Practice Questions—Chapters 9 to 11

Producer Theory

ECON 203

Kevin Hasker

These questions are to help you prepare for the exams only. Do not turn them in. Note that not all questions can be completely answered using the material in the chapter in which they are asked. These are all old exam questions and often the answers will require material from more than one chapter. Questions with lower numbers were asked in more recent years.

1 Chapter 9—Production Functions

1. Write down and define the three assumptions that we always make about the production function. For the production function $Q = 3K^{\frac{1}{2}} + 9L^{\frac{1}{2}}$ show that it satisfies two of these assumptions. (You can choose which two to show.)

2. Consider the following production function $Q = K^{\frac{1}{2}}L^{\frac{1}{4}}$ and answer the following questions about it.

   (a) Define *free disposal*. Does this production function satisfy this assumption? Prove your answer.

   (b) Define *convex isoquants*. Does this production function satisfy this assumption? Argue why your answer is true. *Note: I do not require you to prove your answer for this part.*

   (c) Define *decreasing returns to scale*. Does this production function satisfy this assumption? Prove your answer.

3. In general when do we expect firms to have Increasing Returns to Scale? Constant Returns to Scale? Explain why we expect this in both of these cases.

2 Chapter 10—Cost Functions

1. Assume that the total cost function has the arbitrary form $c(q)$, let F_{st} be the fixed startup costs and F_{su} be the fixed sunk costs. Please note that $c(q)$ includes these costs. In this question I want you to prove that when average variable costs (AVC) are minimized average variable costs equal marginal costs (MC).

   (a) In terms of this cost function write down average costs or average total costs (AC), AVC and MC.

   (b) What is the objective function that you need to minimize in order to minimize average variable costs?

(c) Find the first order condition of this objective function.

(d) Prove that $AVC = MC$ when AVC is minimized.

2. Assume a firm has the production function $Q = 3K^{\frac{1}{2}} + 9L^{\frac{1}{2}}$.

   (a) Write down the definition of decreasing returns to scale. Does this production function satisfy decreasing returns to scale?

   (b) The Long Run Cost Function.

      i. Set up the objective function for the long run cost function.
      ii. Find the first order conditions.
      iii. Find a function for the optimal amount of labor, L, in terms of K and input prices.
      iv. Find the optimal demand for capital, K.
      v. Find the optimal demand for labor, L.
      vi. Find the long run cost function, and simplify if possible.

   (c) The Short Run cost function. Assume throughout that the optimal demand for labor, L, is positive.

      i. Set up the objective function for the short run cost function.
      ii. Find the first order conditions.
      iii. Find the optimal amount of labor, L. Explain your methodology.
      iv. Find the short run cost function.
      v. State the general envelope theorem and explain the basic reason it is true. What does this theorem tell us that $\frac{\partial C^{sr}}{\partial K}$ is equal to?
      vi. Using the short run cost function find the optimal long run demand for capital (K), verify that it is the same as when you found the long run cost function above.
      vii. Find the long run cost function, verify that it is the same as when you found it above.

3. For the general envelope theorem:

   (a) Explain what the general envelope theorem tells us.

   (b) Consider the production function $f(L, K, M)$ where M is materials, K is capital, and L is labor. Let the price of a unit of labor be w, the price of a unit of capital be r and the price of a unit of materials be μ.

      i. Set up the objective function we would use to find the short run cost function, C^{sr}.
      ii. Prove the envelope theorem by finding $\frac{\partial C^{sr}}{\partial K}$.
      iii. How can we use $\frac{\partial C^{sr}}{\partial K}$ to find the long run cost function?

4. Assume that a firm has the production function $Q = K^{\frac{2}{3}}L^{\frac{1}{6}}$. (Let the price of a unit of K be r and the price of a unit of L be w).

   (a) Define *decreasing returns to scale*, does this production function satisfy decreasing returns to scale?

   (b) Set up the objective function to find the short run cost function, C^{SR}.

   (c) Find the short run cost function.

   (d) Show that this cost function is non-increasing in input prices and homogeneous of degree one in input prices.

   (e) Find the long run demand for Capital.

   (f) Find the long run cost function and show that it can be simplified to the form $C^{lr} = w^{\delta}r^{\tau}Q^{\gamma}A$ where δ, τ, γ and A are numbers that depend on the coefficients of the production function. *Note: Credit will only be given for finding the proper values of* δ, τ, γ and A.

   (g) Show that the long run cost function is non-decreasing in input prices and homogeneous of degree one in input prices. *Partial credit will be given for proving this using the general form I gave you in the last part of this problem.*

   (h) Find the long run demand for labor. *Partial credit will be given for finding this using the general form for the cost function I gave you above.*

5. In this question I want you to prove that for a specific production function and in general that the cost function is homogeneous of degree one in input prices, or $C(tw, tr, Q) = tC(w, r, Q)$ for a firm that has two inputs, K or capital and L or labor.

   (a) For the production function $Q = K^{\frac{2}{3}}L^{\frac{1}{6}}$.

      i. In a graph, graph an isoquant when $Q = 1$. Be sure to list at least three points on this isoquant below your graph.

      ii. In the same graph find the cost minimizing isocost curve when $w = 1$ and $r = 4$.

      iii. Now in the same graph find the cost minimizing isocost curve when $w = 3$and $r = 12$.

      iv. Show in the graph that this means $C(3, 12, 1) = 3C(1, 4, 1)$. Explain how you could generalize this result to show that $C(tw, tr, Q) = tC(w, r, Q)$.

   (b) Now I want you to show the same thing for general production processes. Let (K^*, L^*) be the cost minimizing bundle at the input prices (w, r) and output Q.

      i. If arbitrary (K, L) produce at least Q units of output is $wL^* + rK^* \geq wL + rK$, $wL^* + rK^* \leq wL + rK$, or are we unable to tell? Explain.

ii. Prove that the cost function is homogenous of degree one in input prices, or that for $t > 0$, $tC(w, r, Q) = C(tw, tr, Q)$.

6. A firm has three inputs, capital (K), labor (L) and pollution or bad air (B) their production function is $Q = f(K, L, B)$ the government regulates the firm buy saying that B must be less than some given $\bar{B}$, or mathematically that $B \leq \bar{B}$. Note that you will get severely reduced credit if you assume a functional form for $f(K, L, B)$.

   (a) What standard assumption do we need to make on the production function to insure that $B = \bar{B}$ in any cost minimizing solution?

   (b) Set up the firm's cost minimizing objective function, be sure that you include *two* constraints. You may assume that the assumption you needed for the last part of this question is true.

   (c) Find the first order conditions of the cost minimization problem.

   (d) State the envelope theorem, or tell me what the envelope theorem implies.

   (e) Prove the envelope theorem by finding the derivative of this firm's cost function with respect to $\bar{B}$.

   (f) The government wants to find out the firm's marginal benefit of pollution (B) is not balanced with the marginal social cost of pollution. Assuming they do not know the firm's production function how can they find out what this marginal benefit is? What would they need to do to find this benefit?

7. In this question I want you to prove properties of the cost function for a firm that has two inputs, K or capital and L or labor. Let (K^*, L^*) be the cost minimizing bundle at the input prices (w, r) and output Q.

   (a) If arbitrary (K, L) produce at least Q units of output is $wL^* + rK^* \geq wL + rK$, $wL^* + rK^* \leq wL + rK$, or are we unable to tell? Explain.

   (b) Prove that input demands are homogenous of degree zero in input prices, or that for $t > 0$, $K(w, r, Q) = K(tw, tr, Q)$ and $L(w, r, Q) = L(tw, tr, Q)$.

   (c) Prove that the cost function is homogenous of degree one in input prices, or that for $t > 0$, $tC(w, r, Q) = C(tw, tr, Q)$.

8. Given that a firm has the production function $Q = 8\sqrt{K} + 2\sqrt{L}$ find the long run cost function. (Let the price of a unit of K be r and the price of a unit of L be w).

   (a) Define *free disposal*, does this production function satisfy free disposal?

   (b) Set up the objective function.

(c) Find the first order conditions.
(d) Find the "buck for the bang" of Capital (K) and Labor (L).
(e) Find a formula for Labor in terms of Capital and input prices.
(f) Find the demand for Capital.
(g) Find the demand for Labor.
(h) Solve for the long run cost function.

9. For the production function $Q = 8\sqrt{K} + 2\sqrt{L}$

    (a) Find the firm's short run demand for labor and their short run cost function, $C^{sr}(r, w, K, Q)$. You may assume that output is high enough that labor must be used.
    (b) Using the short run cost function and the envelope theorem find the optimal amount of capital.
    (c) Verify that the optimal amount of capital you just found is the same as the long run demand for capital you found in the last question.

10. I claim that in general the total cost function contains both economic costs and accounting costs. Define *economic costs* and *accounting costs* and explain.

11. About Economists and Accountants. **Define all technical terms you use and explain all of your answers.**

    (a) What is the fundamental difference between economists and accountants?
    (b) Which type of costs do accountants think are important but economists do not? Which type of costs do economists think are important but economists do not?
    (c) What does this tell us about the relationship between accounting profit and economic profit in the long run?
    (d) What relationship would we expect between accounting profit and economic profit in the short run? (Your answer can be that you can not tell, but you must explain why each case could be true.)

12. Given that a firm has the production function $Q = K^{\frac{1}{2}}L^{\frac{1}{4}}$ find the long run cost function. (Let the price of a unit of K be r and the price of a unit of L be w).

    (a) Set up the objective function.
    (b) Find the first order conditions.
    (c) Find the "buck for the bang" of Capital (K) and Labor (L).
    (d) Using the first order conditions find a formula for Labor in terms of Capital and input prices.

(e) Find the demand for Capital.

(f) Find the demand for Labor.

(g) Solve for the long run cost function.

(h) Show that the cost function is homogenous of degree one in input prices and increasing in input prices.

13. For the production function $Q = K^{\frac{1}{2}} L^{\frac{1}{4}}$

    (a) Set up the objective function for the short run cost function.

    (b) Find the firm's short run demand for labor and their short run cost function, $C^{sr}(r, w, K, Q)$.

    (c) Using the short run cost function and the envelope theorem find the optimal amount of capital.

    (d) Verify that the optimal amount of capital you just found is the same as the long run demand for capital you found in the last question.

14. In a graph:

    (a) Draw and label the isoquant where $Q = 8$ for the production function $Q = K^{\frac{1}{2}} L^{\frac{1}{4}}$.

    (b) If $w = 3$ and $r = 6$ draw and label the isocost line that is just tangent to the isoquant you found. Be sure to label the points where the isocost curve crosses the axes and the optimal amount of Labor and Capital.

    (c) In the same graph draw the cost minimizing isocost line when w decreases to $\frac{3}{2^{\frac{3}{2}}} \tilde{} 1.06$ (and $r = 6$).

    (d) Using this graph prove that if $w \geq \tilde{w}$ then $C(w, r, Q) \geq C(\tilde{w}, r, Q)$, or that the cost function is non-decreasing in input prices.

    (e) Explain how you could generalize this graph to show that if $w \geq \tilde{w}$ and $r \geq \tilde{r}$ then $C(w, r, Q) \geq C(\tilde{w}, \tilde{r}, Q)$.

15. Assume that a firm has a production function $Q = f(K, L, M, E)$ where K is capital with a price of r, L is labor with a price of w, M is materials with a price of p_m and E is energy with a price of p_e.

    (a) State the envelope theorem, or tell me what the envelope theorem implies.

    (b) Set up this firms' cost minimization problem.

    (c) Prove the envelope theorem by finding the derivative of this firm's cost function with respect to p_m. You will not get credit if you assume that the production function has a specific functional form.

16. About the envelope theorem.

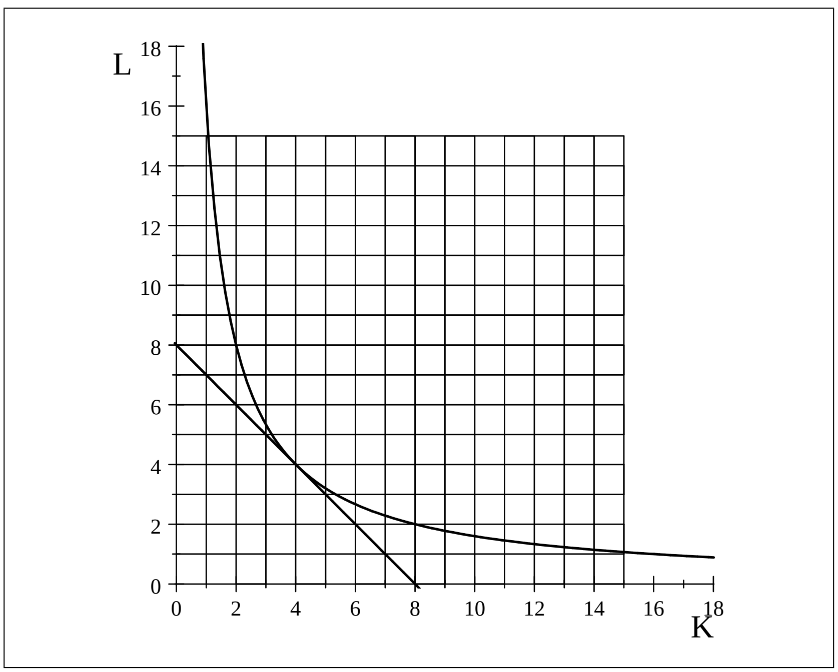
(a) Explain what the envelop theorem tells us simply enough that your 10 year old sibling could understand. (2 points will only be awarded if you do not use the term "derivative" in your answer. You may use the terms "function" and "optimized" or "optimal.")

(b) Explain why the statement you made above is true, you may either use math or English.

(c) If $C^{SR}(w, r, K, Q)$ is a short run cost function what does this theorem tell us $\frac{\partial C^{SR}}{\partial K}$ is equal to? How can we use this to find the long run cost function $C(w, r, Q)$?

17. This question is about proving the fact that if $w_n \geq w_o$ and $r_n \geq r_o$ then $C(w_n, r_n, Q) \geq C(w_o, r_o, Q)$.

(a) In the graph below the straight line is the isocost curve given they need to produce the level of output represented by the isoquant. Find and label the optimal level of labor and capital L_o and K_o respectively, also label the point where the isocost curve crosses the L axis in terms of the general costs, C_o and the price of a unit of labor, w_o.



(b) In the same graph draw a new isocost curve where the only difference is that the price of capital (r) increases. Prove that the new cost (C_n) must be higher than the old cost (C_o) using this graph.

(c) Explain why the above graph also proves that if both r and w increase then we must have $C(w_n, r_n, Q) \geq C(w_o, r_o, Q)$.

(d) Does the isoquant in the graph on the last page satisfy free disposal? Convexity? Explain your answer. *(Note that the fact that isoquants in general may not satisfy these properties that makes the formal proof below necessary.)*

(e) Let $\{L_n, K_n\}$ be the cost minimizing inputs at $\{w_n, r_n, Q\}$. For other $\{L, K\}$ that will produce at least Q units of output is it true that $w_nL_n + r_nK_n \leq w_nL + r_nK$, $w_nL_n + r_nK_n \geq w_nL + r_nK$ or are you unable to tell? Why is the inequality the way that it is?

(f) Explain why $w_nL_n+r_nK_n \geq w_oL_n+r_oK_n$. *(This is an easy question, answering it does not require understanding or answering the rest of the question.)*

(g) Let $\{L_o, K_o\}$ be the cost minimizing inputs at $\{w_o, r_o, Q\}$. Is $w_oL_n + r_oK_n \leq w_oL_o+r_oK_o$, $w_oL_n+r_oK_n \geq w_oL_o+r_oK_o$ or are you unable to tell? Why is the inequality the way that it is?

(h) Combining the statements you just proved; prove the general statement, that if $w_n \geq w_o$ and $r_n \geq r_o$ then $C(w_n, r_n, Q) \geq C(w_o, r_o, Q)$.

(i) In this proof why did I start with $\{L_n, K_n\}$ instead of $\{L_o, K_o\}$?

18. In the long run accounting costs are lower than economic costs, in the short run economic costs are often lower than accounting costs. Explain why this is true.

19. Assume that the production function is $Q = 2L^{\frac{1}{2}} + 3K$ and that the input demand for both capital and labor is positive.

(a) Set up the objective function for finding the cost function.

(b) Find the first order conditions of the objective function you just set up.

(c) Solve for the buck for the bang for labor and capital.

(d) By equalizing the buck for the bang conditions find the input demand for Labor.

(e) From the production constraint find the input demand for capital.

(f) Find the cost function.

(g) Explain the general envelope theorem. You may use the cost function above to illustrate the concept.

(h) Using the envelope theorem and the cost function found in part *(f)* find the demand for labor and capital. You will get partial credit for this question even if you did not find the cost function in *(f)*.

20. Prove that the Cost function is concave in input prices. Or: $C(\alpha w + (1-\alpha)\tilde{w}, \alpha r + (1-\alpha)\tilde{r}, Q) \geq \alpha C(w, r, Q) + (1-\alpha) C(\tilde{w}, \tilde{r}, Q)$.

(a) In intuitive terms explain why this relationship should hold.

(b) Let $\{L^*, K^*\}$ be cost minimizing at $\{w, r\}$ and $\{L_\alpha, K_\alpha\}$ be cost minimizing at $\{\alpha w + (1-\alpha)\tilde{w}, \alpha r + (1-\alpha)\tilde{r}\}$. Is $wL_\alpha + rK_\alpha \geq wL^* + rK^*$, $wL_\alpha + rK_\alpha \leq wL^* + rK^*$, or can we not tell? Why is this?

(c) Using algebra show that: $C(\alpha w + (1-\alpha)\tilde{w}, \alpha r + (1-\alpha)\tilde{r}, Q) = \alpha(wL_\alpha + rK_\alpha) + (1-\alpha)(\tilde{w}L_\alpha + \tilde{r}K_\alpha)$

(d) Combine the insights of the last two parts to prove the statement.

21. Assume that the production function is $Q = \frac{5}{2}L^{\frac{1}{2}} + K$ and that the input demand for both labor and capital is positive.

(a) Solve for the short run cost function.

(b) Let $g(r, w, K) = \min_{L,\lambda} f(L, \lambda, r, w, K)$ and $\{L(w, r, K), \lambda(r, w, K)\}$ be the minimizing values of the right hand side, or $g(r, w, K) = f(L(w, r, K), \lambda(r, w, K), r, w, K)$. State and prove the general envelope theorem using this function. What does this function tell us about the short run cost function when $\frac{\partial C^{SR}(w,r,K)}{\partial K} = 0$?

(c) Use the envelope theorem to find the long run cost function.

(d) Using the envelope theorem and the cost functions you found in part a and c find the demand for labor. You will get partial credit for explaining how to do this even if you did not find the cost functions.

22. If a firm has a cost function $c(w, r, Q)$ show that it is non-decreasing in input prices. Or that if $w > \tilde{w}$ and $r > \tilde{r}$ then $c(w, r, Q) \geq c(\tilde{w}, \tilde{r}, Q)$. Notice you can not make any assumptions about the production function.

(a) If $\{L^*, K^*\}$ are cost minimizing at the prices $\{w, r\}$ show that producing using $\{L^*, K^*\}$ costs less at the input prices $\{\tilde{w}, \tilde{r}\}$.

(b) If $\left\{\tilde{L}, \tilde{K}\right\}$ are cost minimizing at the prices $\{\tilde{w}, \tilde{r}\}$ show a relationship between the costs of producing using $\left\{\tilde{L}, \tilde{K}\right\}$ and the costs of producing using $\{L^*, K^*\}$ at the prices $\{\tilde{w}, \tilde{r}\}$.

(c) Use the previous two arguments to conclude that $c(w, r, Q) \geq c(\tilde{w}, \tilde{r}, Q)$

23. What is the difference between accountants and economists? Name and define a cost that economists think is a cost and accountants don't, and a cost accountants think is a cost and economists don't.

24. Define Fixed Start Up costs. Are they part of variable costs? What is the difference between these costs and Fixed Sunk costs?

25. Let a firm's average costs be $\frac{C(q)}{q}$, and their marginal costs be $\frac{dC(q)}{dq}$. Prove that this firm's average costs equal their marginal costs at the quantity that minimizes average costs.

(a) Set up the objective function you would use to minimize average costs.

(b) Find the first order condition.

(c) Find a term in the first order condition that is equal to marginal costs, and another term that is equal to average costs.

(d) With some algebra prove the statement above.

26. Assume that a firm has a short run cost function given by $C^{SR}(w, r, K, Q) = \frac{1}{4}K^2 - 3\frac{QK}{rw}$.

(a) Find the firm's short run demand for labor using the envelope theorem. (Remember that w is the wage, or the cost of a unit of labor, and that this should be a function of w, r, K, and Q).

(b) Find the firm's long run cost function by using the envelope theorem.

(c) Define the general envelope theorem.

27. Assume that you produce output, Q, with two inputs: capital (K) and labor (L). The price of a unit of capital is r and the price of a unit of labor is w. Assume that $\{L, K\}$ is cost minimizing at the input prices $\{w, r\}$ and that $\left\{\hat{L}, \hat{K}\right\}$ is cost minimizing at $\{\hat{w}, \hat{r}\}$.

(a) What is the relationship between the cost of producing using inputs $\{L, K\}$ and using inputs $\left\{\hat{L}, \hat{K}\right\}$ at the prices $\{\hat{w}, \hat{r}\}$? Is the cost of using $\{L, K\}$ less than or more than the cost of using $\left\{\hat{L}, \hat{K}\right\}$?

(b) Assume that $\hat{w} \leq w$ and that $\hat{r} \leq r$. Prove that $C(w, r, Q) \geq C(\hat{w}, \hat{r}, Q)$, or that the cost function is non-decreasing in input prices.

28. Assume that the production function is $Q = 2L^{\frac{1}{2}} + K$ and that $Q > K$ for parts a through f.

(a) Set up the objective function you need to solve in order to find the short run cost function (where capital (K) is fixed).

(b) Find the first order conditions of the objective function you just set up.

(c) Explain why you do not need to actually find the first order conditions to find the short run cost function.

(d) Find the short run cost function.

(e) Explain what the envelope theorem tells us, you may use the short run cost function to illustrate the concept..

(f) Find the derivative of the short run cost function with respect to capital, K.

(g) Find the long run cost function, you can only get half credit if you do not use the envelope theorem. (You can assume that the optimal amount of labor and capital are both positive.)

29. If a firms production function is:

$$Q = (L+6)^{\frac{1}{3}} K^{\frac{2}{3}}$$

(a) Set up and solve the cost minimization problem, where the price of L (labor) is w and the price of K (capital) is r.
ASSUME THAT BOTH L AND K ARE DEMANDED IN STRICTLY POSITIVE QUANTITIES. (Or the solution is interior.)

i. Set up the Lagrangian.
ii. Solve for the first order conditions.
iii. Solve for the "buck for the bang" for both Labor and Capital.
iv. Solve for K in terms of L and input prices.
v. Solve for the demand curve for L using the production constraint.
vi. Solve for the demand curve for K

(b) If $r = 1$ find a condition on w such that the firm will use only capital in production.

30. Assume that you have a short run cost function:

$$C^{SR}(w, r, Q, K) = \frac{wQ^3}{K^{\frac{2}{3}}} + rK$$

(a) Find the marginal cost of the short run cost function, and establish whether it is increasing, decreasing or constant.
(b) Write down and explain the general envelope theorem.
(c) Using the envelope theorem solve for the input demand for capital and the long run cost function.

31. If a firm has the production function $Q = \ln L + 8 \ln K$ and the price of labor is w and the price of capital is r.

(a) Find the short run cost function (where K is fixed.)
(b) Explain the *general* envelope theorem. How can this be applied to find the long run cost function from the short run cost function?
(c) Using the envelope theorem find the long run cost function. (Note you should use the envelop theorem, you will only receive minor credit for deriving it directly.)

32. If your production function is $Q = f(L, K)$, the price of labor is w and the price of capital is r prove that if L^* is the cost minimizing demand given $\{Q, w, r\}$ then:

$$\frac{\Delta L^*}{\Delta w} \leq 0$$

Note you can only assume this person is a rational cost minimizer.

33. Guclu runs a cloth dyeing plant with three main inputs, labor—L, materials (cloth)—M, and capital (the physical plant)—K. His capital is fixed at $\bar{K}$. The cost of labor is w, of materials is w_m and the capital costs r dollars. His production function is:

$$Q = L^{\frac{1}{2}} M^{\frac{1}{4}} K^{\frac{1}{3}}$$

   (a) Set up his short run cost minimization problem, i.e. his Lagrangian.
   (b) Solve for his short run cost function.
   (c) Using the envelope theorem find his optimal level of capital and his long run cost function.
   (d) Explain the envelope theorem using cost minimization as an example. Why is it a surprising result and why is it true?

34. Why do economists and accountant differ on what is, and what is not a cost? What major class of costs do accountants consider not costs? What major class of accounting costs do economists consider not costs?

35. Consider the two cost functions

$$C_1(q) = 25 + \frac{1}{9}q^2$$
$$C_2(q) = 16 + 2q$$

   (a) If I tell you one is a long run cost function and one is a short run cost function associated with the same technology, which one is the long run cost function? And why?
   (b) What important result in producer theory is illustrated by the above?

36. Henry's Hippo Hippodrome (visualize it) uses 3 gardeners for each lawn mower they buy. The gardeners cost \$60 a day and lawn mowers rent for \$10 a day. If the MP of a gardener is 80 and of the lawn mower are 30 are they minimizing their cost? If not, how should they change their usage of gardeners and lawn mowers?

37. The production function of firm A is $Q = 10K^{.5}L^{.5}$ and of it's competitor (firm B) is $Q = 10K^{.2}L^{.8}$.

   (a) In the long run show that if they both use the same level of capital and labor ($K = L$) then they are equally efficient competitors.

(b) In the short run, (when capital is fixed, and equal to one for both firms) which of them is a better competitor (has a higher marginal product)?

38. You are in charge of cost control for a metropolitan bus company. A consultant finds that the cost of running a bus is independent of the number of people it carries, \$30 per run whether you have two people or fifty (the capacity). Thus during rush hour the average cost per passenger is 60 cents (when the buses are full) and off-hours the average cost \$1.67 because on average 18 people ride on each run. She suggests that you encourage more rush-hour business when costs are low, and discourage off-peak business. Do you agree with her? Discuss your answer.

3 Chapter 11—Profit Maximization

1. Assume that the cost function is $c(q) = 15q + 5q^3 + 32$, the fixed startup costs are $F_{st} = 10$.

   (a) Find the average total costs, average variable costs, and the marginal costs.

   (b) Find the price at which the firm will shut down.

   (c) Find the firm's supply curve.

2. Let the total costs of a firm be $TC(Q) = c(Q) + F_{st} + F_{su}$ where $F_{st} > 0$ are fixed start up costs and $F_{su} > 0$ are fixed sunk costs. Assume that $c'(Q) > 0$ and that $c''(Q) \geq 0$.

   (a) Define marginal costs, average costs, and average variable costs for this general cost function.

   (b) In a graph draw marginal costs, average costs, and average variable costs, make sure that the relationship between the functions is precise. If you do not trust your skills at drawing the relationship you may want to mention key points of your graph below.

   (c) In the same graph show the firm's supply curve.

3. Hulusi's Hen House produces eggs (E) using the production function $E = 12H^8F$, if the price of a Hens (H) is w_h and the price of Feed (F) is w_f.

   (a) Set up the Lagrangian and solve for the cost function.

   (b) Using the cost function find the profit function—be careful to check the second order conditions.

4. Assume that a firm has the production function

$$f(K, L) = K^{\frac{1}{2}} L^{\frac{3}{2}}$$

(a) If $K = L$, and the wage is 3, then what is the price of capital (r)?

(b) If the price of output is 1, and profits are 5 ($\pi = 5$) what is the level of output of the firm?

(c) Now solve for the profit function, being sure to check the second order conditions.

5. Assume there are four firms in an industry all with the same total cost function $C(q) = 2q^2 + 8$. Type a firms have a fixed sunk cost of 8 type b firms have a fixed start up cost of 8.

   (a) Find the marginal cost, average variable cost, and supply curve of firms of type a.

   (b) Find the marginal cost, average variable cost, and supply curve of firms of type b.

   (c) Explain why the two types of firms have different levels of sunk costs when they are using the same technology.