpha > 0$ and $\beta > 0$.

   (a) Find the elasticity of X and Y with regards to p_x, p_y, and I. To be clear you need to find six elasticities.

$$\begin{aligned} e_x(p_x) &= \frac{\partial X}{\partial p_x}\frac{p_x}{X} = -\frac{\alpha I}{p_x^2}\frac{p_x}{X} = -\frac{X}{p_x}\frac{p_x}{X} = -1 \\ e_y(p_x) &= \frac{\partial Y}{\partial p_x}\frac{p_x}{Y} = 0 * \frac{p_x}{Y} = 0 \\ e_y(p_y) &= \frac{\partial Y}{\partial p_y}\frac{p_y}{Y} = -\frac{Y}{p_y}\frac{p_x}{Y} = -1 \\ e_x(p_y) &= \frac{\partial X}{\partial p_y}\frac{p_y}{X} = 0 * \frac{p_x}{Y} = 0 \\ e_x(I) &= \frac{\partial X}{\partial I}\frac{I}{X} = \frac{\alpha}{p_x}\frac{I}{X} = \frac{X}{X} = 1 \\ e_y(I) &= \frac{\partial Y}{\partial I}\frac{I}{Y} = \frac{\beta}{p_y}\frac{I}{Y} = \frac{Y}{Y} = 1 \end{aligned}$$

   (b) Find the share of your income that you spend on X and on Y. To be clear you need to find two income shares. What constraints can you place on α and β based on these shares?

$$\begin{aligned} s_x &= \frac{p_x X}{I} = \frac{p_x \frac{\alpha I}{p_x}}{I} = \alpha \\ s_y &= \frac{p_y Y}{I} = \frac{p_y \frac{\beta I}{p_y}}{I} = \beta \end{aligned}$$

   Since the sum of income shares must be less than one by definition we have

$$\alpha + \beta \leq 1$$

   if we assume there are only two goods then $\beta = 1 - \alpha$.

   (c) Are X and Y gross compliments or gross substitutes? Are X and Y net compliments or net substitutes? Warning: You must support your answer with clear mathematics.

$$e_y(p_x) = e_x(p_y) = 0$$

   so one can say that they are gross compliments, gross substitues, or neither. I prefer the latter but as long as your reasoning is correct I will accept any answer.

To find out whether they are net substitutes or compliments we have to use the slutsky equation in elasticity form.

$$\begin{aligned} e_X(p_y) &= e_{h_X}(p_y) - e_X(I)\, s_Y \\ 0 &= e_{h_X}(p_y) - (1)\,\beta \\ \beta &= e_{h_X}(p_y) > 0 \end{aligned}$$

thus they are net substitutes.

2. Assume that this person's demand for X is given by:

$$X = \frac{1}{2p_x p_y^2} I^3$$

(a) Find the elasticities of X with respect to p_x, p_y, and I. Is Y a gross substitute or complement of X? Is X a normal, inferior, or luxury good? (Hint: Guesses will get no credit, but the answer should be a constant, like 3 or $\frac{1}{2}$.)

$$\begin{aligned} \frac{\partial X}{\partial p_x} &= -\frac{X}{p_x}, e_X(p_x) = \left(-\frac{X}{p_x}\right)\frac{p_x}{X} = -1 \\ \frac{\partial X}{\partial p_y} &= (-2)\frac{X}{p_y}, e_X(p_y) = -2\frac{X}{p_y}\frac{p_y}{X} = -2 \\ \frac{\partial X}{\partial I} &= 3\frac{X}{I}, e_X(I) = \left(3\frac{X}{I}\right)\frac{I}{X} = 3 \end{aligned}$$

Y is a gross compliment of X.
X is a luxury good

(b) Write down the Slutsky Equation in elasticity form.

$$e_X(p_y) = e_{h_x}(p_y) - e_X(I)\, s_y$$

it is fine if you wrote down the equation derived by taking the derivative with respect to X, or if you wrote:

$$e_X(I)\, s_y + e_X(p_y) = e_{h_x}(p_y)$$

(c) What is the share of income this person spends on X? The answer will be a function of prices and income. Do not assume that you know the values of prices or income.

$$s_x = \frac{p_x X}{I} = \frac{p_x \frac{1}{2} p_x^{-1} p_y^{-2} I^3}{I} = \frac{1}{2} p_y^{-2} I^2$$

For the rest of the question assume that $I = 1$ and $p_y = 1$.

(d) Assume that $p_x = 1$, are X and Y net substitutes or net compliments? (Notice that the share of income spent on Y is $s_y = 1 - s_x$.)

$$\begin{aligned} e_X(I)\, s_y + e_X(p_y) &= e_{h_x}(p_y) \\ 3(1 - s_x) + (-2) &= e_{h_x}(p_y) \end{aligned}$$

$$s_x = \frac{1}{2} p_y^{-2} I^2 = \frac{1}{2} 1^{-2} 1^2 = \frac{1}{2}$$

$$1 - \frac{1}{2} 3 = e_{h_x}(p_y)$$

they are net compliments.

For the rest of the problem assume that p_x has risen, the old price is $p_x^o = 1$ while the new price is $p_x^n = 2$.

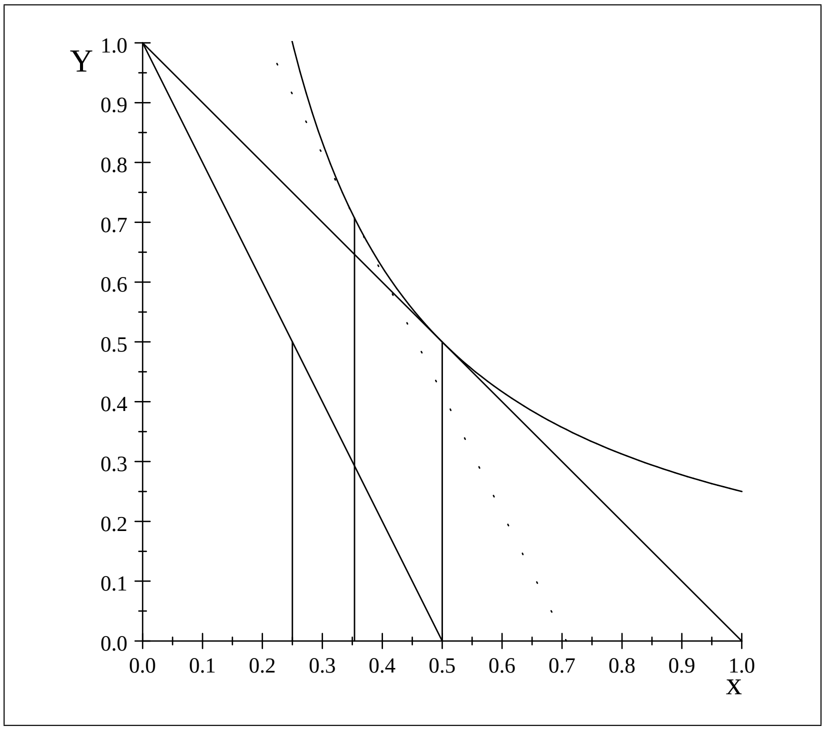
(e) Using the demand curve find the old and new quantities of X, label your answers as X_0 and X_n

$$\begin{aligned} X_o &= \frac{1}{2} \\ X_n &= \frac{1}{2^2} \end{aligned}$$

(f) In the graph below graph the two budget sets and the old and new consumption points. Indicate X_0 and X_n.

(g) In the same graph now find a possible indifference curve and the imaginary budget line that indicates the quantity this person would buy at the new prices if they had enough income to stay on the same indifference curve as before the price change. Label the quantity of

X they would consume in this case as X_i.



The right most solid vertical line is where X_0 should be, the left most solid vertical line is where X_n should be and the solid vertical line in between is where X_i should be. The dotted line is the imaginary budget constraint, and the other budget constraints are as shown.

Notice that to answer this question you do not have to calculate Y it is enough to draw the budget constraints and then just go up in a vertical line from where X_0 and X_n are.

By the way, to produce this graph I used the utility function $U(X, Y) = XY$. This utility function would not have this demand curve, but it does have $X = Y$ when $p_x = p_y$. Your X_i would differ of course, but it should be between X_0 and X_n.

(h) Indicate the income and the substitution effect of the price change using these three quantities ($\{X_o, X_i, X_n\}$). Does the income effect have the same sign as the substitution effect? Why or why not?

The substitution effect is

$$X_o - X_i$$

the Income effect is

$$X_i - X_n$$

they both have the same sign because this is a normal good.

3. Why can X be a gross substitute of Y while Y is a gross compliment of X? Why can X NOT be a net substitute of Y if Y is a net compliment of X?

ANSWER:

Because of the income effect. We can have $e_{h_x}(p_y) - e_X(I)\, s_x = e_X(p_y) > 0$, *i.e. Y is a gross compliment of X, but on the other hand Y being an inferior good (i.e.$e_Y(I) < 0$)and having large $e_Y(I)$or s_y with respect to $e_{h_y}(p_x)$ we can have* $e_{h_y}(p_x) - e_Y(I)\, s_y = e_Y(p_x) < 0$, *hence X is a gross substitute of Y.*

If X is a net substitute of Y, then Y must be a net substitute of X . We have $\frac{\partial h_x}{\partial p_y} = \frac{\partial h_x}{\partial h_y}\frac{\partial h_y}{\partial p_y}$, *since* $\frac{\partial h_y}{\partial p_y} > 0$, $\frac{\partial h_x}{\partial p_y} > 0$ *implies* $\frac{\partial h_x}{\partial h_y} > 0$. *Substituting this result into the equation* $\frac{\partial h_y}{\partial p_x} = \frac{\partial h_y}{\partial h_x}\frac{\partial h_x}{\partial p_x}$ *we have* $\frac{\partial h_y}{\partial p_x} > 0$.

4. Assume that the demand curve for food is $F = I^3 p_c^{-2} p_f^{-2}$ and the demand for clothing is $C = I^{-4} p_f^1 p_c^{-3}$.

   (a) Find out if each good is normal, inferior, or a luxury good.

$$\begin{aligned} e_f(I) &= 3 \geq 1, \text{ luxury} \\ e_c(I) &= -4 \leq 0, \text{ inferior} \end{aligned}$$

   (b) Is food a gross compliment or gross substitute of clothing? Is clothing a gross substitute or a gross complement of Food?

$$\begin{aligned} e_f(p_c) &= -2 \leq 0 \text{ gross complements} \\ e_c(p_f) &= 1 \geq 0 \text{ gross substitutes} \end{aligned}$$

   (c) Write down the slutsky equation.

$$e_f^h(p_c) = e_f(p_c) + e_f(I)\, s_c$$

   (d) If the share of income spent on clothing ($\frac{p_c C}{I}$) is $\frac{1}{8}$ and the share spent on food ($\frac{p_f F}{I}$) is $\frac{1}{2}$ are these goods net substitutes or complements?

$$\begin{aligned} e_f^h(p_c) &= -2 + \frac{3}{8} \leq 0 \text{ net complements} \\ e_c^h(p_f) &= 1 - \frac{4}{2} \leq 0 \text{ net complements} \end{aligned}$$

5. Assume that the demand for F and C are given by:

$$\begin{aligned} F &= 10 - 2p_f + p_c + I \\ C &= 15 - 2p_c - p_f + I \end{aligned}$$

where p_f is the price of food, p_c is the price of clothing, and I is income.

(a) What is the difference between net and gross substitutes? Use the slutsky equation in your answer.

ANSWER:

A net substitute is a substitute holding utility constant, $\frac{\partial F}{\partial C}|_{u=\bar{u}} < 0$ is an adequate definition.

A gross substitute is a substitute holding income constant, or $\frac{\partial F}{\partial C}|_{I=\bar{I}} < 0$. The first definition is equivalent to $\frac{\partial h_f}{\partial p_c} > 0$ the latter to $\frac{\partial F}{\partial p_c} > 0$ and the relationship between them can be expressed using the slutsky equation as:

$$\frac{\partial F}{\partial p_c} = \frac{\partial h_f}{\partial p_c} - \frac{\partial F}{\partial I} C$$

from this we can notice that while net substitutes are symmetric (If C is a net substitute of F then F is a net substitute of C) gross substitutes very well might not be.

(b) Are F and C gross substitutes or complements?

ANSWER:

$$\frac{\partial F}{\partial p_c} = 1 > 0$$

C is a gross substitute of F.

$$\frac{\partial C}{\partial p_f} = -1 < 0$$

but F is a gross complement of C.

(c) Are F and C net substitutes or complements? Assume that $I = 5$ and $p_c = p_f = 1$.

ANSWER:

>From the slutsky equation

$$\begin{aligned} \frac{\partial h_f}{\partial p_c} &= \frac{\partial F}{\partial p_c} + \frac{\partial F}{\partial I} C \\ &= 1 + C \end{aligned}$$

and $C = 15 - 2p_c - p_f + I = 15 - 2(1) - (1) + 5 = 17$ thus F and C are net substitutes.

If I check:

$$\begin{aligned} \frac{\partial h_c}{\partial p_f} &= \frac{\partial C}{\partial p_f} + \frac{\partial C}{\partial I} F \\ &= -1 + F \end{aligned}$$

$F = 10 - 2p_f + p_c - I = 10 - 2(1) + (1) - 5 = 4$
thus again $\frac{\partial h_c}{\partial p_f} > 0$ and these are net substitutes.