

Practice Questions—Chapter 12 and 13

Equilibrium in Competitive Markets

ECON 203

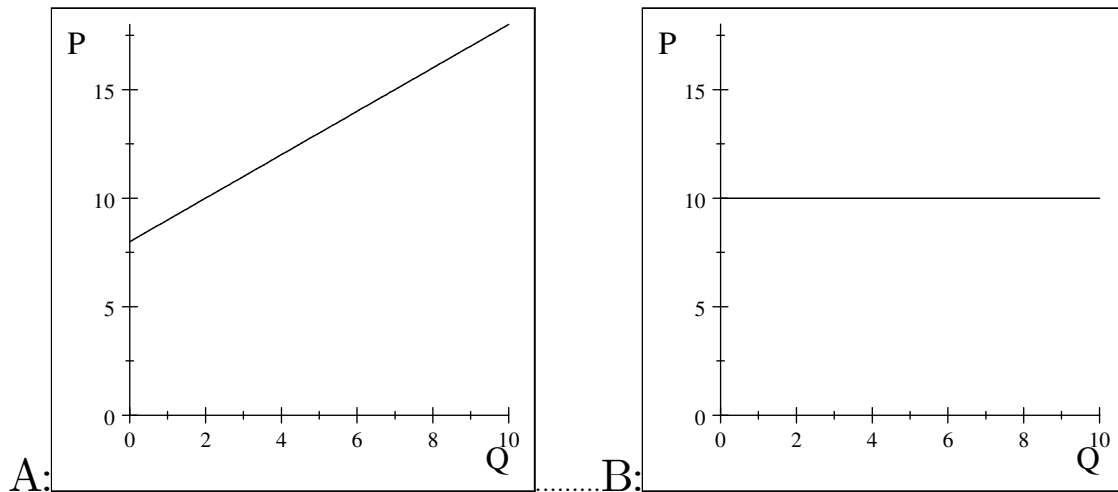
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These questions are to help you prepare for the exams only. Do not turn them in. Note that not all questions can be completely answered using the material in the chapter in which they are asked. These are all old exam questions and often the answers will require material from more than one chapter. Questions with lower numbers were asked in more recent years.

1 Chp 12—The Partial Equilibrium Competitive Model

1. In a given market the market demand curve is $D(P) = 64 - 2P$ and the (domestic) market supply curve is $S(P) = -22 + P$. International firms will supply any amount desired at a price of 24.
 - (a) Solve for the equilibrium price and quantity in this market.
 - (b) Now the government decides to impose an import tariff of τ per unit imported.
 - i. For all τ , find the relationship between τ and the price in this market? *Hint: Be very careful to consider very high and very low τ .*
 - ii. If $\tau = 2$ solve for the equilibrium price and quantity in this market. What is the quantity imported?
 - iii. If $\tau = 2$ find the Consumer Surplus, Producer Surplus, Government Revenue, and Dead Weight Loss in this market. Partial credit will be given for an accurately labelled graph.
 - iv. Define a quota.
 - v. Find a quota that will result in the same price as the tariff you found above. What is the dead weight loss if the government imposes a quota instead of a tariff? (Consider only the welfare of the nation.)
 - vi. Does a quota or a tariff produce a higher dead weight loss? Why might the government want to use the method that produces a higher dead weight loss?

2. Consider the two supply curves ($S(P)$) below:



- A: B:
- Which supply curve has a higher slope ($\frac{\partial S(P)}{\partial P}$), A or B? Explain your answer.
 - Both of these could represent long run supply curves. Explain under what conditions each could be a long run supply curve. Which is of these conditions are the ones we normally assume?
3. In a given industry the total costs are $c(q) = 200 + 2q + 2q^2$. There are $J_A = 12$ firms in the industry with a fixed start up costs of $F_{stA} = 2$. There are $J_B = 8$ firms in the industry with a fixed sunk costs of $F_{suB} = 182$.
- I claim that this it is fairly normal in industries that firms have different fixed start up costs. Why is this? Which type of firms (A or B) have probably been in the industry for longer? Why?
 - Find the marginal costs, average variable costs, and supply curve of a representative type A firm.
 - Find the marginal costs, average variable costs, and supply curve of a representative type B firm.
 - Find the short run supply curve of this industry.
 - Find the short run equilibrium in this industry if the demand is $D(P) = 86 - P$.
 - Find the price at which firms will enter the market.
 - If the quantity supplied in the medium run is 210 what is the market price and the number of firms in the industry?
 - If the quantity supplied in the medium run is 190 what is the market price and the number of firms in the industry?

4. Consider the market for tomatoes.
 - (a) What problem in this market (and other agricultural markets) often leads to government intervention?
 - (b) What is the normal form of government intervention in this market?
 - (c) If governments could costlessly store the product what is another method of intervention in this market? Could this theoretically produce a superior outcome? Why doesn't this method work in reality?
 - (d) Why might governments want to pay farmers not to produce in this market?

5. In a given market the market demand curve is $D(P_d) = 24 - P_d$ and the market supply curve is $S(P_s) = -36 + 3P_s$.
 - (a) Solve for the equilibrium price and quantity in this market.
 - (b) Now the government is considering a per-unit tax of $t = 4$.
 - i. What relationship will hold in equilibrium between P_d and P_s independent of the supply and demand curve?
 - ii. Solve for the equilibrium prices and quantity in this market.
 - iii. Carefully graph the supply, demand, equilibrium prices and quantity, consumer surplus, producer surplus, and dead weight loss in a graph.
 - iv. What is the Consumer Surplus, Producer Surplus, and Dead weight loss?
 - (c) Now consider a general per-unit tax of t .
 - i. Solve for the equilibrium prices and quantity in this market.
 - ii. What is the Dead weight loss in this market? (The answer should be a function of t).

6. In a given market the firms have a cost of $c(q) = 25 + 11q + q^2$ with a fixed sunk cost of 21. There are currently 14 firms in this market.
 - (a) Find the marginal costs, average variable costs, average costs, the price at which firms will shut down and the price at which firms will enter.
 - (b) Find the short run supply curve of a firm and this industry. If the demand curve is $D(P) = 59 - P$ find the equilibrium price and quantity in this market.
 - (c) Assume that in the medium run the costs of the firms do not change (including the sunk costs). Find the industry's medium run supply curve. If the equilibrium quantity in the medium run is 56 what is the price and number of firms? If the equilibrium quantity in the medium run is 75 what is the price and the number of firms?

- (d) Find a firm and the industry's long run supply curve. If the equilibrium quantity supplied is 80 what is the equilibrium price and number of firms?
7. In a given industry the supply curve is $Q_s = -23 + 3 \ln P$ and the demand curve is $Q_d = \delta - P^3$.
- (a) Verify that $\frac{\partial Q_s}{\partial P} > 0$ and that $\frac{\partial Q_d}{\partial P} < 0$.
- (b) Find $\frac{\partial P}{\partial \delta}$ and $\frac{\partial Q}{\partial \delta}$.
- (c) Verify that $\frac{\partial P}{\partial \delta} = \frac{\frac{\partial Q_d}{\partial \delta}}{\frac{\partial Q_s}{\partial P} - \frac{\partial Q_d}{\partial P}}$.
8. How can a long run supply curve be upward sloping? Give an example of an industry where you think it should be upward sloping. How can it be downward sloping? Again give an example. Why do we generally assume that it is neither upward or downward sloping? Please note that we are discussing $\frac{\partial P}{\partial Q}$, or a graph where quantity is on the horizontal axis.
9. In a given market the market demand curve is $D(P_b) = 60 - 4P_b$ and the (domestic) market supply curve is $S(P_s) = -6 + 2P_s$.
- (a) Solve for the equilibrium prices and quantity in this market.
- (b) Now the government is considering a price ceiling of 9.
- What constraint does this impose on the market price?
 - Solve for the equilibrium price and quantity in this market.
 - Find the Consumer Surplus, Producer Surplus, and Dead Weight Loss in this market. You will get partial credit for a graph.
- (c) Now the government decides to allow imports, international firms will supply any amount desired at a price of 9. The government will also impose a import tariff of τ per unit imported.
- What is the relationship between τ and the price in this market?
Hint: Be very careful to consider very high and very low τ .
 - Solve for the equilibrium price and quantity in this market for all τ .
 - In a graph indicate the Consumer Surplus, (Domestic) Producer Surplus, Government Revenue, and Dead Weight Loss in this market when τ is low. Your scale does not have to be precise.
 - Find the Consumer Surplus, (Domestic) Producer Surplus, Government Revenue, and Dead Weight Loss in this market for all τ .
10. What are the arguments in favor of an import quota versus an import tariff? What are the arguments in favor of an import tariff versus an import quota?

11. Hoozools are a very rare and tasty bird that grows to its best on high mountains, where the air is thin and the weather cool. Only 20 farms can produce in the mountains, and they have a cost function of $c_m(q) = 50 + 2q^2$, and their fixed start up costs are 8. There are 6 farms raising hoozools in the valleys, their costs are $c_v(q) = 50 + 4q + 2q^2$, and their fixed start up costs are zero.
 - (a) Find the marginal costs, average variable costs, and supply curve of a representative mountain firm (type m).
 - (b) Find the marginal costs, average variable costs, and supply curve of a representative valley firm (type v).
 - (c) Find the short run supply curve of this industry.
 - (d) Find the short run equilibrium in this industry if the demand for Hoozool meat is $D(P) = 9 - P$. *Hint: This could be considered a trick question, check your answer carefully.*
 - (e) Assume that in the medium run the costs of the firms do not change (including the start up costs). Notice that there can not be entry by mountain farms. Find the industry's medium run supply curve.
 - (f) Find the industry's long run supply curve. Notice that there still can not be entry by mountain farms.
 - (g) Find the long run equilibrium in this industry if the demand for Hoozool meat is $D(P) = 260 - 5P$. To be specific find the price and quantity per firm, the quantity in the market, and the number of valley farms in equilibrium.
 - (h) Find the profits of both types of firms in equilibrium. What do we call the difference between their profits? Define the term you use.
12. Why might a long run supply curve be downward sloping? Provide examples and explain the general concept. Given the total output in the industry, is the shape of a given firm's average and marginal costs necessarily different than in an industry with a constant long run supply curve?
13. Economists have long known that taxes cause dead weight loss in markets. Explain why this does not mean that taxes are bad, you should give two different types of reasons.
14. In the market for Labor in Ankara the supply of labor is $L = S(W) = -2 + 2W$ and the demand for labor is $L = D(W) = 78 - 6W$.
 - (a) What is the equilibrium wage and quantity of Labor in this market? For full credit you must check either the wage or quantity using both equations.
Now assume that the government decides to impose a minimum wage of $\underline{W} = 12$ in this market.

- (b) What will be the quantity of Laborers that will be hired at the minimum wage?
 - (c) What is the Consumer Surplus, the Producer's surplus and the dead weight loss in this market with this minimum wage? You will get partial credit for a correctly labeled graph.
 - (d) A person is unemployed if they would like to work at the current wage and can not. What is the quantity of unemployed people at the minimum wage?
15. Assume that there are 4 firms in an industry with a cost function of $c(q) = 2q + q^2 + 16$, their fixed startup costs are 9. *Hint:* In this question all answers are integer values.
- (a) Find a representative firm's Average Costs, Average Variable Costs, and Marginal costs.
 - (b) At which quantity will a firm shut down? Explain why they shut down at that level.
 - (c) Find each firm's short run supply curve.
 - (d) Find the industry short run supply curve.
 - (e) If market demand is $D(P) = 32 - 2P$ what is the equilibrium price and quantity.
 - (f) If the government imposes a price floor of $\underline{p} = 13$ in this industry.
 - i. What quantity will be traded in this market?
 - ii. What will be the Dead Weight Loss?
 - (g) If the demand curve is $D(P) = 32 - 2P$ what will be the long run equilibrium in this market?
 - i. How much will each firm produce and what will be the market price?
 - ii. What will be the market quantity and how many firms will there be?
16. The Long run and Costs.
- (a) Explain why we assume that $p \leq AC$ in the long run.
 - (b) Explain why we assume that $p \geq AC$ in the long run. Your answer should make explicit mention of sunk costs.
 - (c) Explain why we assume that $p = MC$ in the long run.
 - (d) Prove that if $MC = AC$ then average costs are minimized. In the proof you should use $AC = \frac{C(q)}{q}$ when you minimize average costs.

17. In a given industry there are two completely different technologies represented by the cost functions, $c_a(q) = 4q^2 + 100$ and $c_b(q) = 6q^2 + 54$. The fixed start up costs of firms using technology a is 4 and the fixed startup costs of firms using technology b is 24. There are 8 firms using technology a and 12 firms using technology b .
- For firms using technology a find the marginal costs, average variable costs, price at which these firms will shut down, and the firm's supply curve.
 - For firms using technology b find the marginal costs, average variable costs, price at which these firms will shut down, and the firm's supply curve.
 - Find the industry's short run supply curve.
 - In the medium run find the price that firms using technology a will enter and the price at which firms using technology b will enter.
 - In the medium run assume that the quantity supplied is 81. What is the price in the market? What quantity are firms using technology a supplying? What quantity are firms using technology b supplying? How many firms using technologies a and b are there in the market?
 - In the long run what will be the market price? If the quantity supplied is 72 how many firms will there be and which technology are they using? (*Hint: The number of firms should be an integer.*)
 - One could argue that one of these technologies is superior in the short run since it has a lower marginal and average variable cost. However no one uses it in the long run, or it is "driven out."
Which technology is driven out in the long run? Why is this technology driven out? Which technology could be considered "better" and why is this technology better?
18. In a given market there are two types of firms. Firms of type one have a total cost of $c_1(q) = q + \frac{1}{2}q^2 + 32$, and a fixed start up cost of 8. Firms of type two have a total cost of $c_2(q) = 2q + q^2 + 9$ and a fixed start up cost of 4.
- What is the marginal cost, average variable cost and supply curve of firms of type one?
 - What is the marginal cost, average variable cost and supply curve of firms of type two?
 - If there are 7 firms of type one and 4 firms of type two what is the short run aggregate supply in this industry?
 - In the medium or long run at what price will firms of type one enter? At what price will firms of type two enter?
 - Which type of firm will not produce in the long run?

19. Economic theory states that in the long run all firms will be making zero profits. Why don't these firms shut down since they're making zero profits? Relate your answer to the difference between accountants and economists.
20. Assume there are 10 firms in an industry all with the same total cost function $C(q) = q + \frac{q^2}{2} + 8$. These firms have a fixed sunk cost of 6. All answers for this problem should be whole numbers (integers).
- Find the marginal cost, average variable cost, and supply curve of each firm.
 - Define the *medium run*.
 - Find the price at which firms will enter this industry.
 - Find the medium run supply curve.
 - Assume that the demand curve in this market is $D(P_d) = 50 - 5P_d$ find the medium run price, quantity, and number of firms that will produce.
 - If the demand curve increases to $D(P_d) = 90 - 2P_d$ find the new medium run price, quantity, and number of firms that will produce.
 - Find the long run price, quantity, and number of firms that will produce if the long run demand is $D(P_d) = 90 - 2P_d$.
21. Economists assert that in a normal competitive industry all firms are making zero profits in the long run. Why would anyone invest in a firm that is making zero profits?
22. Consider a short run market where the demand curve is given by $D(P_b)$ and the supply curve is given by $S(P_s)$. There is a per-unit tax so $P_b = P_s + t$, where P_b is the price buyers pay and P_s is the price sellers receive.
- Find $\frac{\partial P_s}{\partial t}$ and $\frac{\partial P_b}{\partial t}$.
 - Prove that $\left| \frac{\partial P_s}{\partial t} \right| + \left| \frac{\partial P_b}{\partial t} \right| = 1$, where $|a|$ is the absolute value of a . Hint: You do not actually need to answer part *a* to answer this part. Explain your answer.
 - If the slope of the supply curve increases ($S_p = \frac{\partial S}{\partial P_s}$) how will this change the graph of the supply curve?
 - Discuss what will happen to $\frac{\partial P_s}{\partial t}$ and $\frac{\partial P_b}{\partial t}$ if in the graph of the supply curve the supply curve gets flatter
23. Consider the supply of wheat in Turkey.
- If a firm has a cost function of $C(q) = q^2 + 100$ where the fixed costs are all fixed sunk costs, what is its short run supply function?

- (b) If a firm has a cost function of $C(q) = 2q^2 + 32$ where the fixed costs are all fixed start up costs, what is its short run supply function?
 - (c) If there are two firms with the costs in part *a* and two firms with the costs in part *b*, what is the short run aggregate supply? (Assume these firms are perfect competitors.)
 - (d) If the demand is $D(P) = 276 - 10P$ what will be the market equilibrium? You should find that in equilibrium all firms produce.
 - (e) If firms could enter this market would they? Why or why not?
 - (f) Assuming these firms use the same production technology in the short and long run, what is the long run costs of the firms in part *a* and part *b*?
 - (g) What quantity will these two types of firms produce in a long run equilibrium? Which of the two types of firm will not produce in the long run and why?
 - (h) Assuming the long run demand is $D(P) = 276 - 10P$ what will be the long run equilibrium price, quantity and number of firms? Note the number of firms must be an integer.
24. There are two technologies used to produce honey with exactly the same costs. In one you situate the hives near orchards, and the bees pollinate the fruit trees, in the other you situate the bees in fields of wildflowers. Since the first method produces a positive externality the government wants to subsidize this method, so they offer them a per-quart subsidy of 20 cents.
- Assume throughout this question that there is an unlimited supply of orchards and fields of wildflowers and that both types of honey are identical.
- (a) What will happen to the price of honey in the short run?
 - (b) What will be the mix of wildflower/orchard honey in the long run?
 - (c) What will happen to the price of a quart of honey in the long run? (Warning: there is a precise answer.)
25. A competitive industry is in a long run equilibrium, and then a sales tax is placed on the output. What do you expect to happen to the price the producers receive, the price the consumers pay, and the number of firms in the industry?
26. In the market for Labor in Ankara the supply of labor is $L = -8 + 4W$ and the demand for labor is $L = 40 - 2W$.
- (a) What is the equilibrium wage and quantity of Labor in this market? For full credit you must check either the wage or quantity using both equations.
- Now assume that the government decides to impose a minimum wage of $\underline{W} = 10$ in this market.

- (b) What will be the quantity of Laborers that will be hired at the minimum wage?
 - (c) What is the Consumer Surplus, the Producer's surplus and the dead weight loss in this market with this minimum wage? You will get partial credit for a correctly labeled graph.
 - (d) A person is unemployed if they would like to work at the current wage and can not. What is the quantity of unemployed people at the minimum wage?
27. Assume there are four firms in an industry all with the same total cost function $C(q) = 2q^2 + 8$. Type a firms have a fixed sunk cost of 8 type b firms have a fixed start up cost of 8.
- (a) Find the marginal cost, average variable cost, and supply curve of firms of type a .
 - (b) Find the marginal cost, average variable cost, and supply curve of firms of type b .
 - (c) Explain why the two types of firms have different levels of sunk costs when they are using the same technology.
 - (d) If there are 4 firms of type a and 8 firms of type b what is the short run market supply curve?
 - (e) Assume that the demand curve in this market is $D(P_d) = 108 - 6P_d$ and that there is a per-unit tax of t . Find the price demanders pay (P_d), the price suppliers receive (P_s), and the market quantity (Q_t) for all t . (Hint: In equilibrium all firms will produce output, and the only part of your answers that will not be integers will be the coefficients on t in the two prices.)
 - (f) In a graph indicate the consumer surplus, producer surplus, government revenue, and dead weight loss in the market. Your graph does not have to be precise.
 - (g) Find the dead weight loss in this market for all t . (Hint: You have already found the equilibrium quantity when $t = 0$, and the coefficient will be a integer.)
28. Economists have long known that taxes cause dead weight loss in markets. Explain why this does not mean that taxes are bad, you should give two different types of reasons.
29. Assume that there are 20 firms in an industry, each of which has sunk fixed costs of $F_{su} = 6$, Fixed start up costs of $F_{st} = 2$ and pure variable costs of $VC(q) = 2q^2$
- In this question you may rest assured that all answers are integer values, except possibly the shut down quantity.

- (a) Find this firm's Average Costs, Average Variable Costs, and Marginal costs.
 - (b) At which quantity will the firm shut down? Explain why they shut down at that level.
 - (c) Find each firm's short run supply curve.
 - (d) Find the industry short run supply curve.
 - (e) If market demand is $D(P) = 54 - 4P$ what is the equilibrium price and quantity.
 - (f) If the government imposes a price floor of $\underline{p} = 10$ in this industry.
 - i. What quantity will be traded in this market?
 - ii. What will be the Dead Weight Loss?
 - (g) If the demand curve is $D(P) = 54 - 4P$ what will be the long run equilibrium in this market?
 - i. How much will each firm produce?
 - ii. What will be the market price?
 - iii. What will be the market quantity?
 - iv. How many firms will there be?
30. There are four firms that all have the same technology, represented by the total cost function $C(q) = 18 + \frac{q^2}{2}$. Type *A* firms (there are two) have a sunk cost of 18, Type *B* firms (again there are two) have no sunk costs.
- (a) Explain why the firms have different levels of sunk costs when they are using the same technology.
 - (b) For each type of firm write down the short run supply function.
 - (c) Write down the short run aggregate supply.
 - (d) If the demand curve is $Q = 30 - 4P$.
 - i. What is the short run equilibrium Price and Quantity? How many firms produce?
 - ii. If the government imposes a price floor of 7, what is the quantity sold? What is the dead weight loss?
 - iii. What is the long run equilibrium Price and Quantity? How many firms produce?
31. The total cost function of a typical soft coal producer is:

$$TC = 100,000 + \frac{1}{10}q^2$$

where Q is measured in railroad cars per year. The industry contains 60 identical producers.

- (a) Calculate the short run supply curves of a firm and the industry. Assume that fixed start up costs are zero.
- (b) If the price of output is 250 calculate and illustrate graphically producer surplus at this price, and firm's profit.
- (c) Calculate the long run supply curve.
- (d) If the federal government is considering imposes a \$32 per carload tax on soft coal producers, calculate the new total and marginal cost of the firm.

32. In a particular industry, domestic demand is given by

$$Q^d = 80 - \frac{1}{2}P$$

and domestic supply is

$$Q^s = -4 + \frac{3}{2}P$$

there is also a world supply curve, and they will supply any quantity at the price of 4.

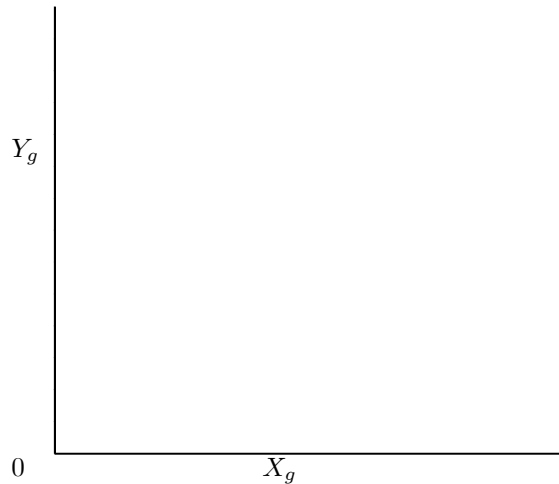
- (a) What is the equilibrium if there are no tariffs or quotas? What quantity is imported and what quantity is produced domestically? What is total domestic welfare at this equilibrium?
- (b) Domestic producers are upset with the current state of affairs, and demand an import tariff. They are arguing for a tariff of twelve dollars per unit imported. What will be the new equilibrium if this is the tariff? What will be the effect on total domestic welfare? On producer surplus?
- (c) They point out that they are being very reasonable, as a tariff that shuts out the imports would be much higher. What would that tariff be? What would be the domestic welfare at that tariff?
- (d) The international producers are upset about this, they offer a counter proposal of a quota of forty two units imported. Why are they in favor of a quota over the tariff? What is the equilibrium price and quantity? How does it compare with in case *c*? What would be the domestic surplus in this situation? (And a hard question) Why did they choose that particular quota?

2 Chapter 13

1. About Pareto Efficiency.

- (a) Define an *allocation*, is it possible for one person to have allocation *A* and another to have allocation *B*? Why or why not?

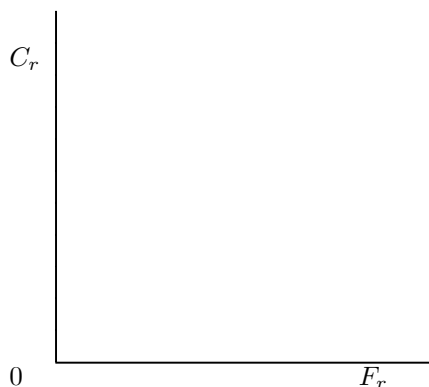
- (b) Define *Pareto Dominance*.
- (c) Define *Pareto Efficiency* using the definition of Pareto Dominance you have just written.
2. Gizem and Volkan consume nothing but Xylophones (X) and Yogurt (Y). Gizem has all of the Xylophones, 30 units, $((X_g^e, Y_g^e) = (30, 0))$ and Volkan has all of the Yogurt, 8 units $((X_v^e, Y_v^e) = (0, 8))$. Gizem's preferences over Xylophones and Yogurt are $U_g(X_g, Y_g) = X_g Y_g^2$ and Volkan's preferences are $U_v(X_v, Y_v) = X_v Y_v^3$.
- (a) Find the Pareto Efficient allocations in this exchange economy.
- (b) Find the General Equilibrium in this exchange economy, you may assume that $p_x = 1$. Be sure to solve for both the prices and the final allocation of goods.
- (c) In the graph provided graph the Edgeworth Box, the Contract Curve, the Initial Endowment, and the final equilibrium in this economy. Let Gizem's consumption be increasing as you move to the upper right.



- (d) Gökçe runs a trading company and is trying to convince Gizem and Volkan to allow imports and exports. After all, she points out, the price of yogurt is $p_y = 8$ in the rest of the world so they could both consume Yogurt more cheaply. (The price of xylophones is the same.) *In these questions none of the answers have to be supported with careful mathematical analysis.*
- Who will do better with free trade? Who will do worse?
 - Is there any way for the person who is doing better to change the other person's mind?
 - Is it Pareto Improving in this example to allow free trade? Be precise in your answer.

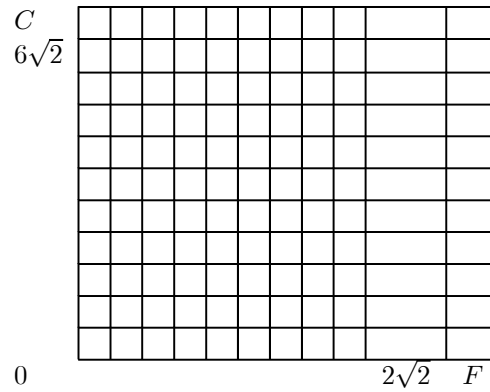
3. Robinson Crusoe consumes only Coconuts (C) and Flip Flops (F), he uses labor to produce both goods and his total endowment of labor is 9. His production function for Coconuts is $C = 4\sqrt{L_c}$ and his production function for Flip Flops is $F = 2\sqrt{L_f}$. He does not enjoy his Coconuts unless he has a new pair of Flip Flops for every 2 Coconuts, or his utility function is $U_r(F_r, C_r) = \min(2F_r, C_r)$.

- (a) Find the production possibilities frontier of Robinson Crusoe's economy and draw it (approximately) in the graph below.
- (b) In the graph below draw an indifference curve for Robinson where his utility is equal to 4. From the indifference curve what do you know about the ratio in which Robinson will consume F and C ?



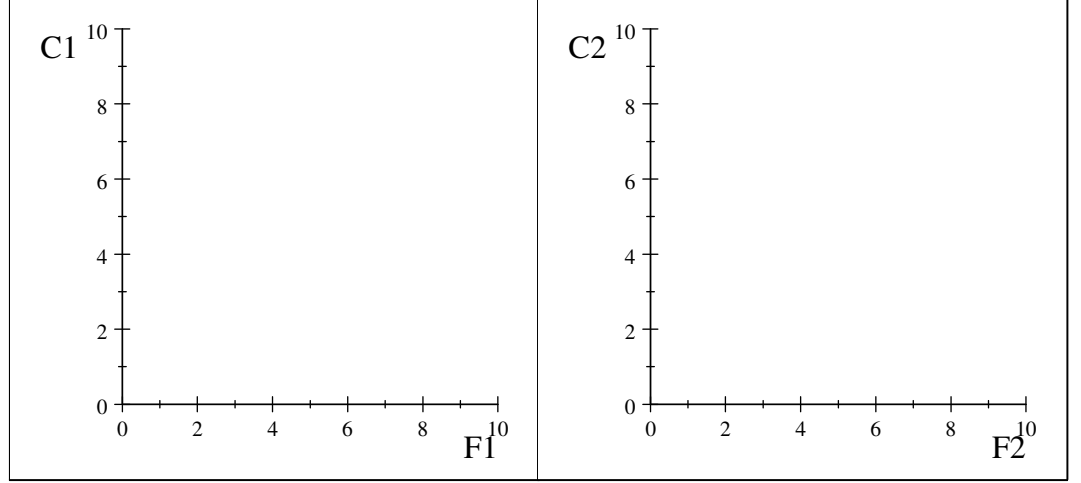
- (c) Find the Pareto Efficient allocation in this economy, and the implicit price of flip flops and coconuts in this allocation, you may set $p_c = 1$ if you want.
- (d) The Native Americans on the next island have just discovered that Robinson Crusoe exists. Being a highly advanced civilization they are more efficient, to be precise their production function for Coconuts is $C = 8\sqrt{L_{nc}}$ and their production function for Flip Flops is $F = 4\sqrt{L_{nf}}$ and their total supply of labor is 9. Their preferences, however, are different. They do not consume Flip Flops—they only make them for the tourists—so $U_n(F_n, C_n) = C_n$.
- One of their Senators makes a passionate speech stating that since they have a clear technological superiority over Robinson they would not benefit from trading with him. Solve for the PPF in this economy and tell me whether he is right or not, explaining your answer carefully.
4. Consider the following economy. The production function for food is $F = (L_f)^{\frac{1}{2}}$, the production function for clothing is $C = (9L_c)^{\frac{1}{2}}$, and the total amount of labor available is $\bar{L} = 8$. There is only one consumer in this economy, his utility function is $U = FC$

- (a) Find the Production Possibility Frontier in this economy.
- (b) Find the Marginal Rate of Transformation and the Marginal Rate of Substitution in this economy.
- (c) Find the Pareto Efficient outcome in this economy.
- (d) Find the price of food in this economy, you may set the price of clothing to one.
- (e) In the graph below graph the production possibilities frontier and the indifference curve of the consumer at the Walrasian equilibrium. Illustrate how you could find the price vector. (*Note I do not expect exact precision in this part of the question, but I expect your graph to be reasonably accurate.*) Then draw what would happen if this economy opened up to international trade. Internationally the price of food is higher than it is in this country. You should explain your answer below the graph.



5. In an exchange economy the utility function of person one is $U_1(F_1, C_1) = F_1^3 C_1^2$, the utility function of person two is $U_2(F_2, C_2) = \min\{4F_2, 2C_2\}$ and the total amount of food available is $\bar{F} = 8$ and of clothing is $\bar{C} = 16$.
 - (a) In the two graphs below draw a representative indifference curve for

both parties.



Indifference Curve for Consumer 1

Indifference Curve for Consumer 2

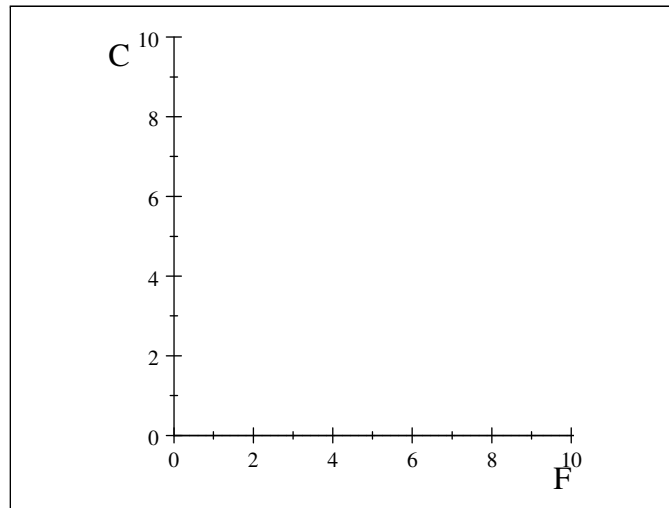
- (b) Write the contract curve in this economy as a function of C_1 in terms of F_1 .
 - (c) Find the price of food, p_f , in every Walrasian or general equilibrium, you may set the price of clothing to one.
 - (d) If the initial endowment of food and clothing for consumer 1 is $(F_1^0, C_1^0) = (5, 10)$ find the final allocation in the general equilibrium.
6. Consider a general equilibrium exchange economy where there are three goods: food (F), clothing (C), and housing (H). Prove that if supply equals demand in the market for food and clothing then supply must equal demand in the market for housing.
 7. Consider the following economy. Person one has the utility function $U_1(F_1, C_1) = \min\{\frac{3}{4}F_1, \frac{1}{4}C_1\}$ and person two has the utility function $U_2(F_2, C_2) = \frac{3}{4}F_2 + \frac{1}{4}C_2$. The total amount of food is $F^0 = 8$, and the total amount of clothing is $C^0 = 24$.
 - (a) Find the set of Pareto Efficient allocations. Write this set as a function of C_1 on F_1 **and** as a function of C_2 on F_2 .
 - (b) Find the prices in any general equilibrium in this economy, you may normalize the price of C to one.
 - (c) If the initial endowment for person one is $(F_1^0, C_1^0) = (6, 12)$, what are the equilibrium consumptions of both people?
 8. Consider an economy where the production function for food is $F = (6 \min\{\frac{3}{4}K_1, \frac{1}{4}L_1\})^{\frac{1}{2}}$ and the production function for clothing is $C = ((\frac{3}{4}K_2 + \frac{1}{4}L_2))^{\frac{1}{2}}$, the total endowment of capital is $K^0 = 8$ and of labor is $L^0 = 24$.

- (a) Explain why you can make extensive use of your answers in the last question to find the Pareto Efficient allocation of labor and capital in this economy.
 - (b) Show that the production possibilities frontier is $\frac{2}{3}C^2 + \frac{2}{9}F^2 = 8$.
9. Consider an economy with one person who's preferences are $U(F, C) = FC^3$ and who has the production possibilities frontier $\frac{2}{3}C^2 + \frac{2}{9}F^2 = 8$.
- (a) Find the Marginal Rate of Substitution for this person.
 - (b) Find the Marginal Rate of Transformation in this economy.
 - (c) Find the Pareto Efficient allocation in this economy.
 - (d) Find the price of food and clothing that can support this Pareto Efficient allocation as a General equilibrium.
 - (e) Explain how you can support the Pareto Efficient allocation as a General equilibrium using the prices you found in the last part of the question. I am not looking for a mathematical solution, merely for a verbal explanation of how it can be done.
10. About Walras's Law.
- (a) Define Walras's Law for an economy with m goods.
 - (b) Assume there are three goods in the economy, F (food), C (clothing) and H (housing). Prove that you can normalize one of the prices to one without loss of generality. (You may assume that in the general equilibrium all prices are strictly positive.)
11. About Walras's Law.
- (a) Define Walras's Law for an economy with m goods.
 - (b) We say that an economy faces *inflation* of $i > 0$ if instead of the price vector $P = [p_f, p_c, p_h, \dots]$ they face the price vector $(1 + i)P = [(1 + i)p_f, (1 + i)p_c, (1 + i)p_h, \dots]$. Explain how Walras's law proves that this will have absolutely no impact on the economy.
 - (c) I assert that in reality inflation does have a real impact on the economy, what is the difference between real world inflation and the type of inflation I just described that causes this impact?
12. Consider an economy with one person who's preferences are $U(F, C) = FC^2$. The production function for food is $F = \sqrt{L_f}$ and for clothing is $C = \sqrt{2L_c}$, the total endowment of labor is $L^0 = 12$.
- (a) Find the production possibilities frontier.
 - (b) Find the Marginal Rate of Substitution for this person and the Marginal Rate of Transformation for this Economy.

- (c) Find the Pareto efficient allocation in this economy.
 - (d) Using only words, describe how you could decentralize this economy so that agents would only have to maximize their objectives given prices.
 - (e) In this decentralized economy explain carefully how you would calculate the consumer's income. You may assume the consumer supplies all of the labor.
 - (f) Find the price of food, clothing, and labor that would support this Pareto efficient point of a decentralized economy.
13. Consider a production exchange economy. The production function for food is $F = \frac{1}{2}\sqrt{2}\sqrt{\min\left(\frac{1}{5}L_f, \frac{4}{5}K_f\right)}$, the production function for clothing is $C = \frac{1}{15}\sqrt{15}K_c^{\frac{1}{4}}L_c^{\frac{1}{4}}$. The total endowment of labor is $L^0 = 120$, the total endowment of capital is $K^0 = 30$.
- (a) Find the contract curve for this economy as a function of L_f in terms of K_f . Verify that it passes through the points $(0,0)$ and (L^0, K^0) .
 - (b) Find the equilibrium prices in any general equilibrium, let the price of labor be w and the price of capital be r . Hint: This can **not** be found from the slope of the contract curve.
 - (c) If the initial endowment of labor and capital of food is $(L_f^0, K_f^0) = (40, 30)$ find out how much labor and capital will be used to produce food in the general equilibrium.
 - (d) Verify that the production possibilities frontier can be written as: $\frac{15}{2}C^2 + \frac{5}{2}F^2 = 30$.
14. Consider a Robinson Crusoe economy where his preferences are written as $U(F, C) = CF^3$ and his initial endowment of labor and capital and his production function for food and clothing are given in the last question.
- (a) Find his marginal rate of substitution and the economy's marginal rate of transformation.
 - (b) Find the Pareto Efficient allocation.
 - (c) Explain how this problem can be decentralized in one of two ways.
 - i. One consumer maximizing his utility given his income and one firm maximizing it's revenue give the production possibilities set.
 - ii. One consumer maximizing his utility given his income, one firm maximizing it's profits from food, and another maximizing it's profits from clothing.

Critically define the income in each variation, and show that in equilibrium the income in both variations will be the same.
 - (d) In the graph below:

- i. Graph the production possibilities set, the Pareto Efficient point, and his indifference curve at the Pareto Efficient allocation. *Note: One of the extreme values of the production possibilities set will not be an integer.*
- ii. Show that he will be better off if he allows international trade, specifically assume that the world price ratio is not the same as his local price ratio. Show his imports and exports in this graph.



- (e) Now prove that as long as the ratio of world prices is not the same as the ratio of local prices he must be strictly better off.