Practice Questions—Chapter 12 and 13

Equilibrium in Competitive Markets

ECON 203

Kevin Hasker

These questions are to help you prepare for the exams only. Do not turn them in. Note that not all questions can be completely answered using the material in the chapter in which they are asked. These are all old exam questions and often the answers will require material from more than one chapter. Questions with lower numbers were asked in more recent years.

1 Chp 12—The Partial Equilibrium Competitive Model

1. In a given market the market demand curve is $D(P) = 64 - 2P$ and the (domestic) market supply curve is $S(P) = -22 + P$. International firms will supply any amount desired at a price of 24.

   (a) Solve for the equilibrium price and quantity in this market.

$$\begin{aligned} 64 - 2P &= -22 + P \\ P &= \frac{86}{3} \\ Q &= -22 + \left(\frac{86}{3}\right) = \frac{20}{3} \end{aligned}$$

   obviously there was a mistake in this question, but it happens.

   (b) Now the government decides to impose an import tariff of τ per unit imported.

      i. For all τ, find the relationship between τ and the price in this market? *Hint: Be very careful to consider very high and very low τ.*

$$P = \begin{cases} 24 + \tau & \tau \le \frac{86}{3} - 24 = \frac{14}{3} \\ \frac{86}{3} & \tau \ge \frac{14}{3} \end{cases}$$

      :

      ii. If $\tau = 2$ solve for the equilibrium price and quantity in this market. What is the quantity imported?

$$\begin{aligned} P &= 24 + 2 \\ Q_d &= 64 - 2(26) = 12 \\ Q_{sd} &= -22 + (26) = 4 \\ Q_i &= Q_d - Q_{sd} \\ &= 12 - 4 = 8 \end{aligned}$$

iii. If $\tau = 2$ find the Consumer Surplus, Producer Surplus, Government Revenue, and Dead Weight Loss in this market. Partial credit will be given for an accurately labelled graph.

$$\begin{aligned} CS &= \frac{1}{2}\left(\frac{64}{2} - 26\right) 12 = 36 \\ PS &= \frac{1}{2}(26 - 22) 4 = 8 \\ R &= \tau Q_i = 2 * 8 = 16 \\ DWL &= \frac{1}{2}(26 - 24)(4 - (-22 + 24)) + \frac{1}{2}(26 - 24)(64 - 2(24) - 12) = 6 \end{aligned}$$

iv. Define a quota.

Solution 1 *A* quota *is a restriction on the quantity that can be imported.*

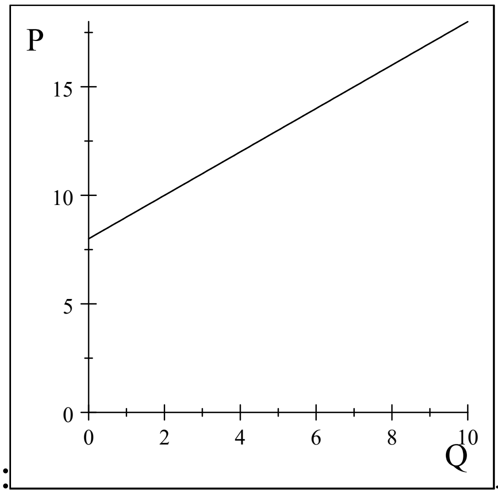
v. Find a quota that will result in the same price as the tariff you found above. What is the dead weight loss if the government imposes a quota instead of a tariff? (Consider only the welfare of the nation.)

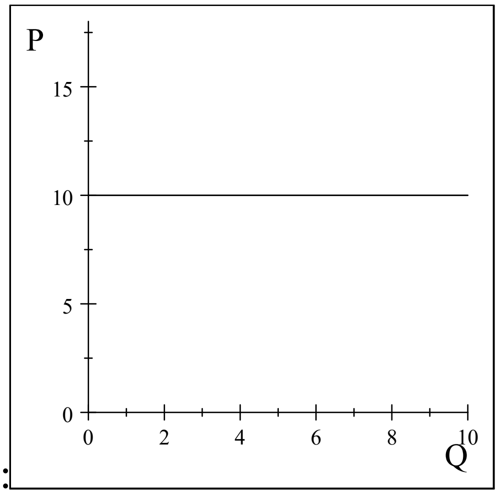
Solution 2 *The amount imported when the tariff is two is* 8*, so the quota is 8.*

vi. Does a quota or a tariff produce a higher dead weight loss? Why might the government want to use the method that produces a higher dead weight loss?

Solution 3 *This question is wrong if we are talking about the world's welfare. The profit to foreign firms should count as a good. From the point of view of National welfare, the tariff has a smaller DWL because the government gets a revenue of* 16*. However the fact that a tariff results in foreign firms making strictly positive profits makes it desirable for those firms, and easier to handle internationally.*

2. Consider the two supply curves ($S(P)$) below:

A:


B:


(a) Which supply curve has a higher slope ($\frac{\partial S(P)}{\partial P}$), A or B? Explain your answer.

Solution 4 *Since the slope we see in the graph is* $\frac{\partial P}{\partial Q}$ *and we want* $\frac{\partial Q}{\partial P}$ *graph B has a higher slope. In fact it looks infinite.*

(b) Both of these could represent long run supply curves. Explain under what conditions each could be a long run supply curve. Which is of these conditions are the ones we normally assume?

Solution 5 *Under the replication hypothesis in order to increase output we just build another plant, which will have the same cost as the previous plants. Thus flat long run supply curves (B) are most normal. In resource extraction industries increasing output will increase costs, and thus graph A is more normal for those industries.*

3. In a given industry the total costs are $c(q) = 200 + 2q + 2q^2$. There are $J_A = 12$ firms in the industry with a fixed start up costs of $F_{stA} = 2$. There are $J_B = 8$ firms in the industry with a fixed sunk costs of $F_{suB} = 182$.

(a) I claim that this it is fairly normal in industries that firms have different fixed start up costs. Why is this? Which type of firms (A or B) have probably been in the industry for longer? Why?

Solution 6 *Fixed costs are generally capital investments, as the capital gets older it will have to be replaced and sunk costs will change to start up costs. In the "long run" all the sunk costs are start up costs. Given this the start up costs for type B firms is* 18*, so they probably have been in the industry longer.*

(b) Find the marginal costs, average variable costs, and supply curve of a representative type A firm.

Solution 7

$$\begin{aligned} MC &= 2+4q \\ AVC &= \frac{2+2q+2q^2}{q} \end{aligned}$$

$$\begin{aligned} MC &\geq AVC \\ 2+4q &\geq \frac{2+2q+2q^2}{q} \\ (2+4q)\,q &\geq 2+2q+2q^2 \\ q &\geq 1 \end{aligned}$$

$$P_a^{sd} = 2+4\,(1) = 6$$

$$\begin{aligned} P &= 2+4q \\ q &= \frac{1}{4}P - \frac{1}{2} \end{aligned}$$

$$s_A\,(p) = \begin{cases} \frac{1}{4}P - \frac{1}{2} & P \geq 6 \\ 0 & P \leq 6 \end{cases}$$

(c) Find the marginal costs, average variable costs, and supply curve of a representative type B firm.

Solution 8

$$\begin{aligned} MC &= 2+4q \\ AVC &= \frac{200+2q+2q^2-182}{q} \end{aligned}$$

:

$$\begin{aligned} MC &\geq AVC \\ 2+4q &\geq \frac{1}{q}\left(2q^2+2q+18\right) \\ (2+4q)\,q &\geq 2q^2+2q+18 \\ q &\geq 3 \end{aligned}$$

$$P_a^{sd} = 2+4\,(3) = 14$$

:

$$\begin{aligned} P &= 2+4q \\ q &= \frac{1}{4}P - \frac{1}{2} \end{aligned}$$

$$s_B\,(p) = \begin{cases} \frac{1}{4}P - \frac{1}{2} & P \geq 14 \\ 0 & P \leq 14 \end{cases}$$

(d) Find the short run supply curve of this industry.

Solution 9

$$\begin{aligned}
S(p) &= J_A s_A(p) + J_B s_B(p) \\
&= \begin{cases} 12\left(\frac{1}{4}P - \frac{1}{2}\right) + 8\left(\frac{1}{4}P - \frac{1}{2}\right) & P > 14 \\ 12\left(\frac{1}{4}P - \frac{1}{2}\right) & 14 > P > 6 \\ 0 & P > 6 \end{cases} \\
&= \begin{cases} 5P - 10 & P > 14 \\ 3P - 6 & 14 > P > 6 \\ 0 & P > 6 \end{cases}
\end{aligned}$$

(e) Find the short run equilibrium in this industry if the demand is $D(P) = 86 - P$.

Solution 10 *We have to check several cases, we might have* $P > 14$ *or* $14 > P > 6$

Case 11 $P > 14$

$$\begin{aligned}
5P - 10 &= 86 - P \\
P &= 16
\end{aligned}$$

So this is the solution since the equilibrium price is above 14.

Case 12 $14 > P > 6$

$$\begin{aligned}
3P - 6 &= 86 - P \\
P &= 23 > 14
\end{aligned}$$

So this can not be the solution since the equilibrium price is above 14.

(f) Find the price at which firms will enter the market.

Solution 13

$$\begin{aligned}
MC &= 2 + 4q \\
AC &= \frac{200 + 2q + 2q^2}{q}
\end{aligned}$$

$$\begin{aligned}
MC &= AC \\
2 + 4q &= \frac{200 + 2q + 2q^2}{q} \\
q &= 10 \\
P_e &= 2 + 4(10) = 42
\end{aligned}$$

(g) If the quantity supplied in the medium run is 210 what is the market price and the number of firms in the industry?

Solution 14 *If no firms enter then we need:*

$$\begin{aligned} 210 &= 5P - 10 \\ P &= 44 \end{aligned}$$

since this is above the price of entry firms will enter, driving the price down to 42. Each firm will produce 10 *units so the number of firms will be:*

$$\begin{aligned} 210 &= J10 \\ J &= 21 \end{aligned}$$

(h) If the quantity supplied in the medium run is 190 what is the market price and the number of firms in the industry?

Solution 15 *If no firms enter then we need*

$$\begin{aligned} 190 &= 5P - 10 \\ P &= 40 < 42 \end{aligned}$$

so there will be $J_A + J_B = 20$ *firms in the industry.*

4. Consider the market for tomatoes.

   (a) What is the general problem in this market that creates political pressure on governments to intervene? A detailed answer is expected.
   The basic problem in this and all agricultural markets is that output is stochastic. After inputs have been determined output may be either high or low. Even worse than this is the fact that this may result in farmers getting less revenue in good years (when price is low even though the quantity is high) than in bad years (when the price is high, but the quantity is low.)

   (b) What is the normal form of government intervention in this market?
   Governments generally impose price floors in these markets, and furthermore enters the market to buy up the excess output to maintain the price floor.

   (c) If governments could costlessly store the product what is another method of intervention in this market? Could this theoretically produce a superior outcome? Why doesn't this method work in reality?
   They could buy in good years and store it until bad years. Theoretically this would result in farmers getting the same price in every year which would be better for both consumers and farmers. Unfortunately farmers usually do not like having a lower price in bad

years, so governments end up not selling the produce. Furthermore if it really was possible for the government to make a profit doing this then private firms would, and they already do to a certain extent. Thus this method (when implemented) generally just results in the government never selling, making it into a price support.

(d) Why might governments want to pay farmers not to produce in this market?

If the price floor is too high then there will be a permanent oversupply of output. If the government pays the farmers to not produce they are saving themselves from having to dispose of the output after the harvest and the farmers do not have to pay for the cost of production, a clear win win situation. A twisted solution to a twisted problem, but a clear win win solution.

5. In a given market the market demand curve is $D(P_d) = 24 - P_d$ and the market supply curve is $S(P_s) = -36 + 3P_s$.

(a) Solve for the equilibrium price and quantity in this market.

$$\begin{aligned} 24 - P &= -(36) + (3)P \\ P &= \frac{24 + (36)}{1 + (3)} \\ Q &= 24 - P = 24 - \frac{24 + (36)}{1 + (3)} \\ &= \frac{24(3) - (36)}{1 + (3)} \end{aligned}$$

(b) Now the government is considering a per-unit tax of t =4

i. What relationship will hold in equilibrium between $P_{(3)}$ and P_s independent of the supply and demand curve?

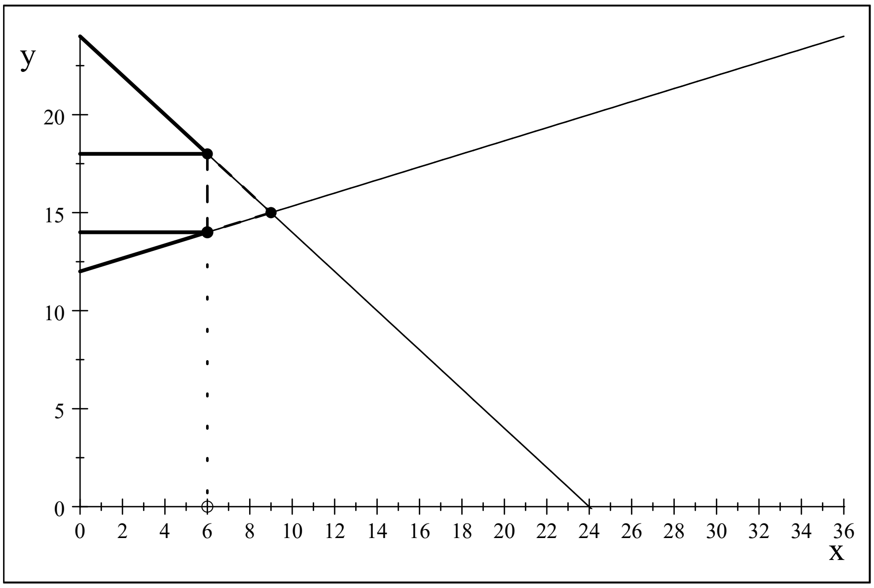
$$P_d = P_s + 4$$

ii. Solve for the equilibrium prices and quantity in this market.

$$\begin{aligned} 24 - P_d &= -(36) + (3)P_s \\ 24 - (P_s + 4) &= -(36) + (3)P_s \\ P_s &= \frac{1}{1 + (3)}(24 + (36) - 4) \\ P_d &= \frac{1}{1 + (3)}(24 + (36) - 4) + 4 = \frac{1}{1 + (3)}(24 + (36) + (3)4) \\ Q &= -(36) + (3)P_s = -(36) + (3)\left(\frac{1}{1 + (3)}(24 + (36) - 4)\right) \\ &= -\frac{1}{1 + (3)}((36) - 24(3) + (3)4) \end{aligned}$$

iii. Carefully graph the supply, demand, equilibrium prices and quantity, consumer surplus, producer surplus, and dead weight loss in the graph below.

I will graph this for $D(P_d) = 24 - P_d$, $S(P_s) = -36 + 3P_s$. *and* $\tau = 4$.



The upper dark triangle is consumer surplus, the lower dark triangle is producer surplus, the dashed triangle on the right is dead weight loss, and the box between CS, PS, DWL and the vertical axis is government revenue. The equilibrium prices are the lower border of CS and the upper border of PS (18 and 14 respectively) and the equilibrium quantity is the dotted line where output is equal to 6.

iv. What is the Consumer Surplus, Producer Surplus, and Dead weight loss?

If the graph is drawn properly these numbers can be read off of *it, if it is not then below are the general formulas.*

$$\begin{aligned}
CS &= \frac{1}{2}Q\left(\frac{24}{(1)}-\left(\frac{24}{(1)}-\frac{1}{(1)}Q\right)\right)\\
&= \frac{1}{2}\frac{Q^2}{(1)}\\
&= \frac{1}{2(1)\left((1)+(3)\right)^2}\left((1)(36)-24(3)+(1)(3)4\right)^2\\
PS &= \frac{1}{2}Q\left(\frac{(36)}{(3)}+\frac{1}{(3)}Q-\frac{(36)}{(3)}\right)\\
&= \frac{1}{2}\frac{Q^2}{(3)}\\
&= \frac{1}{2(3)\left((1)+(3)\right)^2}\left((1)(36)-24(3)+(1)(3)4\right)^2\\
DWL &= \frac{1}{2}4\left(Q(0)-Q(4)\right)\\
&= \frac{1}{2}4\left(-\frac{1}{(1)+(3)}\left((1)(36)-24(3)\right)-\left(-\frac{1}{(1)+(3)}\left((1)(36)-24(3)+(1)(3)4\right)\right)\right)\\
&= \frac{1}{2}(1)(3)\frac{4^2}{(1)+d}
\end{aligned}$$

(c) Now consider a general per-unit tax of t.

i. *(4 points)* Solve for the equilibrium prices and quantity in this market.

$$\begin{aligned}
24-(1)P_d &= -(36)+(3)P_s\\
24-(1)\left(P_s+t\right) &= -(36)+(3)P_s\\
P_s &= \frac{1}{(1)+(3)}\left(24+(36)-(1)t\right)\\
P_d &= \frac{1}{(1)+(3)}\left(24+(36)-(1)t\right)+t=\frac{1}{(1)+(3)}\left(24+(36)+(3)t\right)\\
Q &= -(36)+(3)P_s=-(36)+(3)\left(\frac{1}{(1)+(3)}\left(24+(36)-(1)t\right)\right)\\
&= -\frac{1}{(1)+(3)}\left((1)(36)-24(3)+(1)(3)t\right)
\end{aligned}$$

ii. *(3 points)* What is the Dead weight loss in this market? (The answer should be a function of t)

$$\begin{aligned} DWL &= \frac{1}{2}t\left(Q\left(0\right)-Q\left(t\right)\right) \\ &= \frac{1}{2}t\left(-\frac{1}{(1)+(3)}\left((1)(36)-24(3)\right)-\left(-\frac{1}{(1)+(3)}\left((1)(36)-24(3)+(1)(3)t\right)\right)\right) \\ &= \frac{1}{2}(1)(3)\frac{t^2}{(1)+(3)} \end{aligned}$$

6. In a given market the firms have a cost of $c\left(q\right)=25+11q+q^2$ with a fixed sunk cost of 21. There are currently 14 firms in this market.

   (a) Find the marginal costs, average variable costs, average costs, the price at which firms will shut down and the price at which firms will enter.

$$\begin{aligned} MC &= 11+2(1)q \\ AVC &= \frac{25-21+11q+(1)q^2}{q} \\ AC &= \frac{25+11q+(1)q^2}{q} \\ MC\left(q_{sd}\right) &= 11+2(1)q_{sd}=\frac{25-21+11q_{sd}+(1)q_{sd}^2}{q_{sd}}=AVC\left(q_{sd}\right) \\ q_{sd} &= \sqrt{\frac{25-21}{(1)}} \\ P_{sd} &= MC\left(q_{sd}\right)=11+2(1)\sqrt{\frac{25-21}{(1)}} \\ MC\left(q_e\right) &= 11+2(1)q_e=\frac{25+11q_e+(1)q_e^2}{q_e}=AC\left(q_e\right) \\ q_e &= \sqrt{\frac{25}{(1)}} \\ P_e &= MC\left(q_e\right)=11+2(1)\sqrt{\frac{25}{(1)}} \end{aligned}$$

   (b) Find the short run supply curve of a firm and this industry. If the equilibrium demand curve is $D\left(P\right)=59-P$ find the equilibrium price and quantity in this market.

$$\begin{aligned}
P &= MC\left(q\right) = 11 + 2(1)q \\
q\left(P\right) &= \frac{1}{2(1)}\left(P-11\right) \\
s\left(P\right) &= \begin{cases} \frac{1}{2}\left(P-11\right) & P \geq 15 \\ 0 & P \leq 15 \end{cases} \\
S\left(P\right) &= 14s\left(P\right) \\
S\left(P\right) &= \begin{cases} \frac{14}{2(1)}\left(P-11\right) & P > 15 \\ 0 & P < 15 \end{cases}
\end{aligned}$$

If $P > 11 + 2\left(1\right)\sqrt{\frac{25-21}{(1)}} = 15$

$$\begin{aligned}
\frac{1}{2(1)}\left(P-11\right)14 &= 59 - P \\
P &= \frac{(14)11 + 2(59)(1)}{(14) + 2(1)(1)} = 17 > 15 = 11 + 2(1)\sqrt{\frac{25-21}{(1)}} \\
Q &= 59 - (1)\frac{(14)11 + 2(59)(1)}{(14) + 2(1)(1)} = (14)\frac{(59) - (1)11}{(14) + 2(1)(1)} \\
&= 42
\end{aligned}$$

(c) Assume that in the medium run the costs of the firms do not change (including the sunk costs). Find the industry's medium run supply curve. If the quantity supplied in the medium run is Q_{m1} what is the price and number of firms? If the quantity in the medium run is Q_{m2} what is the price and the number of firms?

$$S\left(P\right) = \begin{cases} \infty & P > 11 + 2(1)\sqrt{\frac{25}{(1)}} \\ \frac{K}{2(1)}\left(P-11\right) = K\sqrt{\frac{25}{(1)}} & P = 11 + 2(1)\sqrt{\frac{25}{(1)}} \\ \frac{(14)}{2(1)}\left(P-11\right) & 11 + 2(1)\sqrt{\frac{25}{(1)}} > P > 11 + 2(1)\sqrt{\frac{25-21}{(1)}} \\ 0 & P < 11 + 2(1)\sqrt{\frac{25-21}{(1)}} \end{cases}$$

for $K \geq (14)$. *If* $11+2(1)\sqrt{\frac{25}{(1)}} > P > 11+2(1)\sqrt{\frac{25-21}{(1)}}$

$$\begin{aligned}
\frac{1}{2(1)}\left(P_{m1}-11\right)(14) &= Q_{m1} \\
11 + 2(1)\sqrt{\frac{25-21}{(1)}} &< P_{m1} = 11 + \frac{2}{(14)}(1)Q_{m1} < 11 + 2(1)\sqrt{\frac{25}{(1)}}
\end{aligned}$$

If $11{+}2(1)\sqrt{\frac{25}{(1)}} > P > 11{+}2(1)\sqrt{\frac{25-21}{(1)}}$

$$\begin{aligned}
\frac{1}{2(1)}\left(P_{m2}-11\right)(14) &= Q_{m2} \\
P_{m2} &= 11+\frac{2}{(14)}(1)Q_{m2} > 11+2(1)\sqrt{\frac{25}{(1)}}
\end{aligned}$$

Thus $P_{m2} = 11{+}2(1)\sqrt{\frac{25}{(1)}}$ *and there must be firms entering, exactly how many can be found by:*

$$\begin{aligned}
\sqrt{\frac{25}{(1)}}(14)_{m2} &= Q_{m2} \\
(14)_{m2} &= \frac{Q_{2m}}{\sqrt{\frac{25}{(1)}}}
\end{aligned}$$

7. In a given market the firms have a cost of $c(q) = 25 + 11q + q^2$ with a fixed sunk cost of 21. There are currently 14 firms in this market.

   (a) Find the marginal costs, average variable costs, average costs, the price at which firms will shut down and the price at which firms will enter.

$$\begin{aligned}
MC &= 11+2(1)q \\
AVC &= \frac{25-21+11q+(1)q^2}{q} \\
AC &= \frac{25+11q+(1)q^2}{q} \\
MC\left(q_{sd}\right) &= 11+2(1)q_{sd} = \frac{25-21+11q_{sd}+(1)q_{sd}^2}{q_{sd}} = AVC\left(q_{sd}\right) \\
q_{sd} &= \sqrt{\frac{25-21}{(1)}} \\
P_{sd} &= MC\left(q_{sd}\right) = 11+2(1)\sqrt{\frac{25-21}{(1)}} \\
MC\left(q_e\right) &= 11+2(1)q_e = \frac{25+11q_e+(1)q_e^2}{q_e} = AC\left(q_e\right) \\
q_e &= \sqrt{\frac{25}{(1)}} \\
P_e &= MC\left(q_e\right) = 11+2(1)\sqrt{\frac{25}{(1)}}
\end{aligned}$$

(b) Find the short run supply curve of a firm and this industry. If the equilibrium demand curve is $D(P) = 59 - P$ find the equilibrium price and quantity in this market.

$$\begin{aligned}
P &= MC(q) = 11 + 2(1)q \\
q(P) &= \frac{1}{2(1)}(P - 11) \\
s(P) &= \begin{cases} \frac{1}{2}(P-11) & P \geq 15 \\ 0 & P \leq 15 \end{cases} \\
S(P) &= 14s(P) \\
S(P) &= \begin{cases} \frac{14}{2(1)}(P-11) & P > 15 \\ 0 & P < 15 \end{cases}
\end{aligned}$$

If $P > 11 + 2(1)\sqrt{\frac{25-21}{(1)}} = 15$

$$\begin{aligned}
\frac{1}{2(1)}(P-11)\,14 &= 59 - P \\
P &= \frac{(14)11 + 2(59)(1)}{(14) + 2(1)(1)} = 17 > 15 = 11 + 2(1)\sqrt{\frac{25-21}{(1)}} \\
Q &= 59 - (1)\frac{(14)11 + 2(59)(1)}{(14) + 2(1)(1)} = (14)\frac{(59) - (1)11}{(14) + 2(1)(1)} \\
&= 42
\end{aligned}$$

(c) Assume that in the medium run the costs of the firms do not change (including the sunk costs). Find the industry's medium run supply curve. If the quantity supplied in the medium run is Q_{m1} what is the price and number of firms? If the quantity in the medium run is Q_{m2} what is the price and the number of firms?

$$S(P) = \begin{cases} \infty & P > 11 + 2(1)\sqrt{\frac{25}{(1)}} \\ \frac{K}{2(1)}(P-11) = K\sqrt{\frac{25}{(1)}} & P = 11 + 2(1)\sqrt{\frac{25}{(1)}} \\ \frac{(14)}{2(1)}(P-11) & 11 + 2(1)\sqrt{\frac{25}{(1)}} > P > 11 + 2(1)\sqrt{\frac{25-21}{(1)}} \\ 0 & P < 11 + 2(1)\sqrt{\frac{25-21}{(1)}} \end{cases}$$

for $K \geq (14)$. *If* $11+2(1)\sqrt{\frac{25}{(1)}} > P > 11+2(1)\sqrt{\frac{25-21}{(1)}}$

$$\begin{aligned}
\frac{1}{2(1)}(P_{m1} - 11)(14) &= Q_{m1} \\
11 + 2(1)\sqrt{\frac{25-21}{(1)}} &< P_{m1} = 11 + \frac{2}{(14)}(1)Q_{m1} < 11 + 2(1)\sqrt{\frac{25}{(1)}}
\end{aligned}$$

If $11{+}2(1)\sqrt{\frac{25}{(1)}} > P > 11{+}2(1)\sqrt{\frac{25-21}{(1)}}$

$$\begin{aligned}\frac{1}{2(1)}\left(P_{m2}-11\right)(14) &= Q_{m2}\\ P_{m2} &= 11+\frac{2}{(14)}(1)Q_{m2} > 11+2(1)\sqrt{\frac{25}{(1)}}\end{aligned}$$

Thus $P_{m2} = 11{+}2(1)\sqrt{\frac{25}{(1)}}$ *and there must be firms entering, exactly how many can be found by:*

$$\begin{aligned}\sqrt{\frac{25}{(1)}}(14)_{m2} &= Q_{m2}\\ (14)_{m2} &= \frac{Q_{2m}}{\sqrt{\frac{25}{(1)}}}\end{aligned}$$

(d) Find a firm and the industry's long run supply curve. If the quantity supplied is Q_l what is the equilibrium price and number of firms?

The equilibrium price is $P_l = 11{+}2(1)\sqrt{\frac{25}{(1)}}$, *the equilibrium number of firms is:*

$$\begin{aligned}J_l\sqrt{\frac{25}{(1)}} &= Q_l\\ J_l &= \frac{Q_l}{\sqrt{\frac{25}{(1)}}}\end{aligned}$$

8. In a given industry the supply curve is $Q_s = -23+3\ln P$ and the demand curve is $Q_d = \delta - P^3$.

(a) Verify that $\frac{\partial Q_s}{\partial P} > 0$ and that $\frac{\partial Q_d}{\partial P} < 0$.

$$\begin{aligned}\frac{\partial Q_s}{\partial P} &= \frac{(3)}{P} > 0\\ \frac{\partial Q_d}{\partial P} &= -(1)(3)P^{(3)-1} < 0\end{aligned}$$

(b) Find $\frac{\partial P}{\partial \delta}$ and $\frac{\partial Q}{\partial \delta}$.

$$\begin{aligned}
(-23) + (3)\ln P &= \delta - (1)P^{(3)} \\
\frac{(3)}{P}\frac{\partial P}{\partial \delta} &= 1 - (1)(3)P^{(3)-1}\frac{\partial P}{\partial \delta} \\
1 &= \left(\frac{(3)}{P} + (1)(3)P^{(3)-1}\right)\frac{\partial P}{\partial \delta} \\
\frac{\partial P}{\partial \delta} &= \frac{1}{\frac{(3)}{P} + (1)(3)P^{(3)-1}} \\
&= \frac{P}{(3) + P^{(3)}(1)(3)} \\
Q &= (-23) + (3)\ln P \\
\frac{\partial Q}{\partial \delta} &= \frac{(3)}{P}\frac{\partial P}{\partial \delta} \\
&= \frac{(3)}{P}\frac{1}{\frac{(3)}{P} + (1)(3)P^{(3)-1}} \\
&= \frac{(3)}{(3) + P^{(3)}(1)(3)}
\end{aligned}$$

(c) Verify that $\frac{\partial P}{\partial \delta} = \frac{\frac{\partial Q_d}{\partial \delta}}{\frac{\partial Q_s}{\partial P} - \frac{\partial Q_d}{\partial P}}$.

$$\begin{aligned}
\frac{\partial Q_d}{\partial \delta} &= 1 \\
\frac{\partial Q_s}{\partial P} &= \frac{(3)}{P} \\
\frac{\partial Q_d}{\partial P} &= -(1)(3)P^{(3)-1} \\
\frac{\frac{\partial Q_d}{\partial \delta}}{\frac{\partial Q_s}{\partial P} - \frac{\partial Q_d}{\partial P}} &= \frac{1}{\frac{(3)}{P} + (1)(3)P^{(3)-1}} = \frac{\partial P}{\partial \delta}
\end{aligned}$$

9. How can a long run supply curve be upward sloping? Give an example of an industry where you think it should be upward sloping. How can it be downward sloping? Again give an example. Why do we generally assume that it is neither upward or downward sloping? Please note that we are discussing $\frac{\partial P}{\partial Q}$, or a graph where quantity is on the horizontal axis.

   It can be upward sloping if some inputs are in limited supply. For example all natural resource extraction industries have upward sloping supply because the marginal cost of extracting yet more resources is always increasing.

   It can be downward sloping because of network effects, i.e. delivery costs are generally decreasing as output is increasing. For example Chinese food in Turkey seems to have a downward sloping long run supply curve. Since *it is nearly as cheap to ship one case of soy sauce to Ankara as it is to ship 1000 cases costs decrease as the quantity of soy sauce sold increases.*

We generally assume long run supply is flat because of the **replication hypothesis**. *This hypothesis is that the easiest way to expand output is just to build another factory. Since the cost of this factory will be the same as the costs of the firms already in the industry the price should not increase or decrease.*

10. In a given market the market demand curve is $D(P_b) = 60 - 4P_b$ and the (domestic) market supply curve is $S(P_s) = -6 + 2P_s$.

    (a) Solve for the equilibrium price and quantity in this market.

$$\begin{aligned}
P_b &= P_s = P \\
60 - 4P &= -6 + 2P \\
P &= \frac{60+6}{4+2} \\
Q &= 60 - 4\left(\frac{60+(6)}{4+(2)}\right) = \frac{60(2) - 4(6)}{4+(2)} \\
Q &= -(6) + (2)\left(\frac{60+(6)}{4+(2)}\right) = \frac{60(2) - 4(6)}{4+(2)}
\end{aligned}$$

    (b) Now the government is considering a price ceiling of (9).

    i. What constraint does this impose on the market price?

$$P \leq (9)$$

    ii. Solve for the equilibrium price and quantity in this market.

$$Q_{traded} = \min\left(60 - 4(9), -(6) + (2)(9)\right) = -(6) + (2)(9)$$

    iii. Find the Consumer Surplus, Producer Surplus, and Dead Weight Loss in this market. You will get partial credit for a graph.

$$\begin{aligned}
CS &= \frac{1}{2}\left(\bar{P} - \left(\frac{60}{4} - \frac{1}{4}Q_{traded}\right)\right)Q_{traded} + \left[\left(\frac{60}{4} - \frac{1}{4}Q_{traded}\right) - (9)\right]Q_{traded} \\
&= \left[\frac{1}{2}\bar{P} + \frac{1}{2}\left(\frac{60}{4} - \frac{1}{4}Q_{traded}\right) - (9)\right]Q_{traded} \\
&= \left[\frac{1}{2}\frac{60}{4} + \frac{1}{2}\left(\frac{60}{4} - \frac{1}{4}(-(6) + (2)(9))\right) - (9)\right](-(6) + (2)(9)) \\
&= \frac{1}{2(4)}\left(260 - 2(4)(9) + (6) - (2)(9)\right)(-(6) + (2)(9)) \\
&= \frac{1}{2(4)}\left(2D((9)) - S((9))\right)S((9))
\end{aligned}$$

:

$$\begin{aligned} PS &= \frac{1}{2}\left((9)-\underline{P}\right)Q_{traded} \\ &= \frac{1}{2}\left((9)-\frac{(6)}{(2)}\right)Q_{traded} \\ &= \frac{1}{2(2)}\left(Q_{traded}\right)^2 = \frac{1}{2(2)}\left(S\left((9)\right)\right)^2 \end{aligned}$$

$$\begin{aligned} DWL &= \frac{1}{2}\left(\frac{60}{4}-\frac{1}{4}Q_{traded}-(9)\right)\left(Q^*-Q_{traded}\right) \\ &= \frac{1}{2}\left(\frac{60}{4}-\frac{1}{4}\left(-(6)+(2)(9)\right)-(9)\right)\left(\frac{60(2)-4(6)}{4+(2)}-\left(-(6)+(2)(9)\right)\right) \\ &= \frac{1}{24}\frac{(2)}{4+(2)}\left(60-4(9)+(6)-(2)(9)\right)^2 = 2 \\ &= \frac{1}{72}\left(D\left((9)\right)-S\left((9)\right)\right)^2 \end{aligned}$$

(c) Now the government decides to allow imports, international firms will supply any amount desired at a price of (9). The government will also impose a import tariff of τ per unit imported.

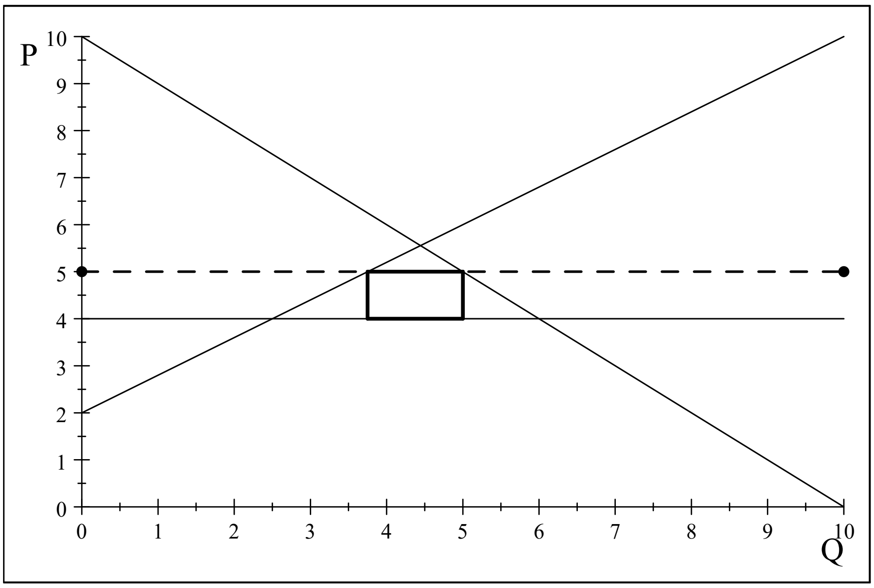
i. What is the relationship between τ and the price in this market? *Hint: Be very careful to consider very high and very low τ.*

$$P = \begin{cases} (9)+\tau & \text{if} \quad \tau \le \frac{60+(6)}{4+(2)}-(9) \\ \frac{60+(6)}{4+(2)} & \text{if} \quad \tau \ge \frac{60+(6)}{4+(2)}-(9) \end{cases}$$

ii. Solve for the equilibrium price and quantity in this market for all τ.
If $\tau \ge \frac{60+(6)}{4+(2)}-(9)$ then we solved this above. Thus assume not.

$$\begin{aligned} Q &= D\left((9)+\tau\right) = 60-4\left((9)+\tau\right) \\ Q_{domestic} &= -(6)+(2)\left((9)+\tau\right) \\ Q_{import} &= 60-4\left((9)+\tau\right)-\left(-(6)+(2)\left((9)+\tau\right)\right) \end{aligned}$$

iii. In the graph below indicate the Consumer Surplus, Producer Surplus, Government Revenue, and Dead Weight Loss in this market when τ is low. Your scale does not have to be precise.



The dark box is the government revenue, the consumer surplus is above the dotted line and below the demand curve, the producer surplus is below the dotted line and above the supply curve. The dead weight loss is the two little triangles to either side of the dark box.

iv. Find the Consumer Surplus, Producer Surplus, Government Revenue, and Dead Weight Loss in this market for all τ.
First if τ is high then we are in case a, except that now there will be dead weight loss.

$$\begin{aligned} DWL &= \frac{1}{2}\left(P^* - (9)\right)\left(D\left((9)\right) - S\left((9)\right)\right) \\ &= \frac{1}{2}\left(\frac{60+(6)}{4+(2)} - (9)\right)\left(60 - 4(9) - (-(6) + (2)(9))\right) \\ &= \frac{1}{2}\frac{1}{4+(2)}\left(60 + (6) - (4+(2))\,(9)\right)^2 \end{aligned}$$

If τ is low then

$$\begin{aligned}
CS &= \frac{1}{2}\left(\frac{60}{4} - ((9)+\tau)\right)(60 - 4((9)+\tau)) \\
&= \frac{1}{2(4)}(60 - 4((9)+\tau))^2 \\
PS_d &= \frac{1}{2}\left(((9)+\tau) - \left(\frac{(6)}{(2)}\right)\right)(-(6) + (2)((9)+\tau)) \\
&= \frac{1}{2((2))}(-(6) + (2)((9)+\tau))^2 \\
GR &= \tau Q_{imported} \\
&= \tau(60 - 4((9)+\tau) - (-(6) + (2)((9)+\tau))) \\
&= \tau(60 + (6) - (4+(2))((9)+\tau)) \\
DWL &= \frac{1}{2}\tau(-(6) + (2)((9)+\tau) - (-(6) + (2)(9))) + \frac{1}{2}\tau(60 - 4(9) - (60 - 4((9)+\tau)) \\
&= \frac{1}{2}\tau^2(4 + (2))
\end{aligned}$$

11. What are the arguments in favor of an import quota versus an import tariff? What are the arguments in favor of an import tariff versus an import quota?

    In favor of a quota is that the profits from the quota go to the importing firm, thus making it more acceptable to the international firms importing to a country.

    In favor of a tariff is that the revenue from the tariff goes to the government, increasing domestic welfare. Furthermore tariffs do not force the government to choose which countries and firms can import, reducing the political difficulty of determining this value.

12. Hoozools are a very rare and tasty bird that grows to its best on high mountains, where the air is thin and the weather cool. Only 20 farms can produce in the mountains, and they have a cost function of $c_m(q) = 50 + 2q^2$, and their fixed start up costs are 8. There are 6 farms raising hoozools in the valleys, their costs are $c_v(q) = 50 + 4q + 2q^2$, and their fixed start up costs are zero.

    (a) Find the marginal costs, average variable costs, and supply curve of a representative mountain firm (type m).

$$\begin{aligned}
MC &= (0) + 2(2)q \\
AVC &= \frac{(8) + (0)q + (2)q^2}{q} \\
(0) + 2(2)q &= \frac{(8) + (0)q + (2)q^2}{q} \\
q &= \sqrt{\frac{(8)}{(2)}} \\
P_{sd} &= (0) + 2(2)\sqrt{\frac{(8)}{(2)}} \\
(0) + 2(2)q &= P \\
q &= -\frac{(0)}{2(2)} + \frac{P}{2(2)} \\
s_m(p) &= \begin{cases} -\frac{(0)}{2(2)} + \frac{P}{2(2)} & \text{if} \quad P \geq (0) + 2(2)\sqrt{\frac{(8)}{(2)}} \\ 0 & \text{if} \quad P \leq (0) + 2(2)\sqrt{\frac{(8)}{(2)}} \end{cases}
\end{aligned}$$

(b) Find the marginal costs, average variable costs, and supply curve of a representative valley firm (type v).

$$\begin{aligned}
MC &= (4) + 2(2)q \\
AVC &= \frac{(4)q + (2)q^2}{q} \\
(4) + 2(2)q &= \frac{(4)q + (2)q^2}{q} \\
q &= 0 \\
P_{sd} &= (4) + 2(2)(0) = (4) \\
(4) + 2(2)q &= P \\
q &= -\frac{(4)}{2(2)} + \frac{P}{2(2)} \\
s_v(p) &= \begin{cases} -\frac{(4)}{2(2)} + \frac{P}{2(2)} & \text{if} \quad P \geq (4) \\ 0 & \text{if} \quad P \leq (4) \end{cases}
\end{aligned}$$

(c) *(3 points)* Find the short run supply curve of this industry.

$$S(P) = \begin{cases} \left(-\frac{(4)}{2(2)} + \frac{P}{2(2)}\right)(6) + \left(-\frac{(0)}{2(2)} + \frac{P}{2(2)}\right)(20) & \text{if} \quad P > (0) + 2(2)\sqrt{\frac{(8)}{(2)}} \\ \left(-\frac{(4)}{2(2)} + \frac{P}{2(2)}\right)(6) & \text{if} \quad (0) + 2(2)\sqrt{\frac{(8)}{(2)}} > P > (4) \\ 0 & \text{if} \quad P < (4) \end{cases}$$

(d) *(4 points)* Find the short run equilibrium in this industry if the demand for Hoozool meat is $D(P) = (9) - (1)P$. *Hint: This could be considered a trick question, check your answer carefully.*

$$\begin{aligned}\left(-\frac{(4)}{2(2)}+\frac{P}{2(2)}\right)(6)+\left(-\frac{(0)}{2(2)}+\frac{P}{2(2)}\right)(20) &= (9)-(1)P \\ P &= \frac{2(2)(9)+(0)(20)+(4)(6)}{(20)+(6)+2(2)(1)} < (0)+2(2)\sqrt{\frac{(8)}{(2)}}\end{aligned}$$

so the mountain firms will not produce.

$$\begin{aligned}\left(-\frac{(4)}{2(2)}+\frac{P}{2(2)}\right)(6) &= (9)-(1)P \\ (0)+2(2)\sqrt{\frac{(8)}{(2)}} &> P = \frac{2(2)(9)+(4)(6)}{(6)+2(2)(1)} > (4)\end{aligned}$$

so this is the equilibrium.

(e) Assume that in the medium run the costs of the firms do not change (including the start up costs). Notice that there can not be entry by mountain farms. Find the industry's medium run supply curve.

$$\begin{aligned}AC &= \frac{50+(4)q+(2)q^2}{q} = (4)+2(2)q = MC \\ q_e &= \sqrt{\frac{50}{(2)}} \\ P_e &= (4)+2(2)\sqrt{\frac{50}{(2)}}\end{aligned}$$

$$S(P) = \begin{cases} \infty & \text{if} \quad P > (4)+2(2)\sqrt{\frac{50}{(2)}} \\ \sqrt{\frac{50}{(2)}}K + \left(\frac{(4)-(0)}{2(2)} + \sqrt{\frac{1}{(2)}50}\right)(20) & \text{if} \quad P = (4)+2(2)\sqrt{\frac{50}{(2)}} \\ \left(-\frac{(4)}{2(2)}+\frac{P}{2(2)}\right)(6)+\left(-\frac{(0)}{2(2)}+\frac{P}{2(2)}\right)(20) & \text{if} \quad P > (0)+2(2)\sqrt{\frac{(8)}{(2)}} \\ \left(-\frac{(4)}{2(2)}+\frac{P}{2(2)}\right)(6) & \text{if} \quad (0)+2(2)\sqrt{\frac{(8)}{(2)}} > P > (4) \\ 0 & \text{if} \quad P < (4) \end{cases}$$

where $K \geq (6)$.

(f) Find the industry's long run supply curve. Notice that there still can not be entry by mountain farms.

$$\begin{aligned} AC_m &= \frac{50+(0)q+(2)q^2}{q} = (0)+2(2)q = MC_m \\ q_{sd} &= \sqrt{\frac{50}{(2)}} \\ P_{sd} &= (0)+2(2)\sqrt{\frac{50}{(2)}} \end{aligned}$$

$$S(P) = \begin{cases} \infty & \text{if} \quad P > (4)+2(2)\sqrt{\frac{50}{(2)}} \\ \sqrt{\frac{50}{(2)}}K + \left(\frac{(4)-(0)}{2(2)} + \sqrt{\frac{1}{(2)}50}\right)(20) & \text{if} \quad P = (4)+2(2)\sqrt{\frac{50}{(2)}} \\ \left(-\frac{(0)}{2(2)} + \frac{P}{2(2)}\right)(20) & \text{if} \quad (4)+2(2)\sqrt{\frac{50}{(2)}} > P > (0)+2(2)\sqrt{\frac{50}{(2)}} \\ 0 & \text{if} \quad P < (0)+2(2)\sqrt{\frac{50}{(2)}} \end{cases}$$

where $K \geq 0$.

(g) Find the long run equilibrium in this industry if the demand for Hoozool meat is $D(P) = (260) - (5)P$. To be specific find the price and quantity per firm, the quantity in the market, and the number of valley farms in equilibrium.

$$(260)-(5)\left((4)+2\sqrt{(2)50}\right) > \left(\frac{(4)-(0)}{2(2)} + \sqrt{\frac{1}{(2)}50}\right)(20)$$

thus valley firms produce.

$$\begin{aligned} P &= (4)+2\sqrt{(2)50} \\ Q &= (260)-(5)\left((4)+2\sqrt{(2)50}\right) \\ q_v &= \sqrt{\frac{50}{(2)}} \\ q_m &= \frac{(4)-(0)}{2(2)} + \sqrt{\frac{1}{(2)}50} \\ n_v &= \frac{(260)-(5)\left((4)+2\sqrt{(2)50}\right) - (20)\left(\frac{(4)-(0)}{2(2)} + \sqrt{\frac{1}{(2)}50}\right)}{\sqrt{\frac{50}{(2)}}} \end{aligned}$$

(h) Find the profits of both types of firms in equilibrium. What do we call the difference between their profits? Define the term you use.

$$
\begin{aligned}
\pi_v &= \left((4) + 2\sqrt{(2)50}\right)\sqrt{\frac{50}{(2)}} - \left(50 + (4)\left(\sqrt{\frac{50}{(2)}}\right) + (2)\left(\sqrt{\frac{50}{(2)}}\right)^2\right) \\
\pi_v &= \left((4) + 2\sqrt{(2)50}\right)\sqrt{\frac{50}{(2)}} - \left(50 + (4)\left(\sqrt{\frac{50}{(2)}}\right) + (2)\left(\sqrt{\frac{50}{(2)}}\right)^2\right) \\
&= \left((4)\sqrt{\frac{1}{(2)}50} + 250\right) - \left(250 + (4)\sqrt{\frac{1}{(2)}50}\right) \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\pi_m &= \left((4) + 2\sqrt{(2)50}\right)\left(\frac{(4)-(0)}{2(2)} + \sqrt{\frac{1}{(2)}50}\right) - \left(50 + (0)\left(\left(\frac{(4)-(0)}{2(2)} + \sqrt{\frac{1}{(2)}50}\right)\right) + (2 \right. \\
&= \frac{1}{2(2)}\left((4) + 2\sqrt{(2)50}\right)\left((4) - (0) + 2\sqrt{50}\right) - \frac{1}{4(2)}\left((16) - (0) + 4(2)50 + 4(2)50 + 4(4 \right.
\end{aligned}
$$

the difference is **economic rent**, *or the long run profit that is due to the land because it gives a natural economic advantage to the mountain firms.*

13. Why might a long run supply curve be downward sloping? Provide examples and explain the general concept. Given the total output in the industry, is the shape of a given firm's average and marginal costs necessarily different than in an industry with a constant long run supply curve?

    Because of Network Costs, the average cost of production can decrease as industry output increases. A simple example is transportation costs, per unit it is always cheaper to ship in a larger quantity. A better example is a parts maker who supplies many different producers, if output in the industry is low there can only be a few suppliers, and the cost of shipping to your plant may be high. As output increases the number of suppliers will, meaning lower transportation costs to get parts for your production.

    The shape of the average and marginal cost of a given firm are generally the same as in an industry with a constant long run supply curve.

14. Economists have long known that taxes cause dead weight loss in markets. Explain why this does not mean that taxes are bad, you should give two different types of reasons.

    This does not imply taxes are bad because this is a partial analysis of the problem. It is overlooking two different problems:

    (a) *Taxes are necessary to make sure that trade occurs, they are needed to support the legal system. Thus the market could not exist without some taxes in some market.*

(b) *The analysis is only of this one market in isolation, taxes are needed to provide other government services (technically public goods).*

15. In the market for Labor in Ankara the supply of labor is $L = S(W) = -2 + 2W$ and the demand for labor is $L = D(W) = 78 - 6W$.

(a) What is the equilibrium wage and quantity of Labor in this market? For full credit you must check either the wage or quantity using both equations.

$$\begin{aligned} -2 + 2W &= 78 - 6W \\ (6+2)W &= 78 + 2 \\ W^* &= \frac{78+2}{6+2} \\ L^* &= -2 + 2\left(\frac{78+2}{6+2}\right) \\ &= \frac{78*2 - 6*2}{6+2} \\ L^* &= 78 - 6\left(\frac{78+2}{6+2}\right) \\ &= 18 \end{aligned}$$

Now assume that the government decides to impose a minimum wage of $\underline{W} = 12$ in this market.

(b) What will be the quantity of Laborers that will be hired at the minimum wage?

$$\begin{aligned} L(12) &= 78 - 6(10+2) \\ &= \frac{78*2 - 6*2}{6+2} - 6*2 \end{aligned}$$

(c) What is the Consumer Surplus, the Producer's surplus and the dead weight loss in this market with this minimum wage? You will get partial credit for a correctly labeled graph.

$$\begin{aligned} CS &= \frac{1}{2}6\left(W_d(0) - (10+2)\right) \\ W_d(Q) &= \frac{78}{6} - \frac{1}{6}Q \\ CS &= \frac{1}{2}6 + \left(\frac{78}{6} - \frac{78+2}{6+2} - 2\right) \\ &= \frac{1}{2}6\frac{1}{6}\left(\frac{78*2 - 6*2}{2+6} - 6*2\right) \\ &= \frac{1}{2}\frac{1}{6}(6)^2 \end{aligned}$$

$$
\begin{aligned}
PS &= \frac{1}{2}6\left(W_s(6)-W_s\left(0\right)\right)+6\left(10+2-W_s\left(6\right)\right)\\
&= 6\left(\frac{1}{2}\left(W_s\left(6\right)-W_s\left(0\right)\right)+10+2-W_s*6\right)\\
L &= -2+2W\\
W_s &= \frac{L}{2}+\frac{2}{2}\\
W_s\left(6\right) &= \frac{\frac{78*2-6*2}{6+2}-6*2}{2}+\frac{2}{2}=\frac{78+2}{6+2}-\frac{6}{2}*2\\
PS &= 6\left(\frac{1}{2}\left(\frac{78+2}{6+2}-\frac{6}{2}*2-\frac{2}{2}\right)+\frac{78+2}{6+2}+2-\left(\frac{78+2c}{6+2}-\frac{6}{2}*2\right)\right)\\
&= 6\left(\frac{1}{2}\frac{1}{2}\left(\frac{78*2-6*2}{6+2}-6*2\right)+\frac{1}{2}\left(6+2\right)*2\right)\\
&= \frac{1}{2}6\left(\frac{1}{2}6+\left(6+2\right)*2\right)
\end{aligned}
$$

$$
\begin{aligned}
DWL &= \frac{1}{2}(18-6)\left(12-W_s\left(6\right)\right)\\
&= \frac{1}{2}\left(\frac{78*2-6*2}{6+2}-\left(\frac{78*2-6*2}{6+2}-\left(6*2\right)\right)\right)\left(\frac{78+2}{6+2}+2-\left(\frac{78+2}{6+2}-\frac{6}{2}*2\right)\right)\\
&= \frac{1}{2}\frac{6}{2}\left(6+2\right)2^2
\end{aligned}
$$

(d) A person is unemployed if they would like to work at the current wage and can not. What is the quantity of unemployed people at the minimum wage?

$$
\begin{aligned}
U\left(\underline{W}\right) &= S\left(\underline{W}\right)-D\left(\underline{W}\right)\\
&= -2+2*12-\left(78-6*12\right)\\
&= -2-78+\left(6+2\right)12\\
&= -2-78+\left(6+2\right)\left(\frac{78+2}{6+2}+2\right)\\
&= 2\left(6+2\right)
\end{aligned}
$$

16. Assume that there are 4 firms in an industry with a cost function of $c\left(q\right)=2q+q^2+16$, their fixed startup costs are 9. *Hint:* In this question all answers are integer values.

(a) Find a representative firm's Average Costs, Average Variable Costs, and Marginal costs.

$$
\begin{aligned}
MC &= 2 + 2 * 1 * q \\
AVC &= \frac{1}{q}\left(2q + q^2 + 9\right) \\
AC &= \frac{1}{q}\left(2q + q^2 + 16\right)
\end{aligned}
$$

(b) At which quantity will a firm shut down? Explain why they shut down at that level.

They will shut down when price is equal to their average variable costs, because at this point they can not cover the costs they will need to incur when they produce (the variable or avoidable costs.)

$$
q_{sd} = \sqrt{\frac{9}{1}}
$$

(c) Find each firm's short run supply curve.

$$
\begin{aligned}
p_{sd} &= 2 + 2\sqrt{\frac{9}{1}} = 2 + 2\sqrt{9} \\
p &= 2 + 2q \\
q\left(p\right) &= -\frac{1}{2}\left(2 - p\right)
\end{aligned}
$$

$$
s\left(p\right) = \begin{cases} -\frac{1}{2d}\left(c - p\right) & p \geq 2 + 2\sqrt{9} \\ 0 & p \leq 2 + 2\sqrt{9} \end{cases}
$$

(d) Find the industry short run supply curve.

$$
S\left(p\right) = \begin{cases} -\frac{4}{2}\left(2 - p\right) & p > 2 + 2\sqrt{9} \\ 0 & p < 2 + 2\sqrt{9} \end{cases}
$$

(e) If market demand is $D\left(P\right) = 32 - 2P$ what is the equilibrium price and quantity.

$$
\begin{aligned}
32 - 2P &= -\frac{4}{2}\left(2 - P\right) \\
P &= \frac{4 + 2 * 32}{4 + 2 * 2} \\
Q &= -\frac{4}{2}\left(2 - \left(\frac{4 * 2 + 2 * 32}{4 + 2}\right)\right) \\
&= \frac{4 * 32 - 4 * 2 * 2}{4 + 2 * 2}
\end{aligned}
$$

(f) If the government imposes a price floor of $\underline{p} = 13$in this industry.

i. What quantity will be traded in this market?

$$\begin{aligned} Q\left(\underline{p}\right) &= 32 - 2\left(\frac{4*2+2*32}{4+2*2}+4\right) \\ &= \frac{4\left(32-2*2\right)}{4+2*2} - 2*4 \end{aligned}$$

ii. What will be the Dead Weight Loss?

$$\begin{aligned} DWL &= \frac{1}{2}\left(14 - \left(Q(13)\right)\left(13 - P_s\left(Q\left(13\right)\right)\right)\right) \\ P_s\left(Q\right) &= \frac{1}{4}\left(4*2+2\right)*14 \\ &= \frac{1}{2}\left(\frac{14\left(32-2*2\right)}{14+2*2} - \left(\frac{4\left(32-2*2\right)}{4+2*2} - 2*4\right)\right)\left(\left(\frac{4*2+2*32}{4+2*2}+4\right) - \frac{1}{4}\left(4 *\right.\right. \\ DWL &= \frac{\left(4+2*2\right)*2}{24}4^2 = 32 \end{aligned}$$

(g) If the demand curve is $D\left(P\right) = 32 - 2P$ what will be the long run equilibrium in this market?

i. How much will each firm produce and what will be the market price?

$$\begin{aligned} q_e &= \sqrt{\frac{16}{1}} = 4 \\ p_e &= 2 + 2\sqrt{16} = 10 \end{aligned}$$

ii. What will be the market quantity and how many firms will there be?

$$\begin{aligned} Q_{lr} &= 32 - 2\left(2 + 2\sqrt{16}\right) = 12 \\ J_{lr} &= \frac{32 - 2\left(2+2\sqrt{16}\right)}{\sqrt{\frac{16}{1}}} = 3 \end{aligned}$$

17. The Long run and Costs.

(a) Explain why we assume that $p \leq AC$ in the long run.

Because of entry. If $p > AC$ then a firm can make a positive economic profit by entering the industry so firms will enter until the price is driven back under average costs.

(b) Explain why we assume that $p \geq AC$ in the long run. Your answer should make explicit mention of sunk costs.

Because of exit. Since all sunk costs have become fixed start up costs in the long run average variable cost is equal to average costs. Firms exit if the price is lower than average variable costs, and thus we must have $p \geq AVC = AC$

(c) Explain why we assume that $p = MC$ in the long run.

This is the first order condition of profit maximization. If the firm produces they must produce at this quantity.

(d) Prove that if $MC = AC$ then average costs are minimized. In the proof you should use $AC = \frac{C(q)}{q}$ when you minimize average costs.

Note that $MC = C'(q)$, $AC = \frac{C(q)}{q}$

$$\min_q \frac{C(q)}{q}$$

this has the first order condition:

$$\begin{aligned} \frac{C'(q)}{q} - \frac{C(q)}{q^2} &= 0 \\ \left(\frac{C'(q)}{q} - \frac{C(q)}{q^2}\right) q &= 0q \\ C'(q) - \frac{C(q)}{q} &= 0 \end{aligned}$$

and since $MC = C'(q)$, $AC = \frac{C(q)}{q}$ *this means*

$$\begin{aligned} MC - AC &= 0 \\ MC &= AC \end{aligned}$$

18. In a given industry there are two completely different technologies represented by the cost functions, $c_a(q) = 4q^2 + 100$ and $c_b(q) = 6q^2 + 54$. The fixed start up costs of firms using technology a is 4 and the fixed startup costs of firms using technology b is 24. There are 8 firms using technology a and 12 firms using technology b.

(a) For firms using technology a find the marginal costs, average variable costs, price at which these firms will shut down, and the firm's supply curve.

$$\begin{aligned} MC &= 8q \\ AVC &= \frac{1}{q}\left(4q^2 + 4\right) \end{aligned}$$

:

$$\begin{aligned} 2\left(\frac{2}{3}\right)(6)\,q_{sd} &= \frac{\left(\frac{2}{3}\right)(6)\,q_{sd}^2+(4)}{q_{sd}} \\ q_{sd} &= 1 \\ P_{sda} &= 2\left(\frac{2}{3}\right)(6)\,q_{sd} = 8 \end{aligned}$$

: :

$$s_a(P) = \begin{cases} \frac{1}{8}P & P \geq P_{sda} \\ 0 & P \leq P_{sda} \end{cases}$$

(b) For firms using technology b find the marginal costs, average variable costs, price at which these firms will shut down, and the firm's supply curve.

$$\begin{aligned} MC &= 12q \\ AVC &= \frac{(6)\,q^2+(24)}{q} \end{aligned}$$

$$\begin{aligned} 2\,(6)\,q_{sd} &= \frac{(6)\,q_{sd}^2+(24)}{q_{sd}} \\ q_{sd} &= 2 \\ P_{sdb} &= 2\,(6)\,q_{sd} = 24 \end{aligned}$$

: :

$$s_a(P) = \begin{cases} \frac{P}{12} & P \geq P_{sdb} \\ 0 & P \leq P_{sdb} \end{cases}$$

(c) Find the industry's short run supply curve.

$$S(P) = \begin{cases} (8)\frac{P}{2\left(\frac{2}{3}\right)(6)} + (12)\frac{P}{2(6)} & P > P_{sdb} \\ (8)\frac{P}{2\left(\frac{2}{3}\right)(6)} & P_{sda} < P < P_{sdb} \\ 0 & P < P_{sda} \end{cases} = \begin{cases} 2P & P_{bds} < P \\ P & (P_{ads} < P) < P_{bds} \\ 0 & P < P_{ads} \end{cases}$$

(d) In the medium run find the price that firms using technology a will enter and the price at which firms using technology b will enter.

$$\begin{aligned} MC_a &= AC_a \\ 2\left(\frac{2}{3}\right)(6)\,q_e &= \frac{\left(\frac{2}{3}\right)(6)\,q_e^2+(100)}{q_e} \\ q_e &= 5 \\ P_{ea} &= 40 \end{aligned}$$

$$\begin{aligned} MC_b &= AC_b \\ 2(6)\, q_e &= \frac{(6)\, q_e^2 + (54)}{q_e} \\ q_e &= 3 \\ P_{eb} &= 36 < P_{ea} \end{aligned}$$

(e) In the medium run assume that the quantity supplied is 81. What is the price in the market? What quantity are firms using technology a supplying? What quantity are firms using technology b supplying? How many firms using technologies a and b are there in the market?

If the price is the price at which firms with technology b will enter then the quantity the firms in the market will supply is

$$S(P) = 72 < 81$$

: : : *thus the price is P_{eb} and the quantity the firms of type a will supply is* $\frac{P_{eb}}{2\left(\frac{2}{3}\right)(6)} = \frac{2\sqrt{(6)(54)}}{2\left(\frac{2}{3}\right)(6)} = \frac{9}{2}$ *and the quantity firms using technology b will supply is* 3 *(found above). The number of firms in the market will be* (8) *using technology a and* 15 *using technology b.*

(f) In the long run what will be the market price? If the quantity supplied is 72 how many firms will there be and which technology are they using? *(Hint: The number of firms should be an integer.)*

In the long run the price will be $\min_{a,b}\{P_{ea}, P_{eb}\} = P_{eb}$ *and all firms will be using technology b, thus there will be* 24 *firms,* $J_{lr} = \frac{72}{3}$.

(g) One could argue that one of these technologies is superior in the short run since it has a lower marginal and average variable cost. However no one uses it in the long run, or it is "driven out."

Which technology is driven out in the long run? Why is this technology driven out? Which technology could be considered "better" and why is this technology better?

Technology a has lower marginal and average variable costs but it is driven out in the long run.

It is driven out because it has such high fixed sunk costs, so in order to break even in the long run they need a higher price.

You could argue that either technology is better, in the short run technology a is clearly superior, because of lower marginal costs. In the long run technology b is clearly better because it has a lower total fixed cost. One could consider the trade-off as being between a more efficient factory and a less efficient but smaller factory. The less efficient but smaller factory wins in the long run, but which is better is a matter of framework. In the long run the "worse" technology is more efficient because it has a lower capital cost.

19. In a given market there are two types of firms. Firms of type one have a total cost of $c_1(q) = q + \frac{1}{2}q^2 + 32$, and a fixed start up cost of 8. Firms of type two have a total cost of $c_2(q) = 2q + q^2 + 9$ and a fixed start up cost of 4.

(a) What is the marginal cost, average variable cost and supply curve of firms of type one?

$$\begin{aligned} MC_1 &= 1 + 2\left(\frac{1}{2}\right)q \\ AVC_1 &= \frac{(1)q + \left(\frac{1}{2}\right)q^2 + (8)}{q} \end{aligned}$$

$$\begin{aligned} P &= MC_1 = (1) + 2\left(\frac{1}{2}\right)q \\ q &= \frac{P-(1)}{2\left(\frac{1}{2}\right)} = P - 1 \end{aligned}$$

:

$$\begin{aligned} MC_1 &= AVC_1 \\ (1) + 2\left(\frac{1}{2}\right)q &= \frac{(1)q + \left(\frac{1}{2}\right)q^2 + (8)}{q} \\ q_{sd1} &= 4 \end{aligned}$$

:

$$\begin{aligned} \frac{1}{\left(\frac{1}{2}\right)}\sqrt{\left(\frac{1}{2}\right)(8)} &= \frac{P-(1)}{2\left(\frac{1}{2}\right)} = P - 1 \\ P_{sd1} &= 5 \end{aligned}$$

$$s(p) = \begin{cases} P - 1 & P \geq 5 \\ 0 & P \leq 5 \end{cases}$$

(b) What is the marginal cost, average variable cost and supply curve of firms of type two?

$$\begin{aligned} MC_2 &= (2) + 2(1)q \\ AVC_2 &= \frac{(2)q + (1)q^2 + (4)}{q} \end{aligned}$$

$$\begin{aligned} P &= MC_2 = (2) + 2(1)q \\ q &= \frac{P-2}{2} \end{aligned}$$

$$
\begin{aligned}
MC_2 &= AVC_2 \\
(2) + 2\,(1)\,q &= \frac{(2)\,q + (1)\,q^2 + (4)}{q} \\
q_{sd2} &= 2
\end{aligned}
$$

:

$$
\begin{aligned}
\frac{1}{(1)}\sqrt{(1)\,(4)} &= \frac{P-(2)}{2\,(1)} \\
P_{sd2} &= 6
\end{aligned}
$$

:

$$
s\,(p) = \begin{cases} \frac{1}{2}P - 1 & P \geq 6 \\ 0 & P \leq 6 \end{cases}
$$

(c) If there are 7 firms of type one and 4 firms of type two what is the short run aggregate supply in this industry?

$$
S\,(P) = \begin{cases} (7)\,\frac{P-(1)}{2\left(\frac{1}{2}\right)} + (4)\,\frac{P-(2)}{2(1)} & P > 2\sqrt{(1)\,(4)} + (2) \\ (7)\,\frac{P-(1)}{2\left(\frac{1}{2}\right)} & 2\sqrt{\left(\frac{1}{2}\right)(8)} + (1) < P < 2\sqrt{(1)\,(4)} + (2) \\ 0 & P < 2\sqrt{\left(\frac{1}{2}\right)(8)} + (1) \end{cases} = \begin{cases} 9P - 11 & \\ 7P - 7 & (5 \\ 0 & \end{cases}
$$

(d) In the medium or long run at what price will firms of type one enter? At what price will firms of type two enter?

Firms will enter when $P \geq AC$ this occurs when $MC \geq AC$

$$
\begin{aligned}
MC_i &= \alpha_i + 2\beta_i q \\
AC_i &= \frac{\alpha_i q + \beta_i q^2 + F_i}{q}
\end{aligned}
$$

$$
\begin{aligned}
P &= MC_i = \alpha_i + 2\beta_i q \\
q &= \frac{P_i - \alpha_i}{2\beta_i}
\end{aligned}
$$

$$
\begin{aligned}
MC_i &= AVC_i \\
\alpha_i + 2\beta_i q &= \frac{\alpha_i q + \beta_i q^2 + F_i}{q} \\
q_{entry\ i} &= \frac{1}{\beta_i}\sqrt{\beta_i F_i}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\beta_i}\sqrt{\beta_i F_i} &= \frac{P_i - \alpha_i}{2\beta_i} \\
P_{entry\ i} &= 2\sqrt{\beta_i F_i} + \alpha_i
\end{aligned}
$$

so for firms of type 1 this is: $q_{entry\ 1} = 8$ and $P_{entry\ 1} = 9$ *for firms of type 2 this is* $q_{entry\ i} = 3$ and $P_{entry\ i} = 8$.

(e) Which type of firm will not produce in the long run?

In the long run $P \geq AC$ in order to be sure there is no exit, $P \leq AC$ in order to be sure there is no more entry, thus the firm with a lower P_{entry} will be the one that produces, this is type two.

20. Economic theory states that in the long run all firms will be making zero profits. Why don't these firms shut down since they're making zero profits? Relate your answer to the difference between accountants and economists.

These firms don't shut down because their costs are economic costs, not accounting costs. Therefore they are opportunity costs and include a fair wage for the firm's owner and a fair return on any capital he (or others) might invest in this firm. They would be making a strictly positive accounting profit since they do not consider dividends for investors or the salary for the firm's owner as a proper part of costs.

21. Assume there are 10 firms in an industry all with the same total cost function $C(q) = q + \frac{q^2}{2} + 8$. These firms have a fixed sunk cost of 6.

All answers for this problem should be whole numbers (integers).

(a) Find the marginal cost, average variable cost, and supply curve of each firm.

$$\begin{aligned} MC &= \frac{dC}{dq} = 1 + q \\ AVC &= \frac{q + \frac{1}{2}q^2 + 2}{q} \end{aligned}$$

$$\begin{aligned} MC(q_{sd}) &= AVC(q_{sd}) \\ 1 + q &= \frac{q + \frac{1}{2}q^2 + 2}{q} \\ q + q^2 &= q + \frac{1}{2}q^2 + 2 \\ q_{sd} &= 2 \end{aligned}$$

$$\begin{aligned} p_{sd} &= MC(q_{sd}) = 1 + (2) \\ &= 3 \end{aligned}$$

$$\begin{aligned} p &= 1 + q \\ q &= p - 1 \end{aligned}$$

$$s(p) = \begin{cases} p - 1 & p \geq p_{sd} \\ 0 & p \leq p_{sd} \end{cases}$$

(b) Define the *medium run.*

The medium run is when firms in the industry might have fixed sunk costs, but firms can enter the industry.

(c) Find the price at which firms will enter this industry.

$$AC = \frac{q + \frac{1}{2}q^2 + 8}{q}$$

$$\begin{aligned} MC(q_e) &= AC(q_e) \\ 1 + q &= \frac{q + \frac{1}{2}q^2 + 8}{q} \\ q + q^2 &= q + \frac{1}{2}q^2 + (8) \\ q_e &= 4 \\ p_e &= MC(q_e) = 1 + (4) = 5 \end{aligned}$$

(d) Find the medium run supply curve.

$$S(p) = \begin{cases} \infty & p > p_e \\ 4N & p = p_e \\ 10p - 10 & p_e > p > p_{sd} \\ 0 & p < p_{sd} \end{cases}$$

(e) Assume that the demand curve in this market is $D(P_d) = 50 - 5P_d$ find the medium run price, quantity, and number of firms that will produce.

There are two cases to check:

i. $p = p_e$, $Q = 50 - 5(3) = 35$, $N = \frac{Q}{q} = \frac{35}{4} = 8.75 < 10$ *since* $N < 10$ *this can not be the equilibrium.*

ii. $N = 10$, $p_e > p > p_{sd}$

$$\begin{aligned} 10P - 10 &= 50 - 5P \\ P^* &= 4 \end{aligned}$$

and one can readily verify that $p_e > P^* > p_{sd}$.

$$\begin{aligned} Q^* &= 50 - 5P^* \\ &= 50 - 5(4) \\ &= 30 \end{aligned}$$

:

$$\begin{aligned} Q^* &= 10P^* - 10 \\ &= 10(4) - 10 = 30 \end{aligned}$$

(f) If the demand curve increases to $D(P_d) = 90 - 2P_d$ find the new medium run price, quantity, and number of firms that will produce.

There are two cases to check:

i. $p = p_e$, $Q_m^* = 90 - 2(5) = 80$, $N_m^* = \frac{Q_m^*}{q_m^*} = 20$ *and one can readily verify that* $N > 10$ *so this is the equilibrium.*

ii. $N = 10$, $p_e > p > p_{sd}$ *there is no need to check this case since the one above worked.*

(g) Find the long run price, quantity, and number of firms that will produce if the long run demand is $D(P_d) = 90 - 2P_d$.

Since with this demand curve there was entry in the equilibrium above

$$\begin{aligned} P &= P_e \\ Q &= Q_m^* \\ N &= N_m^* \end{aligned}$$

22. Economists assert that in a normal competitive industry all firms are making zero profits in the long run. Why would anyone invest in a firm that is making zero profits?

Because economic costs are opportunity costs, thus every investor is getting the opportunity cost of the money they have invested, and thus would not make more by investing elsewhere.

23. Consider a short run market where the demand curve is given by $D(P_b)$ and the supply curve is given by $S(P_s)$. There is a per-unit tax so $P_b = P_s + t$, where P_b is the price buyers pay and P_s is the price sellers receive.

(a) Find $\frac{\partial P_s}{\partial t}$ and $\frac{\partial P_b}{\partial t}$.

$$\begin{aligned} D(P_s + t) &= S(P_s) \\ D_p \frac{\partial P_s}{\partial t} + D_p &= S_p \frac{\partial P_s}{\partial t} \\ D_p &= (S_p - D_p) \frac{\partial P_s}{\partial t} \\ \frac{\partial P_s}{\partial t} &= \frac{D_p}{S_p - D_p} \end{aligned}$$

$$\begin{aligned} P_b &= P_s + t \\ \frac{\partial P_b}{\partial t} &= \frac{\partial P_s}{\partial t} + 1 \\ \frac{\partial P_b}{\partial t} &= \frac{D_p}{S_p - D_p} + \frac{S_p - D_p}{S_p - D_p} \\ \frac{\partial P_b}{\partial t} &= \frac{S_p}{S_p - D_p} \end{aligned}$$

(b) Prove that $\left|\frac{\partial P_s}{\partial t}\right| + \left|\frac{\partial P_s}{\partial t}\right| = 1$, where $|a|$ is the absolute value of a. Hint: You do not actually need to answer part a to answer this part. Explain your answer.

$$P_b = P_s + t$$

$$\begin{aligned} \frac{\partial P_b}{\partial t} &= \frac{\partial P_s}{\partial t} + 1 \\ \frac{\partial P_b}{\partial t} - \frac{\partial P_s}{\partial t} &= 1 \end{aligned}$$

$\frac{\partial P_b}{\partial t} \geq 0$ *and* $\frac{\partial P_s}{\partial t} \leq 0$ *as shown above, thus:*

$$\left|\frac{\partial P_b}{\partial t}\right| + \left|\frac{\partial P_s}{\partial t}\right| = 1$$

This must be true since someone must pay the per unit tax, thus P_b *must increase and* P_s *must decrease so that the difference is the amount the tax has increased.*

(c) If the slope of the supply curve increases ($S_p = \frac{\partial S}{\partial P_s}$) how will this change the graph of the supply curve?

It will make it flatter, since what you see in the graph of the supply curve is $\frac{1}{S_p}$.

(d) Discuss what will happen to $\frac{\partial P_s}{\partial t}$ and $\frac{\partial P_b}{\partial t}$ if in the graph of the supply curve the supply curve gets flatter

$$\begin{aligned} \frac{\partial P_s}{\partial t} &= \frac{D_p}{S_p - D_p} \\ \frac{\partial P_b}{\partial t} &= \frac{S_p}{S_p - D_p} \end{aligned}$$

$$\begin{aligned} \frac{\partial P_s}{\partial t \partial S_p} &= \frac{-D_p}{(S_p - D_p)^2} \geq 0 \\ \frac{\partial P_b}{\partial t \partial S_p} &= \frac{1}{S_p - D_p} - \frac{S_p}{(S_p - D_p)^2} \\ &= \frac{S_p - D_p}{(S_p - D_p)^2} - \frac{S_p}{(S_p - D_p)^2} \\ &= \frac{-D_p}{(S_p - D_p)^2} \geq 0 \end{aligned}$$

24. Consider the supply of wheat in Turkey.

(a) If a firm has a cost function of $C(q) = q^2 + 100$ where the fixed costs are all fixed sunk costs, what is its short run supply function?

$$\begin{aligned} p &= MC = 2q \\ p &\geq AVC = \frac{q^2}{q} \\ MC &\geq AVC \\ 2q &\geq \frac{q^2}{q} \\ q &\geq 0 \\ p &\geq 0 \end{aligned}$$

$$s(p) = \begin{cases} \frac{p}{2} & p \geq 0 \\ 0 & p \leq 0 \end{cases}$$

(b) If a firm has a cost function of $C(q) = 2q^2 + 32$ where the fixed costs are all fixed start up costs, what is its short run supply function?

$$\begin{aligned} p &= MC = 4q \\ p &\geq AVC = \frac{2q^2 + 32}{q} \\ MC &\geq AVC \\ 4q &\geq \frac{2q^2 + 32}{q} \\ q &\geq 4 \\ p &\geq 4q = 16 \end{aligned}$$

$$s(p) = \begin{cases} \frac{p}{4} & p \geq 16 \\ 0 & p \leq 16 \end{cases}$$

(c) If there are two firms with the costs in part a and two firms with the costs in part b, what is the short run aggregate supply? (Assume these firms are perfect competitors.)

$$S(p) = \begin{cases} 2\left(\frac{p}{2}\right) + 2\left(\frac{p}{4}\right) = \frac{3}{2}p & p > 16 \\ 2\left(\frac{p}{2}\right) = p & p < 16 \end{cases}$$

(d) If the demand is $D(P) = 276 - 10P$ what will be the market equilibrium? You should find that in equilibrium all firms produce.

$$\begin{aligned} \frac{3}{2}P &= 276 - 10P \\ P &= 24 \\ Q &= \frac{3}{2}(24) = 36 \end{aligned}$$

(e) If firms could enter this market would they? Why or why not?

Yes, because price is higher than the average total costs of the firms in part (b). (Notice that this is the same as average variable costs.)

(f) Assuming these firms use the same production technology in the short and long run, what is the long run costs of the firms in part a and part b?

$$\begin{aligned} C_a(q) &= q^2 + 100 \\ C_b(q) &= 2q^2 + 32 \end{aligned}$$

where all fixed costs are fixed start up costs.

(g) What quantity will these two types of firms produce in a long run equilibrium? Which of the two types of firm will not produce in the long run and why?

For type a firms:

$$\begin{aligned} MC_a &= 2q = \frac{q^2 + 100}{q} = ATC_a \\ q &= 10 \\ p &= 2q = 20 \end{aligned}$$

For type b:

$$\begin{aligned} MC_b &= 4q = \frac{2q^2 + 32}{q} = ATC_a \\ q &= 4 \\ p &= 4q = 16 \end{aligned}$$

So type a firms would produce 10 units each, type b would produce 4 units each. However type a firms would not produce because type b firms would be willing to supply at a lower price.

(h) Assuming the long run demand is $D(P) = 276 - 10P$ what will be the long run equilibrium price, quantity and number of firms? Note the number of firms must be an integer.

$$\begin{aligned} P &= 16 \\ Q &= D(P) = 276 - 10P = 276 - 10(16) \\ Q &= 116 \\ n &= \frac{Q}{q} = \frac{116}{4} = 29 \end{aligned}$$

25. There are two technologies used to produce honey with exactly the same costs. In one you situate the hives near orchards, and the bees pollinate

the fruit trees, in the other you situate the bees in fields of wildflowers. Since the first method produces a positive externality the government wants to subsidize this method, so they offer them a per-quart subsidy of 20 cents.

Assume throughout this question that there is an unlimited supply of orchards and fields of wildflowers and that both types of honey are identical.

(a) What will happen to the price of honey in the short run?

The marginal costs of the subsidized firms will effectively decrease by twenty cents, thus shifting their individual supply curve down (to the right). This will shift the total supply of honey down (to the right) and the price of honey will decrease.

(b) What will be the mix of wildflower/orchard honey in the long run?

We know that if $p > LAC$ a firm will produce. Since $AC_o = AC_w - s$, where s is the subsidy near orchards producers have a lower AC and hence they are ready to produce at a lower price than the other type. Considering the fact that if $p = LAC_o$ an infinite supply will come from the identical low cost producers and no supply from the identical high cost producers, the downward sloping demand curve will cut the supply at $p^* = LAC_o$ so there will be no wildflower honey in the long run.

(c) What will happen to the price of a quart of honey in the long run? (Warning: there is a precise answer.)

It will be lowered by exactly twenty cents. First notice that:

$$MC_{new} = MC_{old} - 20, \; AC_{new} = AC_{old} - 20$$

Therefore $AC_{new} = MC_{new}$ at the same quantity as when $AC_{old} = MC_{old}$. This means that $p = AC_{new} = AC_{old} - 20$ or the change will be precisely twenty cents per quart. This follows from the fact that long run supply curve is perfectly elastic

26. A competitive industry is in a long run equilibrium, and then a sales tax is placed on the output. What do you expect to happen to the price the producers receive, the price the consumers pay, and the number of firms in the industry?

Assuming an upward sloping aggregate supply function and a downward sloping demand function, a sales tax will shift the supply curve to left. Hence the new equilibrium price which consumers pay will be higher, the new quantity will be lower and hence the price, suppliers receive will be lower. In aggregate, both the consumer and supplier surplus will decrease and hence tax imposes burden on both. Note that since we assumed that the supply function is upward sloping the firms are not identical, since if they were supply would be horizontal. Now given that the firms are not identical only those, who can survive a price decrease will keep producing *and those who were just breaking even before the price decrease will leave the industry. Hence the number of firms will decrease.*

27. In the market for Labor in Ankara the supply of labor is $L = -8 + 4W$ and the demand for labor is $L = 40 - 2W$.

    (a) What is the equilibrium wage and quantity of Labor in this market? For full credit you must check either the wage or quantity using both equations.

$$\begin{aligned} (40) - (2)\,W &= -8 + (4)\,W \\ W &= 8 \\ L &= -(8) + (4)\,W = 24 \\ L &= (40) - (2)\,W = 24 \end{aligned}$$

    Now assume that the government decides to impose a minimum wage of $\underline{W} = 10$ in this market.

    (b) What will be the quantity of Laborers that will be hired at the minimum wage?

$$L\left(\underline{W}\right) = (40) - (2)\,\underline{W} = 20$$

    (c) What is the Consumer Surplus, the Producer's surplus and the dead weight loss in this market with this minimum wage? You will get partial credit for a correctly labeled graph.

$$\begin{aligned} CS &= \frac{1}{2}\left(\bar{W} - \underline{W}\right) L\left(\underline{W}\right) \\ CS &= 100 \end{aligned}$$

:

$$\begin{aligned} PS &= \frac{1}{2}\left(W_s\left(L\left(\underline{W}\right)\right) - W(0)\right) L\left(\underline{W}\right) + \left(\underline{W} - W_s\left(L\left(\underline{W}\right)\right)\right) L\left(\underline{W}\right) \\ &= \frac{1}{2}\left(\frac{L\left(\underline{W}\right) + (8)}{(4)} - \frac{(8)}{(4)}\right) L\left(\underline{W}\right) + \left(10 - \frac{L\left(\underline{W}\right) + (8)}{(4)}\right) L\left(\underline{W}\right) \\ PS &= \frac{1}{2}\left(\frac{20 + (8)}{(4)} - \frac{(8)}{(4)}\right) 20 + \left(10 - \frac{20 + (8)}{(4)}\right) 20 \\ &= 110 \end{aligned}$$

$$\begin{aligned} DWL &= \frac{1}{2}\left(\underline{W} - W_s\left(L\left(\underline{W}\right)\right)\right)\left(L^* - L\left(\underline{W}\right)\right) \\ &= \frac{1}{2}\left(10 - \frac{L\left(\underline{W}\right) + (8)}{(4)}\right)(24 - 20) \\ &= 6 \end{aligned}$$

(d) A person is unemployed if they would like to work at the current wage and can not. What is the quantity of unemployed people at the minimum wage?

$$\begin{aligned} S(\underline{W}) - D(\underline{W}) &= -(8) + (4)\underline{W} - ((40) - (2)\underline{W}) \\ &= 12 \end{aligned}$$

28. Assume there are four firms in an industry all with the same total cost function $C(q) = 2q^2 + 8$. Type a firms have a fixed sunk cost of 8 type b firms have a fixed start up cost of 8.

(a) Find the marginal cost, average variable cost, and supply curve of firms of type a.

$$\begin{aligned} MC &= 4q \\ AVC &= \frac{2q^2}{q} = 2q \\ MC &\geq AVC \\ 4q &\geq 2q \\ q &\geq 0 \end{aligned}$$

$$\begin{aligned} P_s &= 4q \\ q &= \frac{1}{4}P_s \end{aligned}$$

$$s(P_s) = \frac{1}{4}P_s$$

(b) Find the marginal cost, average variable cost, and supply curve of firms of type b.

$$\begin{aligned} MC &= 4q \\ AVC &= \frac{2q^2 + 8}{q} \\ MC &\geq AVC \\ 4q &\geq \frac{2q^2 + 8}{q} \\ q &\geq 2 \\ P_s &\geq 2(2) = 4 \end{aligned}$$

$$\begin{aligned} P_s &= 4q \\ q &= \frac{1}{4}P_s \end{aligned}$$

$$s(P_s) = \begin{cases} \frac{1}{4}P_s & P_s \geq 4 \\ 0 & P_s \leq 4 \end{cases}$$

(c) Explain why the two types of firms have different levels of sunk costs when they are using the same technology.

These firms are at different points in their "life cycle." Type a firms have already made an irreversible investment in a plant, type b firms have not made that investment.

(d) If there are 4 firms of type a and 8 firms of type b what is the short run market supply curve?

$$S(P_s) = \begin{cases} 3P_s & P_s > 4 \\ P_s & P_s < 4 \end{cases}$$

(e) Assume that the demand curve in this market is $D(P_d) = 108 - 6P_d$ and that there is a per-unit tax of t. Find the price demanders pay (P_d), the price suppliers receive (P_s), and the market quantity (Q_t) for all t. (Hint: In equilibrium all firms will produce output, and the only part of your answers that will not be integers will be the coefficients on t in the two prices.)

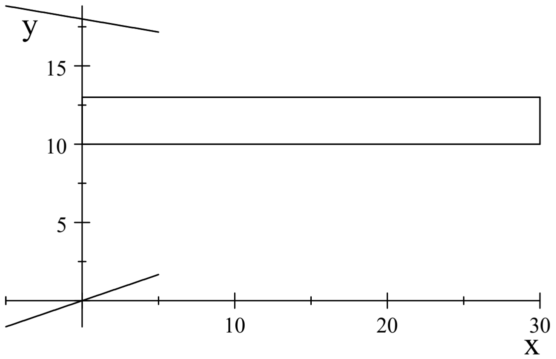
$$\begin{aligned} S(P_s) &= D(P_d) \\ P_d &= P_s + t \\ S(P_s) &= D(P_s + t) \\ 3P_s &= 108 - 6(P_s + t) \\ P_s &= 12 - \frac{2}{3}t \\ P_d &= P_s + t \\ &= 12 - \frac{2}{3}t + t \\ &= 12 + \frac{1}{3}t \end{aligned}$$

$$\begin{aligned} Q_t &= 3\left(12 - \frac{2}{3}t\right) \\ &= 36 - 2t \end{aligned}$$

(f) In a graph indicate the consumer surplus, producer surplus, government revenue, and dead weight loss in the market. Your graph does not have to be precise.

To be precise I am going to use a graphing program to figure out what it would be on exam type (1) when $t = 3$.

$$
\begin{aligned}
S(P) &= Q = 3P \\
P_s(Q) &= \frac{1}{3}Q \\
D(P) &= Q = 108 - 6P \\
P_d(Q) &= 18 - \frac{1}{6}Q \\
Q_{t=3} &= 36 - 2(3) = 30 \\
P_s(t=3) &= 12 - \frac{2}{3}(3) = 10 \\
P_d(t=3) &= P_s(t=3) + 3 = 13
\end{aligned}
$$



The rectangular area is the Government Revenue box = tQ_t. The triangle above the Government Revenue box is Consumer Surplus. The triangle below the Government Revenue box is Producer Surplus, the triangle to the right of the Government Revenue is the Dead Weight Loss, DWL is 9 in this case.

(g) Find the dead weight loss in this market for all t. (Hint: You have already found the equilibrium quantity when $t = 0$, and the coefficient will be a integer.)

$$
\begin{aligned}
DWL &= \frac{1}{2}hl \\
h &= P_d - P_s = t \\
l &= Q_0 - Q_t \\
&= 36 - (36 - 2t) \\
&= 2t \\
DWL &= t^2
\end{aligned}
$$

29. Economists have long known that taxes cause dead weight loss in markets. Explain why this does not mean that taxes are bad, you should give two different types of reasons.

This does not imply taxes are bad because this is a partial analysis of the problem. It is overlooking two different problems:

(a) *Taxes are necessary to make sure that trade occurs, they are needed to support the legal system. Thus the market could not exist without some taxes in some market.*

(b) *The analysis is only of this one market in isolation, taxes are needed to provide other government services (technically public goods).*

30. Assume that there are 20 firms in an industry, each of which has sunk fixed costs of $F_{su} = 6$, Fixed start up costs of $F_{st} = 2$ and pure variable costs of $VC(q) = 2q^2$

In this question you may rest assured that all answers are integer values, except possibly the shut down quantity.

(a) Find this firms Average Costs, Average Variable Costs, and Marginal costs.

$$\begin{aligned} AC &= \frac{2q^2 + 8}{q} \\ AVC &= \frac{2q^2 + 2}{q} \\ MC &= 4q \end{aligned}$$

(b) At which quantity will the firm shut down? Explain why they shut down at that level.

$$\begin{aligned} 4q &= \frac{2q^2 + 2}{q} \\ q^{sd} &= \frac{1}{2}\sqrt{4} \end{aligned}$$

They shut down at this level because they can not even cover their avoidable costs, the pure variable costs plus the fixed start up cost.

(c) Find each firm's short run supply curve.

$$\begin{aligned} p &= 4q \\ q &= \frac{1}{2}\frac{p}{2} \\ p^{sd} &= 4q^{sd} = 4\left(\frac{1}{2}\sqrt{4}\right) = 2 \end{aligned}$$

$$s(p) = \begin{cases} \frac{p}{4} & p \geq 4 \\ 0 & p \leq 4 \end{cases}$$

(d) Find the industry short run supply curve.

$$S(p) = 20s(p) = \begin{cases} 5P & p \geq 4 \\ 0 & p \leq 4 \end{cases}$$

(e) If market demand is $D(P) = 54 - 4P$ what is the equilibrium price and quantity.

$$\begin{aligned} 5P &= 54 - 4P \\ P^* &= \frac{54}{10+8}2 = 6 \\ Q^* &= 54 - 4P = \frac{540}{10+8} = 30 \end{aligned}$$

(f) If the government imposes a price floor of $\underline{p} = 10$ in this industry.

i. What quantity will be traded in this market?

$$Q_{\underline{p}} = \min\left\{\frac{540}{10+8}, 10\frac{P}{2}\right\}$$

ii. What will be the Dead Weight Loss?
the consumer would be willing to pay:

$$\begin{aligned} Q_{\underline{p}} &= 54 - 4P \\ P_{\underline{p}} &= \frac{54 - Q_{\underline{p}}}{4} \end{aligned}$$

so the dead weight loss is:

$$\frac{1}{2}\left(30 - Q_{\underline{p}}\right)\left(\frac{54 - Q_{\underline{p}}}{4} - \underline{10}\right)$$

(g) If the demand curve is $D(P) = 54 - 4P$ what will be the long run equilibrium in this market?

i. How much will each firm produce?

$$\begin{aligned} \frac{2q^2 + 8}{q} &= 4q \\ q &= \frac{1}{2}\sqrt{16} = 2 \end{aligned}$$

ii. What will be the market price?

$$P = 4q = 2\sqrt{16} = 8$$

iii. What will be the market quantity?

$$Q = 54 - 4P = 54 - 8\sqrt{16} = 22$$

iv. How many firms will there be?

$$n = \frac{Q}{q} = \frac{22}{2} = 11$$

31. There are four firms that all have the same technology, represented by the total cost function $C(q) = 18 + \frac{q^2}{2}$. Type A firms (there are two) have a sunk cost of 18, Type B firms (again there are two) have no sunk costs.

(a) Explain why the firms have different levels of sunk costs when they are using the same technology.

Firms of type A have already built their plant, etcetera, thus they have non-reversible investments and sunk costs. Firms of type B have not built their plant, or they are in the long run. Thus the "sunk cost" of type A firms is a start up cost for type B firms.

(b) For each type of firm write down the short run supply function.

$$\begin{aligned}
MC &= q \\
AVC &= \frac{g + \frac{q^2}{2}}{q} \text{ where } g \in \{0, 18\} \\
MC &\geq AVC \\
q &\geq \frac{g + \frac{q^2}{2}}{q} \\
q &\geq \sqrt{2g} \\
P &= MC = q \\
q &= P \\
q &= \begin{cases} 0 & \text{if } P \leq \sqrt{2g} \\ P & \text{if } P \geq \sqrt{2g} \end{cases}
\end{aligned}$$

(c) Write down the short run aggregate supply.

$$Q = \begin{cases} 2P & P \leq 6 \\ 4P & P \geq 6 \end{cases}$$

(d) If the demand curve is $Q = 30 - 4P$.

i. What is the short run equilibrium Price and Quantity? How many firms produce?

If all four firms produce the price is less than 6, so with only two firms:

$$\begin{aligned} 2P &= (30) - (4) P \\ P^* &= 5 \\ Q^* &= 10 = 2P \\ P &\leq 6 \end{aligned}$$

ii. If the government imposes a price floor of 7, what is the quantity sold? What is the dead weight loss?
The quantity sold is the minimum of the quantity demanded and supplied, in this case that will be the quantity demanded, or $\underline{Q} = 30 - 4\underline{P} = 2$.
The deadweight loss is the area of a triangle with a width of $\left(\underline{Q} - Q^*\right)$ *and the height of the distance between the demand and supply when the quantity. This is* $\underline{P} - \frac{\underline{\underline{Q}}}{2}$ *because at the quantity of* $\underline{Q} = 2$ *only two firms will produce.*

$$\begin{aligned} & \frac{1}{2}\left(Q^* - \underline{Q}\right)\left(\underline{P} - \frac{\underline{\underline{Q}}}{2}\right) \\ = & \frac{1}{2}(10 - 2)\left(7 - \frac{2}{2}\right) \\ = & 24 \end{aligned}$$

iii. What is the long run equilibrium Price and Quantity? How many firms produce?

$$\begin{aligned} MC &= AC \\ q &= \frac{18 + \frac{q^2}{2}}{q} \\ q &= 6 \\ P &= 6 \end{aligned}$$

$$\begin{aligned} Q &= 6 \\ n &= 1 \end{aligned}$$

32. The total cost function of a typical soft coal producer is:

$$TC = 100,000 + \frac{1}{10}q^2$$

where Q is measured in railroad cars per year. The industry contains 60 identical producers.

(a) Calculate the short run supply curves of a firm and the industry. Assume that fixed start up costs are zero.

Short-run supply function of the firm

In the short-run the firm will continue to produce as long as price is greater than the minimum average variable cost, and increase the output until the marginal cost is equal to price. Then the short-run supply curve is the part of the MC curve above the minimum AVC. The MC curve of the firm can be found by

$$MC = \frac{\partial TC}{\partial q} = \frac{d[100,000 + \frac{1}{10}q^2]}{dq} = \frac{1}{5}q$$

Minimum AVC can easily be calculated after noting that MC cuts the AVC curve at the minimum AVC. First finding that the AVC curve and then solving for the intersection of AVC and MC curves we find the output level where AVC is minimum.

$$AVC = \frac{VC}{q} = \frac{\frac{1}{10}q^2}{q} = \frac{1}{10}q$$

[Note that 100,000 is the fixed cost of production, and $\frac{1}{10}q^2$ *is the variable cost of production, i.e. it varies with the amount produced.]*
To find the intersection point

$$AVC = MC \Longrightarrow \frac{1}{5}q = \frac{1}{10}q \Longrightarrow q = 0$$

The AVC is at its minimum when $q = 0$*, and at this point AVC is* $\frac{1}{10}(0) = 0$*. Then, as long as* $p > 0$ *the firm will produce and it will increase the output until* $MC = p$*. Then the short-run supply curve of the firm is*

$$q = 5p$$

Short-run supply function of the Industry

Short-run supply function of the industry is the horizontal summation of the firms' short-run supply curves. Horizontal summation means that we are adding the output of each firm at a given price. The firms in the industry have the same MC and the same minimum AVC, and hence the same short run supply function.
Firm i's output is

$$q_i = 5p$$

Summing up over 60 firms:

$$Q = \sum_{i=1}^{60} q_i = 5p * 60 = 300p$$

$$Q = 300p$$

(b) If the price of output is 250 calculate and illustrate graphically producer surplus at this price, and firm's profit.

Producer surplus is the area above the supply curve and below the price. To find the area we first find how much output the firm will supply. Supply curve is given by $q = 5p$*, then* $q = 1250$ *when the price* 250*. The producer surplus is*

$$PS = \frac{250 * 1250}{2} = 156,250$$

Firm's profit:

$$\pi = R - TC = R - FC - VC$$
$$(250 * 1250) - 100,000 - \tfrac{1}{10}(1,250)^2 = 56,250$$

(c) Calculate the long run supply curve.

The long run differs from the short run in the sense that now firms can enter or exit the industry. A firm that is actually in the market will decide to exit if the price is lower than the minimum average cost; but stays if it is greater and produce until $MC = p$*.*

Then the supply in the long run is equal to the part of the MC curve that is above the minimum AC.

Noting that MC curve intersects with the AC curve at its minimum we will solve for the minimum AC by solving the intersection point.

$$AC = \frac{TC}{q} = \frac{100,000}{q} + \frac{1}{10}q$$

Solving for the intersection point of AC and MC

$$\frac{100,000}{q} + \frac{1}{10}q = \frac{1}{5}q$$

$$q^2 = 1,000,000 \qquad \Longrightarrow q = 1000, p = 200$$

Then whenever price is above 200, the firm will supply until $q = 5p$*,and when ever the price is below 200 supply will be zero:*

This supply curve can be represented as

$$q = \begin{cases} 0 & p \leq 200 \\ 5p & p \geqslant 200 \end{cases}$$

Since this is a constant cost industry, the industry's long run supply curve will be horizontal line at the minimum average cost. Then the industry supply curve is horizontal at $p = 200$*.*

(d) If the federal government is considering imposes a \$32 per carload tax on soft coal producers, calculate the new total and marginal cost of the firm.

Since the tax is on per unit output, MC increases by the amount of per unit tax:

$$MC\prime = MC + 35 \Longrightarrow \frac{1}{5}q + 35$$

And total costs increase by the amount of total tax payment

$$TC\prime = TC + 35q \Longrightarrow 100,000 + \frac{1}{10}q^2 + 35q$$

33. In a particular industry, domestic demand is given by

$$Q^d = 80 - \frac{1}{2}P$$

and domestic supply is

$$Q^s = -4 + \frac{3}{2}P$$

there is also a world supply curve, and they will supply any quantity at the price of 4.

(a) What is the equilibrium if there are no tariffs or quotas? What quantity is imported and what quantity is produced domestically? What is total domestic welfare at this equilibrium?

When there are no tariffs or quotas the equilibrium price is 4. *At this price domestic producer supplies*
$-4 + \frac{3}{2}4 = 2$ *units of the good,*
and consumers demand
$80 - \frac{1}{2}4 = 78$ *units of the good.*
Domestic producers only produce 2 *units which is not enough to settle the domestic demand of* 78, *so* $(78 - 2) = 76$ *units of the good is imported.*
Total domestic welfare is the sum of consumer surplus and producer surplus.

$$CS = \int_{4}^{160} (80 - \frac{1}{2}P)dP = 6084$$

$$PS = \int_{8/3}^{4} (-4 + \frac{3}{2}P)dP = \frac{4}{3}$$

Total domestic welfare $= CS + PS = 6085.3$

(b) Domestic producers are upset with the current state of affairs, and demand an import tariff. They are arguing for a tariff of twelve dollars per unit imported. What will be the new equilibrium if this is the tariff? What will be the effect on total domestic welfare? On producer surplus?

With the tariff of 12$ *per unit of the good the domestic price becomes* $(4+12)=16$.

At this price domestic producers produce

$-4+\frac{3}{2}16=20$ *units* ,

and domestic consumers demand

$80-\frac{1}{2}16=72$ *units of the good.*

$(72-20)=52$ *units of the good is imported.*

With the tariff, now the government starts to make revenue from tariff. Assuming that government does not turn this revenue to production of consumption or production goods, the following results will be obtained.

Producer surplus increases to:

$$PS' = \int_{8/3}^{16} (-4+\frac{3}{2}P)dP = \frac{400}{3}$$

Consumer surplus decreases to:

$$CS' = \int_{16}^{160} (80-\frac{1}{2}P)dP = 5184$$

The decrease in total domestic welfare:

$$CS + PS - CS' - PS' = 900 - 132 = 768$$

(c) They point out that they are being very reasonable, as a tariff that shuts out the imports would be much higher. What would that tariff be? What would be the domestic welfare at that tariff?

If the tariff that is set increases the price to the level where the domestic supply equals domestic demand, then there is no room for imports; that is if price with the tariff is equal to the closed economy price all of the domestic demand is then met by the domestic producers so there is no need for imports.

Equating domestic supply and domestic demand, and then solving for the price, we have

$$80-\frac{1}{2}P = -4+\frac{3}{2}P$$

$P=42$ and therefore the required level of tariff is $T=P-4=38$

With P=42 welfare is:

$$CS + PS = \int_{42}^{160} (80 - \frac{1}{2}P)dP + \int_{8/3}^{42} (-4 + \frac{3}{2}P)dP = \frac{13\,924}{3} = 4641.\overline{33}$$

(d) The international producers are upset about this, they offer a counter proposal of a quota of forty two units imported. Why are they in favor of a quota over the tariff? What is the equilibrium price and quantity? How does it compare with in case c? What would be the domestic surplus in this situation? (And a hard question) Why did they choose that particular quota?

Quota is a restriction on quantity traded which in turn indirectly affect the prices. Because of the quota imposed, the price in the country will increase above 4 as we will see below. Now the foreign producer will have non zero profits, this is why he prefers a quota over a tariff. Again the equilibrium level of price and quantity will be found by equating supply and demand in the country; but note that additional to the domestic supply now there is also an additional supply of 42 units from the foreign suppliers.

Considering the possibility of import the total supply will be:

$$Q^s_T = \begin{cases} -4 + \frac{3}{2}P & \text{if} \quad P < 4 \\ -4 + \frac{3}{2}P + 42 & \text{if} \quad P \geq 4 \end{cases}$$

Then by equating demand and supply:

$$Q^d = Q^s_T \text{ we get } \frac{3}{2}P + 38 = 80 - \frac{1}{2}P \Longrightarrow P = 21$$

Note that for $P < 4$, $Q^s_T < Q^d$.

At P=21 domestic consumers demand 69.5 units, domestic producers produce 27.5 units, and 42 units will be imported.

Total welfare is given by:

$$CS + PS = \int_{21}^{160} (80 - \frac{1}{2}P)dP + \int_{8/3}^{21} (-4 + \frac{3}{2}P)dP = \frac{15\,247}{3} = 5082.\overline{3}$$

This result is different from part c for many ways. First of all a different tool is used to restrict the trade, rather than restricting the price, quantity is restricted. In part c there is no import, but here foreign firms can sell their product , and even make a profit out of it. Now the consumer surplus is higher, since now there are some imported goods in the economy which pulls the price downwards; but as you can guess domestic producers' surplus is lower.

To see why they demanded a quota of 42, first we will write the equilibrium price as a function of the imported quantity q_i:

$$Q^d = 80 - \frac{1}{2}P = Q^s_T = -4 + \frac{3}{2}P + q_i \Rightarrow P = \frac{84 - q_i}{2}$$

Then the total revenue of the foreign firms will be: $R = P * q_i = \frac{84-q_i}{2} q_i$ which takes its maximum at $q_i^* = 42$.

2 Chapter 13

1. About Pareto Efficiency.

   (a) Define an *allocation*, is it possible for one person to have allocation A and another to have allocation B? Why or why not?

   Solution 16 *An allocation is an outcome for the society, specifying what each person gets given what occurs. It is not logically possible for one person to have allocation A and another to have B because both specify what each gets in the given outcome. This might be a possible outcome, but it should be written as a seperate allocation.*

   (b) Define *Pareto Dominance.*

   Solution 17 *A Pareto Dominates B if everyone prefers A to B and for at least one person the preferences are strict.*

   (c) Define *Pareto Efficiency* using the definition of Pareto Dominance you have just written.

   Solution 18 *An allocation is Pareto Efficient if there is nothing that Pareto Dominates it.*

2. Gizem and Volkan consume nothing but Xylophones (X) and Yogurt (Y). Gizem has all of the Xylophones, 30 units, $(\left(X_g^e, Y_g^e\right) = (30, 0))$ and Volkan has all of the Yogurt, 8 units $((X_v^e, Y_v^e) = (0, 8))$. Gizem's preferences over Xylophones and Yogurt are $U_g\left(X_g, Y_g\right) = X_g Y_g^2$ and Volkan's preferences are $U_v\left(X_v, Y_v\right) = X_v Y_v^3$.

   (a) Find the Pareto Efficient allocations in this exchange economy.

   Solution 19

$$\begin{aligned}
\frac{MU_x^g}{MU_y^g} &= \frac{\frac{U_g}{X_g}}{2\frac{U_g}{Y_g}} = \frac{Y_g}{2X_g} = \frac{Y_v}{3X_v} = \frac{\frac{U_v}{X_v}}{3\frac{U_v}{Y_v}} = \frac{MU_x^v}{MU_y^v} \\
30 &= X_g + X_v \\
8 &= Y_g + Y_v
\end{aligned}$$

$$\begin{aligned}\frac{Y_g}{2X_g} &= \frac{(8-Y_g)}{3(30-X_g)} \\ Y_g &= \frac{16X_g}{90-X_g}\end{aligned}$$

(b) Find the General Equilibrium in this exchange economy. Be sure to solve for both the prices and the final allocation of goods.

Solution 20

$$\begin{aligned}\frac{MU_x^g}{MU_y^g} &= \frac{Y_g}{2X_g} = \frac{p_x}{p_y} \\ p_xX_g + p_yY_g &= p_x30 \\ Y_g &= \frac{16X_g}{90-X_g}\end{aligned}$$

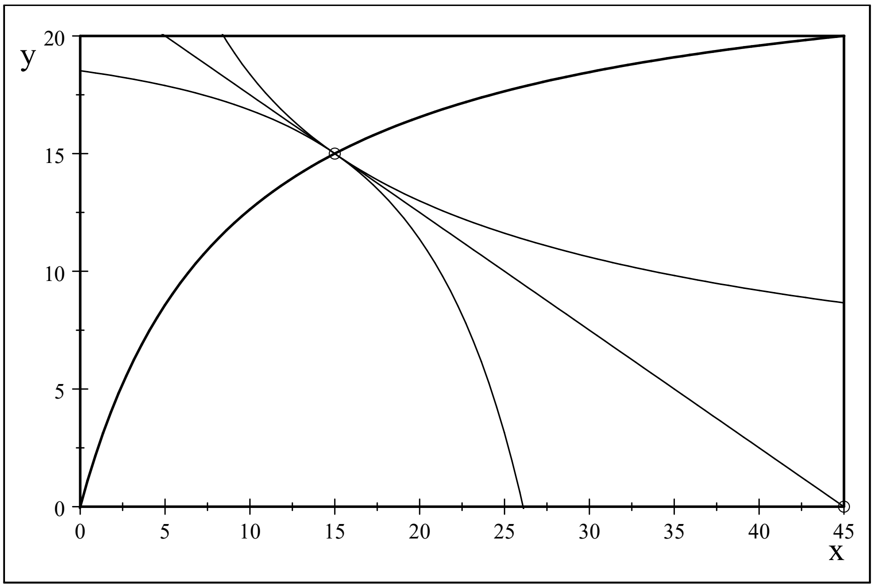
$$\begin{aligned}Y_g &= 2X_g\frac{p_x}{p_y} \\ p_xX_g + p_y\left(2X_g\frac{p_x}{p_y}\right) &= p_x30 \\ X_g &= 10\end{aligned}$$

$$\begin{aligned}Y_g &= \frac{16(10)}{90-(10)} \\ Y_g &= 2 \\ \frac{(2)}{2(10)} &= \frac{p_x}{p_y} \\ \frac{1}{10} &= \frac{p_x}{p_y}\end{aligned}$$

(c) In the graph provided graph the Edgeworth Box, the Contract Curve, the Initial Endowment, and the final equilibrium in this economy. Let Gizem's consumption be increasing as you move to the upper right.

I will graph the answers for a different economy, where

α	β	γ	δ	M	N	PE	$\frac{p_x}{p_y}$	X_g	Y_g
1	2	1	3	45	20	$24\frac{X_g}{X_g+9}$	$\frac{1}{2}$	15	15



to be precise the parts required for the answer are the box itself,the contract curve (these are the two dark lines) the circle in the lower right hand corner indicating the endowment and the circle in the middle indicating the final equilibrium. For the fun of it I have also included the price vector linking the endowment and the final allocation and the indifference curves of the two people through the equilibrium.

(d) Gökçe runs a trading company and is trying to convince Gizem and Volkan to allow imports and exports. After all, she points out, the price of yogurt is $p_y = \rho$ in the rest of the world so they could both consume Yogurt more cheaply. (The price of xylophones is the same.) *In these questions none of the answers have to be supported with careful mathematical analysis.*

i. Who will do better with free trade? Who will do worse?

Solution 21 *Gizem will be better off because the cost of her consumption has fallen but her income has remained constant. Volkan will be worse off because his income will fall more than the cost of his consumption.*

ii. Is there any way for the person who is doing better to change the other person's mind?

Solution 22 *Gizem could transfer some of her new found wealth to Volkan and they could both be better off. Essentially this is what some of you called a "bribe."*

iii. Is it Pareto Improving in this example to allow free trade? Be precise in your answer.

Solution 23 *If Gizem makes the income transfer, then yes. If she does not then no. If she does not then both outcome are*

Pareto efficient, but switching from one PE state to another is not Pareto efficient.

3. Robinson Crusoe consumes only Coconuts (C) and Flip Flops (F), he uses labor to produce both goods and his total endowment of labor is 9. His production function for Coconuts is $C = 4\sqrt{L_c}$ and his production function for Flip Flops is $F = 2\sqrt{L_f}$. He does not enjoy his Coconuts unless he has a new pair of Flip Flops for every 2 Coconuts, or his utility function is $U_r(F_r, C_r) = \min(2F_r, C_r)$.*GREAT EMBARRASMENT! THE ANSWERS I CALCULATED BEFORE THE EXAM WERE NOT RIGHT! SO MOST OF YOU ENDED UP WITH NON-INTEGER ANSWERS. MY APOLOGIES.*

4. *(17 points total)* Robinson Crusoe consumes only Coconuts (C) and Flip Flops (F), he uses labor to produce both goods and his total endowment of labor is (9). His production function for Coconuts is $C = 4\sqrt{L_c}$, and his production function for Flip Flops is $F = 2\sqrt{L_f}$. He does not enjoy his Coconuts unless he has a new pair of Flip Flops for every (2) Coconuts, or his utility function is $U_r(F_r, C_r) = \min(2F_r, C_r)$.

   (a) *(4 points)* Find the production possibilities frontier of Robinson Crusoe's economy and draw it (approximately) in the graph below.

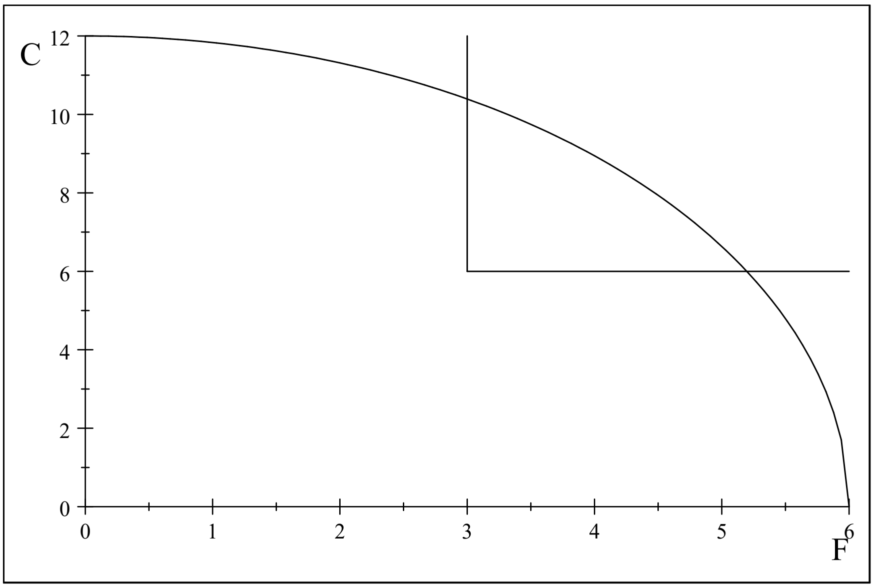
$$\begin{aligned} L_c + L_f &= 9 \\ L_c &= \frac{1}{16}C^2 \\ L_f &= \frac{1}{4}F^2 \end{aligned}$$

$$\frac{1}{16}C^2 + \frac{1}{4}F^2 = 9$$

   (b) *(3 points)* In the graph below draw an indifference curve for Robinson where his utility is equal to 4. From the indifference curve what do you know about the ratio in which Robinson will consume F and C?

Solution 24 *From the indifference curve or the utility function we can see that* $2F_r = C_r$ *.I will graph the answers when:*

A_c	A_f	K	Z
4	2	2	9



(c) *(5 points)* Find the Pareto Efficient allocation in this economy, and the implicit price of flip flops and coconuts in this allocation.

$$\begin{aligned} 2F &= C \\ \frac{1}{16}C^2 + \frac{1}{4}F^2 &= 9 \\ \frac{(2F)^2}{16} + \frac{F^2}{4} &= 9 \\ F^2 &= 18 \\ F &= 3\sqrt{2} \\ C &= 6\sqrt{2} \end{aligned}$$

$$\begin{aligned} C &= 2\sqrt{36 - F^2} \\ MRT &= -\frac{dC}{dF} = 2\left(\frac{1}{2}\right)\left(36 - F^2\right)^{\frac{1}{2}-1}(2F) = 2\frac{F}{\sqrt{36 - F^2}} = \frac{p_f}{p_c} \\ 2\frac{\left(3\sqrt{2}\right)}{\sqrt{36 - \left(3\sqrt{2}\right)^2}} &= 2 = \frac{p_f}{p_c} \end{aligned}$$

(d) *(4 points)* The Native Americans on the next island have just discovered that Robinson Crusoe exists. Being a highly advanced civilization they are more efficient, to be precise their production function for Coconuts is $C = 8\sqrt{L_{nc}}$, and their production function for Flip Flops is $F = 4\sqrt{L_{nf}}$, and their total supply of labor is 9. Their preferences, however, are different. They do not consume Flip Flops—they only make them for the tourists—so $U_n(F_n, C_n) = C_n$.

One of their Senators makes a passionate speech stating that since they have a clear technological superiority over Robinson they would not benefit from trading with him. Solve for the PPF in this economy and tell me whether he is right or not, explaining your answer carefully.

Solution 25

$$
\begin{aligned}
L_{nc} + L_{nf} &= 9 \\
\frac{1}{64}C^2 + \frac{1}{16}F^2 &= 9 \\
\left(\frac{1}{64}C^2 + \frac{1}{16}F^2\right)4 &= 9 * 4 \\
\frac{1}{16}C^2 + \frac{1}{4}F^2 &= 36
\end{aligned}
$$

So the PPF is equivalent just scaling up Robinson's resource constraint. So he is right that they have a clear technological superiority over Robinson, but he is not right in saying this means they will get no benefit from trading with him. A technologically advanced nation can still benefit from trade with a less advanced nation as long as the PPF is concave, which it is in this example. Since Robinson and the Native Americans do not have the same preferences they can benefit from trade.

5. *(22 points total)* Consider the following economy. The production function for food is $F = ((1)\, L_f)^{\left(\left(\frac{1}{2}\right)\right)}$, the production function for clothing is $C = ((9)\, L_c)^{\left(\left(\frac{1}{2}\right)\right)}$, and the total amount of labor available is $\bar{L} = (8)$. There is only one consumer in this economy, his utility function is $U = FC$

   (a) *(3 points)* Find the Production Possibility Frontier in this economy.

$$
\begin{aligned}
F &= (1)\, L_f^{\left(\left(\frac{1}{2}\right)\right)} \\
\left(\frac{F}{(1)}\right)^{\frac{1}{\left(\left(\frac{1}{2}\right)\right)}} &= L_f \\
C &= (9)\, L_c^{\left(\left(\frac{1}{2}\right)\right)} \\
\left(\frac{C}{(9)}\right)^{\frac{1}{\left(\left(\frac{1}{2}\right)\right)}} &= L_c \\
L_f + L_c &= (8) \\
\frac{(F)^{\frac{1}{\left(\left(\frac{1}{2}\right)\right)}}}{(1)} + \frac{(C)^{\frac{1}{\left(\frac{1}{2}\right)}}}{(9)} &= (8)
\end{aligned}
$$

(b) *(4 points)* Find the Marginal Rate of Transformation and the Marginal Rate of Substitution in this economy.

$$MRS = \frac{MU_f}{MU_c} = \frac{\frac{U}{F}}{\frac{U}{C}} = \frac{C}{F}$$

$$MRT = \frac{\frac{1}{\left(\frac{1}{2}\right)}\left(\frac{F}{(1)}\right)^{\frac{1}{\left(\frac{1}{2}\right)}-1}\frac{1}{(1)}}{\frac{1}{\left(\frac{1}{2}\right)}\left(\frac{C}{(9)}\right)^{\frac{1}{\left(\frac{1}{2}\right)}-1}\frac{1}{(9)}} = \left(\frac{F}{C}\right)^{\frac{1}{\left(\frac{1}{2}\right)}-1}\frac{(9)}{(1)}$$

(c) *(6 points)* Find the Pareto Efficient outcome in this economy.

$$\begin{aligned}
\frac{C}{F} &= \left(\frac{F}{C}\right)^{\frac{1}{\left(\frac{1}{2}\right)}-1}\frac{(9)}{(1)} \\
\frac{(1)}{(9)} &= \left(\frac{F}{C}\right)^{\frac{1}{\left(\frac{1}{2}\right)}} \\
\left(\frac{(1)}{(9)}\right)^{\left(\frac{1}{2}\right)} &= \frac{F}{C} \\
F &= \left(\frac{(1)}{(9)}\right)^{\left(\frac{1}{2}\right)} C
\end{aligned}$$

$$\begin{aligned}
\frac{F^{\frac{1}{\left(\frac{1}{2}\right)}}}{(1)} + \frac{C^{\frac{1}{\left(\frac{1}{2}\right)}}}{(9)} &= (8) \\
\frac{\left(\left(\frac{(1)}{(9)}\right)^{\left(\frac{1}{2}\right)} C\right)^{\frac{1}{\left(\frac{1}{2}\right)}}}{(1)} + \frac{C^{\frac{1}{\left(\frac{1}{2}\right)}}}{(9)} &= (8) \\
\frac{1}{9}C^2 + F^2 &= 8 \\
C^2 &= 36 \\
C &= 6 \\
F &= 2
\end{aligned}$$

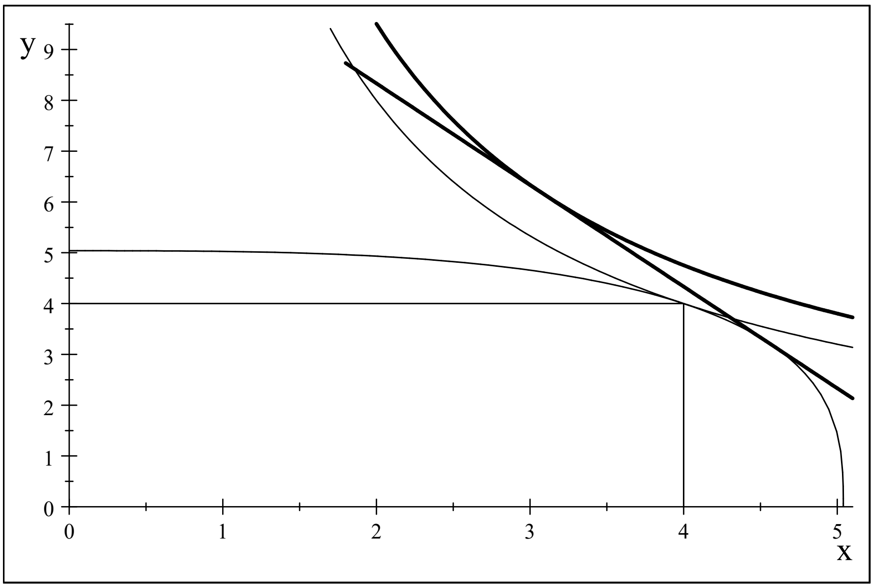
: : : :

(d) *(3 points)* Find the price of food in this economy, you may set the price of clothing to one.

$$\begin{aligned}
\frac{C}{F} &= p_f \\
\frac{\left((9)(8)\frac{1}{2}\right)^{\left(\frac{1}{2}\right)}}{\left((1)(8)\frac{1}{2}\right)^{\left(\frac{1}{2}\right)}} &= p_f \\
\left(\frac{(9)}{(1)}\right)^{\left(\frac{1}{2}\right)} &= 3 = p_f
\end{aligned}$$

: :

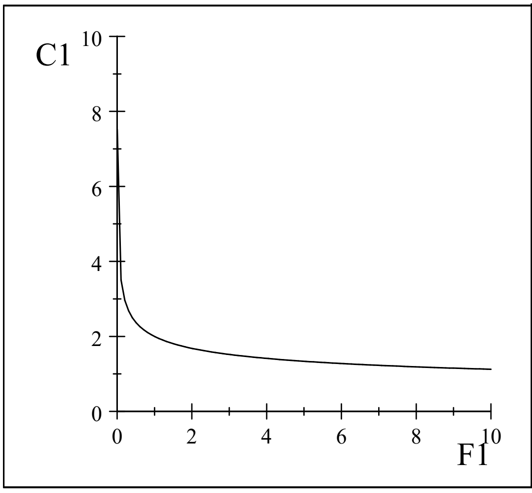
(e) *(6 points)* In the graph below graph the production possibilities frontier and the indifference curve of the consumer at the Walrasian equilibrium. Illustrate how you could find the price vector. *(Note I do not expect exact precision in this part of the question, but I expect your graph to be reasonably accurate.)* Then draw what would happen if this economy opened up to international trade. Internationally the price of food is higher than it is in this country. For clarity you may want to explain your answer below the graph.



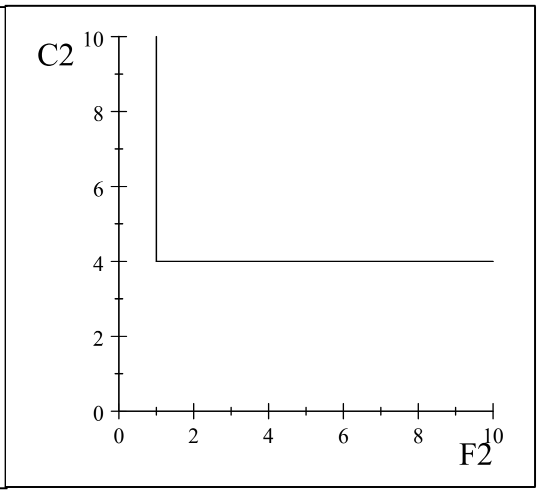
The light lines are the actual solution for the first parameters listed in the table above. The optimal F and C are 4. If the price increases to 2 the consumption and production point will differ, resulting in a budget line like the dark one above. The consumer will be happier because that budget line will cross through the old indifference curve. Society will benefit from international trade.

6. *(18 points total)* In an exchange economy the utility function of person one is $U_1(F_1, C_1) = F_1^3 C_1^2$, the utility function of person two is $U_2(F_2, C_2) = \min\{4F_2, 2C_2\}$ and the total amount of food available is $\bar{F} = 8$ and of clothing is $\bar{C} = 16$.

   (a) *(2 points)* In the two graphs below draw a representative indifference curve for both parties.



Indifference Curve for Consumer 1



Indifference Curve for Consumer 2

   (b) *(6 points)* Write the contract curve in this economy as a function of C_1 in terms of F_1.

$$\begin{aligned}(4)((8) - F_1) &= (2)\left(\frac{(8)}{(2)}(4) - C_1\right) \\ C_1 &= 2F_1\end{aligned}$$

:

   (c) *(4 points)* Find the price of food, p_f, in every Walrasian or general equilibrium.

$$\begin{aligned}\frac{(3)\frac{U}{F_1}}{(2)\frac{U}{C_1}} &= p_f \\ \frac{(3)}{(2)}\frac{C_1}{F_1} &= p_f \\ \frac{(3)}{(2)}\frac{\frac{1}{(2)}(4)F_1}{F_1} &= p_f \\ 3 &= p_f\end{aligned}$$

:: :

(d) *(6 points)* If the initial endowment of food and clothing for consumer 1 is $\left(F_1^0, C_1^0\right) = (5, 10)$ find the final allocation in the general equilibrium.

$$\begin{aligned} C_1^* &= \frac{(2)}{(3)+(2)}\left(p_f(5)+(10)\right) = \frac{(2)}{(3)+(2)}\left(\frac{(3)(4)}{(2)(2)}(5)+(10)\right) = 10 \\ F_1^* &= \frac{(3)}{(3)+(2)}\frac{1}{p_f}\left(p_f(5)+(10)\right) = \frac{(3)}{(3)+(2)}\frac{1}{\frac{(3)(4)}{(2)(2)}}\left(\frac{(3)(4)}{(2)(2)}(5)+(10)\right) = 5 \end{aligned}$$

7. *(10 points)* Consider a general equilibrium exchange economy where there are three goods: food (F), clothing (C), and housing (H). Prove that if supply equals demand in the market for food and clothing then supply must equal demand in the market for housing.

   For each person i it must be true that:

   $$p_f F_i + p_c C_i + p_h H_i = p_f F_i^0 + p_c C_i^0 + p_h H_i^0$$

   thus it must still be true if we sum over i.

   $$p_f\left(\Sigma_i F_i\right) + p_c\left(\Sigma_i C_i\right) + p_h\left(\Sigma_i H_i\right) = p_f\left(\Sigma_i F_i^0\right) + p_c\left(\Sigma_i C_i^0\right) + p_h\left(\Sigma_i H_i^0\right)$$

   since we have market clearing in the market for food and clothes:

   $$\begin{aligned} \left(\Sigma_i F_i\right) &= \left(\Sigma_i F_i^0\right) \\ \left(\Sigma_i C_i\right) &= \left(\Sigma_i C_i^0\right) \end{aligned}$$

   $$\begin{aligned} p_f\left(\Sigma_i F_i^0\right) + p_c\left(\Sigma_i C_i^0\right) + p_h\left(\Sigma_i H_i\right) &= p_f\left(\Sigma_i F_i^0\right) + p_c\left(\Sigma_i C_i^0\right) + p_h\left(\Sigma_i H_i^0\right) \\ p_h\left(\Sigma_i H_i\right) &= p_h\left(\Sigma_i H_i^0\right) \end{aligned}$$

   thus we must have

   $$\Sigma_i H_i = \Sigma_i H_i^0$$

   unless $p_h = 0$, but if you only wrote $p_h\left(\Sigma_i H_i\right) = p_h\left(\Sigma_i H_i^0\right)$ then that is fine.

8. Consider an economy where the production function for food is $F = \left(6\min\left\{\frac{3}{4}K_1, \frac{1}{4}L_1\right\}\right)^{\frac{1}{2}}$ and the production function for clothing is $C = \left(\left(\frac{3}{4}K_2 + \frac{1}{4}L_2\right)\right)^{\frac{1}{2}}$, the total endowment of capital is $K^0 = 8$ and of labor is $L^0 = 24$.

9. *(14 points total)* Consider the following economy. Person one has the utility function $U_1\left(F_1, C_1\right) = \min\left\{\left(\frac{3}{4}\right)F_1, \left(1-\left(\frac{3}{4}\right)\right)C_1\right\}$ and person two has the utility function $U_2\left(F_2, C_2\right) = \left(\frac{3}{4}\right)F_2 + \left(1-\left(\frac{3}{4}\right)\right)C_2$. The total amount of food is $F^0 = 8$, and the total amount of clothing is $C^0 = 24$.

(a) *(6 points)* Find the set of Pareto Efficient allocations. Write this set as a function of C_1 on F_1 **and** as a function of C_2 on F_2.

$$\begin{aligned}\left(1-\left(\frac{3}{4}\right)\right)C_1 &= \left(\frac{3}{4}\right)F_1\\ C_1 &= \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}F_1\end{aligned}$$

$$\begin{aligned}C_1+C_2 &= (24)\\ F_1+F_2 &= (8)\end{aligned}$$

$$\begin{aligned}(24)-C_2 &= \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}\left((8)-F_2\right)\\ \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}F_2 &= \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}(8)-(24)+C_2\end{aligned}$$

$$3F_2 = C_2$$

:

(b) *(4 points)* Find the prices in any general equilibrium in this economy, you may normalize the price of C to one.

$$3 = p_f$$

(c) *(4 points)* If the initial endowment for person one is $\left(F_1^0, C_1^0\right) = (6,(12))$, what are the equilibrium consumptions of both people?

$$\begin{aligned}C_1 &= 3F_1\\ \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}F_1+C_1 &= \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}(6)+(12)\\ \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}F_1+\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}F_1 &= \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}(6)+(12)\\ F_1 &= \frac{\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}(6)+(12)}{\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}+\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}} = 5\\ C_1 &= \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}\left(\frac{\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}(6)+(12)}{\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}+\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}}\right) = 15\end{aligned}$$

: : :

10. *(12 points total)* Consider an economy where the production function for food is $F = \left((6)\min\left\{\frac{3}{4}K_1, \left(1-\left(\frac{3}{4}\right)\right)L_1\right\}\right)^{\frac{1}{2}}$ and the production function for clothing is $C = \left((1)\left(\left(\frac{3}{4}\right)K_2 + \left(1-\left(\frac{3}{4}\right)\right)L_2\right)\right)^{\frac{1}{2}}$, the total endowment of capital is $K^0 = 8$ and of labor is $L^0 = 24$.

(a) *(2 points)* Explain why you can make extensive use of your answers in the last question to find the Pareto Efficient allocation of labor and capital in this economy.

We can re-write the functions in this section as:

$$\frac{F^2}{(6)} = \min\left\{\left(\frac{3}{4}\right)K_1, \left(1-\left(\frac{3}{4}\right)\right)L_1\right\}$$
$$\frac{C^2}{(1)} = \left(\frac{3}{4}\right)K_2 + \left(1-\left(\frac{3}{4}\right)\right)L_2$$

then it is transparent that they are the same as in the last question. In fact if we rewrite the expressions with F for K and *C for L you can see that the problem is exactly the same, thus we know that the PE allocations in that economy are still PE in this one, specifically that* $L_i = \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}K_i$ for $i \in \{1, 2\}$.

(b) *(10 points)* Show that the production possibilities frontier is $\frac{2}{3}C^2 +$

$\frac{2}{9}F^2 = 8.$

$$\begin{aligned}
L_1 &= \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}K_1 \\
F &= \left((6)\min\left\{\left(\frac{3}{4}\right)K_1, \left(1-\left(\frac{3}{4}\right)\right)\left(\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}K_1\right)\right\}\right)^{\frac{1}{2}} = \left((6)\left(\right.\right. \\
\frac{F^2}{\left(\frac{3}{4}\right)(6)} &= K_1 \\
\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}K_2 &= L_2 \\
C &= \left((1)\left(\left(\frac{3}{4}\right)K_2 + \left(1-\left(\frac{3}{4}\right)\right)L_2\right)\right)^{\frac{1}{2}} \\
\frac{C^2}{(1)} &= \left(\left(\frac{3}{4}\right)K_2 + \left(1-\left(\frac{3}{4}\right)\right)\left(\frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}K_2\right)\right) \\
\frac{C^2}{(1)} &= K_2\left(\frac{\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}\right) \\
\frac{C^2}{(1)}\frac{1-\left(\frac{3}{4}\right)}{\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)} &= K_2 \\
K_1 + K_2 &= (8) \\
\frac{2}{3}C^2 + \frac{2}{9}F^2 &= 8
\end{aligned}$$

11. *(16 points total)* Consider an economy with one person who's preferences are $U(F,C) = FC^3$ and who has the production possibilities frontier $\frac{2}{3}C^2 + \frac{2}{9}F^2 = 8$.

    (a) *(2 points)* Find the Marginal Rate of Substitution for this person.

    $$MRS = \frac{(1)\frac{U}{F}}{(3)\frac{U}{C}} = \frac{(1)C}{(3)F}$$

    (b) *(2 points)* Find the Marginal Rate of Transformation in this economy.

    $$MRT = \frac{\frac{2F}{\left(\frac{3}{4}\right)(6)}}{\frac{2C}{(1)}\frac{1-\left(\frac{3}{4}\right)}{\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)}} = \frac{F}{C}\frac{\left(\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right)}{\left(\frac{3}{4}\right)\left(1-\left(\frac{3}{4}\right)\right)}\frac{(1)}{(6)}$$

    :

(c) *(5 points)* Find the Pareto Efficient allocation in this economy.

$$\frac{F}{C}\frac{\left(\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right)}{\left(\frac{3}{4}\right)\left(1-\left(\frac{3}{4}\right)\right)}\frac{(1)}{(6)} = \frac{(1)\,C}{(3)\,F}$$

$$\frac{\left(\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right)(3)}{\left(\frac{3}{4}\right)(1)\left(1-\left(\frac{3}{4}\right)\right)}\frac{(1)}{(6)}F^2 = C^2$$

$$\frac{F^2}{\left(\frac{3}{4}\right)(6)} + \frac{\left(\frac{1}{\frac{(1)}{(3)}}\frac{\frac{2}{\left(\frac{3}{4}\right)(6)}}{2\frac{1}{(1)}\frac{1-\left(\frac{3}{4}\right)}{\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)}}F^2\right)}{(1)}\frac{1-\left(\frac{3}{4}\right)}{\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)} = (8)$$

$$\frac{F^2}{\left(\frac{3}{4}\right)(6)} + \frac{F^2}{\left(\frac{3}{4}\right)(6)}\frac{(3)}{(1)} = (8)$$

$$\left(1+\frac{(3)}{(1)}\right)\frac{1}{\left(\frac{3}{4}\right)(6)}F^2 = (8)$$

$$\frac{(1)+(3)}{(1)}\frac{1}{\left(\frac{3}{4}\right)(6)}F^2 = (8)$$

$$F = \sqrt{\frac{(1)}{(1)+(3)}\left((8)\left(\frac{3}{4}\right)(6)\right.}$$

$$\sqrt{\frac{(3)}{(1)+(3)}\left((8)(1)\frac{\left(\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)-2\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right)}{\left(1-\left(\frac{3}{4}\right)\right)}\right)} = 3 = C$$

: :

(d) *(3 points)* Find the price of food and clothing that can support this Pareto Efficient allocation as a General equilibrium.

$$\frac{1}{3} = \frac{p_f}{p_c}$$

:

(e) *(4 points)* Explain how you can support the Pareto Efficient allocation as a General equilibrium using the prices you found in the last part of the question. I am not looking for a mathematical solution, merely for a verbal explanation of how it can be done.

You can act as if a firm is maximizing its revenue given prices, or the firm solves:

$$R = \max_{F,C} p_f F + p_c C - \lambda\left(\frac{2}{3}C^2 + \frac{2}{9}F^2 - 8\right)$$

: *it's revenue then becomes the income of the consumer, $I = R$, then the consumer maximizes his utility given the prices and income.*

$$\max_{F,C} U(F,C) - \lambda\left(p_f F + p_c C - I\right)$$

12. *(12 points)* About Walras's Law.

   (a) *(6 points)* Define Walras's Law for an economy with m goods.

   In a general equilibrium economy with m goods you only have to solve for $m-1$ prices and set supply equal to demand in $m-1$ markets.

   (b) *(6 points)* Assume there are three goods in the economy, F (food), C (clothing) and H (housing). Prove that you can normalize one of the prices to one without loss of generality. (You may assume that in the general equilibrium all prices are strictly positive.)

   I will show this for exchange economies, the general proof is similar but harder. For each consumer i their budget constraint is:

$$p_f F_i + p_c C_i + p_h H_i \leq p_f F_i^0 + p_c C_i^0 + p_h H_i^0$$

   you can always divide this by a positive constant and have the same inequality. If you divide by p_h you have:

$$\frac{p_f}{p_h} F_i + \frac{p_c}{p_h} C_i + H_i \leq \frac{p_f}{p_h} F_i^0 + \frac{p_c}{p_h} C_i^0 + H_i^0$$

   now if we rewrite the prices as $\tilde{p}_f = \frac{p_f}{p_h}$, $\tilde{p}_c = \frac{p_c}{p_h}$ you have in effect normalized the price of a house to 1.

13. *(14 points)* About Walras's Law.

   (a) *(6 points)* Define Walras's Law for an economy with m goods.

   Walras's Law says that in an economy with m goods we only need to solve for supply equals demand in $m-1$ markets and that we can only solve for $m-1$ prices.

   (b) *(4 points)* We say that an economy faces *inflation* of $i > 0$ if instead of the price vector $P = [p_f, p_c, p_h, ...]$ they face the price vector $(1+i)\,P = [(1+i)\,p_f, (1+i)\,p_c, (1+i)\,p_h, ...]$. Explain how Walras's law proves that this will have absolutely no impact on the economy.

   Walras's Law says can rewrite the prices as: $P' = \left[1, \frac{p_c}{p_f}, \frac{p_h}{p_f}, ...\right]$, or we can rewrite the prices $(1+i)\,P' = \left[1, \frac{(1+i)p_c}{(1+i)p_f}, \frac{(1+i)p_h}{(1+i)p_f}, ...\right] = \left[1, \frac{p_c}{p_f}, \frac{p_h}{p_f}, ...\right] = P'$ thus the two price vectors are the same and this will have no impact on the economy.

   (c) *(4 points)* I assert that in reality inflation does have a real impact on the economy, what is the difference between real world inflation and the type of inflation I just described that causes this impact?

   There are some subtle reasons that this may not be true, but the basic one is that inflation does not affect all prices simultaneously. To give a simple example, the inflation rate for capital and labor are rarely the same. A more concrete example is consider two workers. Worker A *has a long term contract where his wage is determined for a two year period. Worker B gets an hourly wage—to make it more extreme assume she is hired each day. Obviously Worker B's wage will react much more quickly to market forces, namely a general increase in the wage level.*

Because of this inflation shifts the wealth of the economy among individuals, and only in the theoretic extreme does it have no impact on the equilibrium.

14. *(25 points total)* Consider an economy with one person who's preferences are $U(F,C) = FC^{(2)}$. The production function for food is $F = \sqrt{(1)\,L_f}$ and for clothing is $C = \sqrt{(2)\,L_c}$, the total endowment of labor is $L^0 = (12)$.

    (a) *(4 points)* Find the production possibilities frontier.

$$\begin{aligned} F &= \sqrt{(1)\,L_f} \\ (1)\,L_f &= F^2 \\ L_f &= \frac{F^2}{(1)} \end{aligned}$$

$$\begin{aligned} C &= \sqrt{(2)\,L_c} \\ (2)\,L_c &= C^2 \\ L_c &= \frac{C^2}{(2)} \end{aligned}$$

$$\begin{aligned} L_f + L_c &= L^0 \\ \frac{1}{2}C^2 + F^2 &= 12 \end{aligned}$$

:

    (b) *(4 points)* Find the Marginal Rate of Substitution for this person and the Marginal Rate of Transformation for this Economy.

$$\begin{aligned} \frac{\partial U}{\partial F} &= (1)\,\frac{U}{F} \\ \frac{\partial U}{\partial C} &= (2)\,\frac{U}{C} \\ MRS &= \frac{\frac{\partial U}{\partial F}}{\frac{\partial U}{\partial C}} = \frac{(1)\,\frac{U}{F}}{(2)\,\frac{U}{C}} = (1)\,\frac{C}{F\,(2)} \end{aligned}$$

$$\frac{\partial PPF}{\partial F} = \frac{2F}{(1)}$$
$$\frac{\partial PPF}{\partial C} = \frac{2C}{(2)}$$
$$MRT = \frac{\frac{\partial PPF}{\partial F}}{\frac{\partial PPF}{\partial C}} = \frac{\frac{2F}{(1)}}{\frac{2C}{(2)}} = \frac{1}{C}F\frac{(2)}{(1)}$$

(c) *(6 points)* Find the Pareto efficient allocation in this economy.

$$MRS = MRT$$
$$(1)\frac{C}{F(2)} = \frac{1}{C}F\frac{(2)}{(1)}$$
$$\frac{(1)}{(2)}C^2 = F^2\frac{(2)}{(1)}$$
$$C^2 = F^2\frac{1}{(1)}(2)\frac{(2)}{(1)}$$
$$C = F\sqrt{\frac{1}{(1)}(2)\frac{(2)}{(1)}}$$

$$\frac{F^2}{(1)} + \frac{\left(F^2\frac{1}{(1)}(2)\frac{(2)}{(1)}\right)}{(2)} = (12)$$
$$\frac{F^2}{(1)}\frac{((1)+(2))}{(1)} = (12)$$
$$F^2 = (1)(12)\frac{(1)}{(1)+(2)}$$
$$F = \sqrt{(1)(12)\frac{(1)}{(1)+(2)}} = 2$$
$$C = F\sqrt{\frac{1}{(1)}(2)\frac{(2)}{(1)}} = \sqrt{(1)(12)\frac{(1)}{(1)+(2)}}\sqrt{\frac{1}{(1)}(2)\frac{(2)}{(1)}} = 4$$

: :

(d) *(4 points)* Using only words, describe how you could decentralize this economy so that agents would only have to maximize their objectives given prices.

We can set this up as a consumer who gets all the profits from the firms and the wage of labor and firms who simply maximize profits given prices. The prices are given by the slope of the PPF or the indifference curve of the consumer at the Pareto efficient outcome.

(e) *(2 points)* In this decentralized economy explain carefully how you would calculate the consumer's income. You may assume the consumer supplies all of the labor.

$I = wL + \pi_f(p_f, w) + (1)_c(p_c, w)$ *where* $\pi_f(p_f, w) = \max_{F,L_f} p_f F - wL_f,\ L_f = \frac{F^2}{(1)},\ \pi_c(p_c, w) = \max_{C,L_c} p_c C - wL_c,\ L_c = \frac{C^2}{(2)}$. *Alternatively* $I = wL + R(p_f, p_c)$ *where*

$$R(p_f, p_c) = \max_{F,C} p_f F + p_c C,\ \frac{F^2}{(1)} + \frac{C^2}{(2)} = 12$$

(f) *(5 points)* Find the price of food, clothing, and labor that would support this Pareto efficient point of a decentralized economy.

$$\frac{p_f}{p_c} = MRS = (1)\frac{C}{F(2)} = \frac{(1)}{(2)}\frac{\sqrt{(1)(12)\frac{(1)}{(1)+(2)}}\sqrt{\frac{1}{(1)}(2)\frac{(2)}{(1)}}}{\sqrt{(1)(12)\frac{(1)}{(1)+(2)}}} = \sqrt{\frac{(1)(2)}{(2)(1)}} = 1$$

:

$$\max_{F,L_f} p_f F - wL_f,\ L_f = \frac{F^2}{(1)}$$

$$\max_F p_f F - w\frac{F^2}{(1)}$$

$$\begin{aligned} p_f - w\frac{2F}{(1)} &= 0 \\ \frac{p_f}{w} &= \frac{2F}{(1)} = \frac{2\sqrt{(1)(12)\frac{(1)}{(1)+(2)}}}{(1)} = 2\sqrt{\frac{(12)}{(1)}\frac{(1)}{(1)+(2)}} = 4 \end{aligned}$$

15. Consider a production exchange economy. The production function for food is $F = \frac{1}{2}\sqrt{2}\sqrt{\min\left(\frac{1}{5}L_f, \frac{4}{5}K_f\right)}$, the production function for clothing is $C = \frac{1}{15}\sqrt{15}K_c^{\frac{1}{4}}L_c^{\frac{1}{4}}$. The total endowment of labor is $L^0 = 120$, the total endowment of capital is $K^0 = 30$.

16. *(16 points total)* Consider a production exchange economy. The production function for food is $F = \left(\frac{1}{(2)}\min\left\{\frac{(30)}{(120)+(30)}L_f, \left(1 - \frac{(30)}{(120)+(30)}\right)K_f\right\}\right)^{\frac{1}{2}}$, the production function for clothing is $C = \left(\frac{1}{(15)}L_c^{\left(1-\left(\frac{1}{2}\right)\right)}K_c^{\left(\frac{1}{2}\right)}\right)^{\frac{1}{2}}$. The total endowment of labor is $L^0 = (120)$, the total endowment of capital is $K^0 = (30)$.

   (a) *(4 points)* Find the contract curve for this economy as a function of L_f in terms of K_f. Verify that it passes through the points $(0, 0)$ and $\left(L^0, K^0\right)$.

   From the Leontief production function we know that:

$$\begin{aligned} \frac{(30)}{(120)+(30)}L_f &= \left(1 - \frac{(30)}{(120)+(30)}\right)K_f \\ L_f &= 4K_f \end{aligned}$$

Obviously $L_f(0)=0$ *and* $L_f((30))=\frac{(120)}{(30)}(30)=(120)$

(b) *(4 points)* Find the equilibrium prices in any general equilibrium, let the price of labor be w and the price of capital be r. Hint: This can **not** be found from the slope of the contract curve.

The MRS of the production function for clothing can be written as:

$$MRS=\frac{MP_l}{MP_k}=\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)\right)\frac{C}{L_c}}{\frac{1}{2}\left(\frac{1}{2}\right)\frac{C}{K_c}}=\frac{1-\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}\frac{K_c}{L_c}=\frac{w}{r}$$

including that we must be on the contract curve:

$$MRS=\frac{1-\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}\frac{K_c}{\frac{(120)}{(30)}K_c}=\frac{1-\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}\frac{(30)}{(120)}=\frac{w}{r}=\frac{1}{4}$$

:

(c) *(4 points)* If the initial endowment of labor and capital of food is $\left(L_f^0,K_f^0\right)=(40,30)$ find out how much labor and capital will be used to produce food in the general equilibrium.

The budget constraint for the production function for food is:

$$wL_f+rK_f=w(40)+r(30)$$

from the fact that this equilibrium is Pareto Efficient we know that $L_f=4K_f$

$$w4K_f+rK_f=w(40)+r(30)$$

letting $r=1$ *and* $w=\frac{1}{4}$

$$\begin{aligned}\left(\frac{1-\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}\frac{(30)}{(120)}\right)\frac{(120)}{(30)}K_f+K_f &= \frac{1-\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}\frac{(30)}{(120)}(40)+(30)\\ K_f &= \frac{1-\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}\frac{(30)}{(120)}(40)+(30)=40\\ K_f &= \left(\frac{1-\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}\frac{(30)}{(120)}(40)+(30)\right)\left(\frac{1}{2}\right)\\ &= \frac{1}{(120)}\left((40)(30)\left(1-\left(\frac{1}{2}\right)\right)+(30)(120)\left(\frac{1}{2}\right)\right)=20\end{aligned}$$

: :

(d) *(4 points)* Verify that the production possibilities frontier can be written as: $\frac{15}{2}C^2+\frac{5}{2}F^2=30$.

We have that $L_f=\frac{(120)}{(30)}K_f$, *thus* $F=\left(\frac{1}{(2)}\left((120)\frac{K_f}{(120)+(30)}\right)\right)^{\frac{1}{2}}$

$$\begin{aligned}F^2 &= \frac{1}{(2)}\frac{(120)}{(120)+(30)}K_f\\ (2)\frac{(120)+(30)}{(120)}F^2 &= K_f\end{aligned}$$

We also have that $L_c = \frac{(120)}{(30)} K_c$

$$\begin{aligned} C &= \left(\frac{1}{(15)} \left(\frac{(120)}{(30)} K_c \right)^{\left(1-\left(\frac{1}{2}\right)\right)} K_c^{\left(\frac{1}{2}\right)} \right)^{\frac{1}{2}} \\ C^2 &= \left(\frac{1}{(15)} \left(\frac{(120)}{(30)} K_c \right)^{\left(1-\left(\frac{1}{2}\right)\right)} K_c^{\left(\frac{1}{2}\right)} \right) \\ &= \frac{1}{(15)} \left(\frac{(120)}{(30)} \right)^{1-\left(\frac{1}{2}\right)} K_c \\ (15) \left(\frac{(30)}{(120)} \right)^{1-\left(\frac{1}{2}\right)} C^2 &= K_c \end{aligned}$$

By the resource constraint, $K_f + K_c = (30)$*, thus we have* $\frac{15}{2} C^2 + \frac{5}{2} F^2 = 30$

17. *(31 points total)* Consider a Robinson Crusoe economy where his preferences are written as $U(F, C) = \sqrt[3]{C} F$ and his initial endowment of labor and capital and his production function for food and clothing are given in the last question.

    (a) *(4 points)* Find his marginal rate of substitution and the economy's marginal rate of transformation.

$$MRS = \frac{\left(\frac{3}{4}\right)\left(\frac{4}{3}\right)\frac{U}{F}}{\left(1-\left(\frac{3}{4}\right)\right)\left(\frac{4}{3}\right)\frac{U}{C}} = \frac{C}{F} \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)}$$

$$MRT = \frac{\frac{\partial PPF}{\partial F}}{\frac{\partial PPF}{\partial C}} = \frac{2(2)\frac{(120)+(30)}{(120)} F}{2(15)\left(\frac{(30)}{(120)}\right)^{1-\left(\frac{1}{2}\right)} C} = \frac{F}{C} \frac{(2)\left(\frac{(120)}{(30)}\right)^{1-\left(\frac{1}{2}\right)} ((120)+(30))}{(120)(15)}$$

    (b) *(4 points)* Find the Pareto Efficient allocation.

$$\begin{aligned} \frac{C}{F} \frac{\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)} &= \frac{F}{C} \frac{(2)\left(\frac{(120)}{(30)}\right)^{1-\left(\frac{1}{2}\right)} ((120)+(30))}{(120)(15)} \\ C^2 &= F^2 \frac{1-\left(\frac{3}{4}\right)}{(120)\left(\frac{3}{4}\right)(15)} (2) \left(\frac{(120)}{(30)}\right)^{1-\left(\frac{1}{2}\right)} ((120)+(30)) = \frac{1}{9} F^2 \end{aligned}$$

: *putting this into the production possibilities frontier:*

$$(2)\frac{(120)+(30)}{(120)}F^2+(15)\left(\frac{(30)}{(120)}\right)^{1-\left(\frac{1}{2}\right)}C^2$$

$$(2)\frac{(120)+(30)}{(120)}F^2+(15)\left(\frac{(30)}{(120)}\right)^{1-\left(\frac{1}{2}\right)}\left(F^2\frac{1-\left(\frac{3}{4}\right)}{(120)\left(\frac{3}{4}\right)(15)}(2)\left(\frac{(120)}{(30)}\right)^{1-\left(\frac{1}{2}\right)}((120)+(30))\right)$$

$$F^2(2)\frac{((120)+(30))}{(120)\left(\frac{3}{4}\right)}$$

$$F^2$$

$$F^*$$

:

$$\begin{aligned} C^2 &= \left(\frac{(120)\left(\frac{3}{4}\right)}{(2)((120)+(30))}(30)\right)\frac{1-\left(\frac{3}{4}\right)}{(120)\left(\frac{3}{4}\right)(15)}(2)\left(\frac{(120)}{(30)}\right)^{1-\left(\frac{1}{2}\right)}((120)+(30)) \\ &= \frac{1-\left(\frac{3}{4}\right)}{(15)}\left(\frac{(120)}{(30)}\right)^{1-\left(\frac{1}{2}\right)}(30) \\ C^* &= \left(\frac{1-\left(\frac{3}{4}\right)}{(15)}\left(\frac{(120)}{(30)}\right)^{1-\left(\frac{1}{2}\right)}(30)\right)^{\frac{1}{2}}=1 \end{aligned}$$

:

(c) *(8 points)* Explain how this problem can be decentralized in one of two ways.

i. One consumer maximizing his utility given his income and one firm maximizing it's revenue give the production possibilities set.

ii. One consumer maximizing his utility given his income, one firm maximizing it's profits from food, and another maximizing it's profits from clothing.
In the first problem if $MRT(F^*,C^*)=p_f/p_c$ then the firm will produce the Pareto Efficient amount, and if $I=p_fF^*+p_cC^*$, where $MRS(F^*,C^*)=p_f/p_c$ then the consumer will consume his endowment and this will be the Pareto efficient allocation.
In the second problem we have four prices (p_f,p_c,w,r) each firm will maximize their profits giving an $F(p_f,w,r)$ and a $C(p_c,w,r)$, for the appropriate (p_f,p_c) this will give us F^* and C^*, the con-

sumer then will have the income

$$\begin{aligned} I &= \pi_f + \pi_c + wL^0 + rK^0 \\ &= \left(p_f F^* - wL_f^* - rK_f^*\right) + \left(p_c C^* - wL_c^* - rK_c^*\right) + wL^0 + rK^0 \\ &= p_f F^* + p_c C^* + w\left(L^0 - L_f^* - L_c^*\right) + r\left(K^0 - K_f^* - K_c^*\right) \end{aligned}$$

by market clearing, $L^0 = L_f^* + L_c^*$, $K^0 = K_f^* + K_c^*$ *thus*

$$I = p_f F^* + p_c C^*$$

thus if we choose prices like before, the problems are identical.

Critically define the income in each variation, and show that in equilibrium the income in both variations will be the same.

(d) *(9 points)* In the graph below:

i. Graph the production possibilities set, the Pareto Efficient point, and his indifference curve at the Pareto Efficient allocation.

I will graph this for the problem:

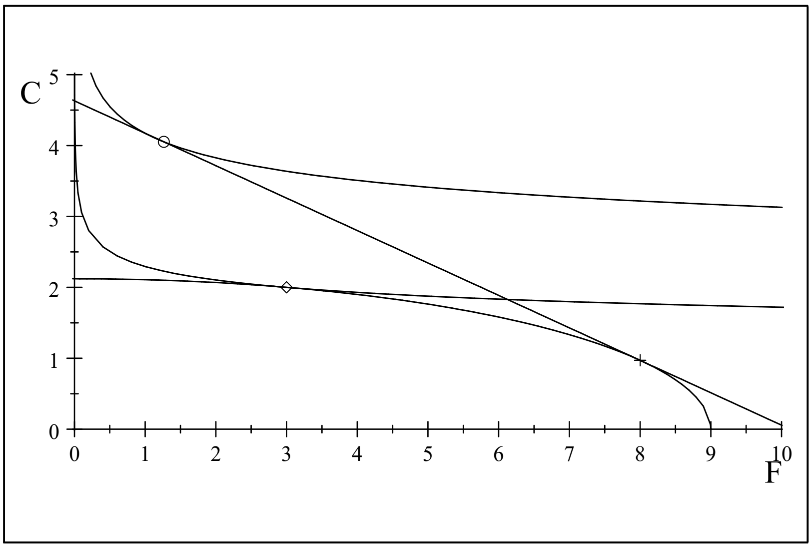
ω_f	X	Y	ω_c	β	γ	ρ
$\frac{1}{2}$	81	81	18	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{9}{8}$

$$\begin{aligned} U &= F^{\frac{1}{8}} C \\ C &= \frac{U}{F^{\frac{1}{8}}} \end{aligned}$$

$$\begin{aligned} U^* &= 3^{\frac{1}{8}}(2) = 2.2944 \\ C &= \frac{3^{\frac{1}{8}}(2)}{F^{\frac{1}{8}}} \end{aligned}$$

of course this is practically impossible to graph without a program, but I have one!! The diamond in the picture is the equilibrium.

ii. Show that he will be better off if he allows international trade, specifically assume that the world price ratio is not the same as his local price ratio. Show his imports and exports in this graph. *I choose a world price ratio that would make a dramatic difference,* $\frac{p_f}{p_c} = 0.45733$, *and then derived the supply* $(F_s^*, C_s^*) = (8, 0.97183)$, *and the demand* $(F_d^*, C_d^*) = (1.2656, 4.0517)$. *Since the program I am using can't graph all of these differences very well, the exports are* $F_s^* - F_d^* = 8 - 1.2656 = 6.7344$, *and the imports are* $C_d^* - C_s^* = 4.0517 - 0.97183 = 3.0799$. *The supply is the cross in the picture, the circle is the demand. As you can see he is on a strictly higher indifference curve thus has a higher utility. In your answer you should show the new budget constraint, and the new consumption point. It is important that* *the indifference curves you draw do not cross.*



(e) *(6 points)* Now prove that as long as the ratio of world prices is not the same as the ratio of local prices he must be strictly better off.

First assume that the economy continues to produce the old output levels, (F^*, C^*)*, at the new prices. With this budget constraint the consumer must be at least as well off as before because he can still consume* (F^*, C^*) *but he may not. Now assume the economy maximizes their revenue, this must create a strictly higher budget constraint, and thus the budget set has strictly expanded. Thus the consumer must be strictly better off.*