1 Imperfect Information and the Impact on the Economy.

Sigh, yet again the book lets me down. Yet again they teach a bunch of formal theory without teaching the basic lessons of a subject. The lesson you need to learn about imperfect information is it’s impact on markets and how people can respond to this problem. The primary impact is that markets can fall far from the goal of Pareto efficiency when there is imperfect information, in fact they can collapse if the problem is too severe. People respond to this problem by signalling, or taking actions that have no economic value (or for which the marginal benefit is less than the marginal cost) in order to establish something about their personal characteristics. The book does not clearly teach either of these topics, so I will be covering them in this handout.

In this handout I will only talk about asymmetric information, moral hazard is covered well in the book and in general it is more complicated than asymmetric information, so I will focus on the simpler case.

1.1 Asymmetric Information and the “Market for Lemons”

The first lesson is that asymmetric information can cause markets to collapse. Generally they always cause markets to work less than efficiently. As an example of this consider the market for used cars. It is a well known fact that used cars sell for much less than new cars. If you drive a car off of the dealer’s lot and then immediately try to sell it you probably will sell the car for between 10% to 20% less than you bought it for. Why is that?

Perhaps the question you really should be asking is why would someone sell a car that they just bought? The most common reason would be that there is some problem with the car. This problem might be that they... don’t like the color? It’s too big for the city? No, usually it is some mechanical problem that the person is just too lazy to fix themselves. In the US (where the theory was written) we call such a car a “lemon,” but don’t ask me why. I don’t know. Now when someone has a lemon they might be honest enough to tell you, but often they won’t be. This means that the quality you should expect from this car will be less than the quality you should expect from a new car, and thus the price will be lower.

How significant can this impact be? Well let’s develop a little model to explain this. Let’s say that buyers have a value for the car of $v_i w$, where $w$ is the quality level of the car. I would like to use $q$ for quality, but that is the quantity in a market, so I use $w$ for worth. Now the buyer won’t know that so they have to find their expected utility. Say, for example that the quality level can be one of three levels: high or $w_h$, middle or $w_m$, and low or $w_l$ and that the probabilities of these three outcomes are $\rho_h$, $\rho_m$ and $\rho_l$ respectively. Then their expected utility will be:

$$E(v_i w) = v_i E(w) = v_i (\rho_h w_h + \rho_m w_m + \rho_l w_l)$$
formally there’s a lot more to expected utility, but for our purposes this is really all you need to know. The key thing to notice is that \( E(w) < w_h \), so the quality level the buyer expects is lower than the highest quality cars on the market. How could this be a problem? Well let’s say that a seller’s value of a car is just it’s quality level, and that the seller knows the quality level. Then the seller with a high quality car will only sell their car if \( v_i E(w) \geq w_h \).

Let’s say that this isn’t true, or that \( v_i E(w) < w_h \). The consumer must know this in equilibrium, so their expected utility is now lower:

\[
E(v_i w) = v_i E(w) = v_i \left( \frac{\rho_m}{\rho_m + \rho_l} w_m + \frac{\rho_l}{\rho_m + \rho_l} w_l \right)
\]

(Notice I have re-weighted the probabilities so they still sum to one.) And at this point again we know that \( E(w) < w_m \) (since this is now the highest quality) and we have to check if \( v_i E(w) \geq w_m \). If it’s false then the middle quality vehicle owners will not enter the market, and \( E(w) = w_l \).

Now compare this to the Pareto efficient outcome. As long as \( v_i > 1 \) (which we will always assume) every type of car should be sold, if only the low quality cars are coming into the market then there could be a lot of buyers who would like to buy. This is the basic problem in this market, and explains the price difference between new cars and “slightly used.” There is a lemons problem in this market, and it can cause a collapse.

I want to now present a continuous model, where we have a continuous demand and supply curve. In order to do this we will assume that we have a continuum of buyers and sellers, which both have uniform distributions. Now some of you who aren’t comfortable with math might get a little intimidated by this assumption, but it really isn’t that difficult. All we are doing is just making our demand and supply curves smooth, if I didn’t do this then the curves would be step functions, jumping down when one person is no longer willing to buy in the market. We can also normalize quantity to be between zero and 100 without any loss. If this is true and the values of consumers are distributed uniformly over \([\bar{v}, \bar{v}]\) our inverse demand curve is:

\[
P = \left( \bar{v} - \frac{\bar{v} - \bar{w}}{100} Q \right) E[w]
\]

where \( E[w] \) is the expected quality level of the car. The quality levels of the car will be distributed uniformly on \([0, \bar{w}]\) so our inverse supply curve will be:

\[
P = \frac{\bar{w}}{100} Q
\]

and the marginal car will have a quality level of \( P \). Now, one final technical detail, what is \( E[w|w \leq P] \)? I.e. given that every car on the market has a quality level below \( P \) what is the expected quality of a car?

\[
E[w|w \leq P] = \frac{1}{P} \int_0^P wdw
\]
\( \frac{1}{P} \) is an adjustment of the probabilities so that the probabilities sum to 1. Now let’s solve this:

\[
\frac{1}{P} \int_0^P w \, dw = \frac{1}{P} \left[ \frac{w^2}{2} \right]_0^P = \frac{P^2}{2P} = \frac{P}{2}
\]

Before I find the equilibrium let’s graph the demand and supply curves. To do this we need to make some assumption about the quality levels of cars on the market, as a first pass let’s assume that all cars are offered for sale. We also need some parameters, so let’s assume that \( \bar{w} = 16, \bar{v} = 1.9 \) and \( v = 1 \). In this case we can see that the supply curve is \( P = \frac{16}{100} Q \) and the demand curve is \( P = (1.9 - .009Q) E[w] \), or our first demand curve is \( :P = (1.9 - .009Q) 8 \).

Now notice that our original guess (that all cars would be sold) is wrong. So what is the value of the marginal car now? Well we can see that

\[
\frac{16}{100} Q = (1.9 - .009Q) 8
\]

\[
Q = 65.517
\]

\[
P = \frac{16}{100} Q = \frac{16}{100} (65.517) = 10.483
\]

So this gives us a new demand curve, \( P = (1.9 - .009Q) \frac{10.483}{2} \)
which is lower than before. Uh ohh. You see what is happening... Let’s go through several further steps in one graph. Each lower demand curve is the result of figuring out the actual quality of cars that will be sold using the previous demand curve.

and you can see that with each iteration of the process the demand curve shifts further and further down. What will be the final result? Well to do that we have to find out what the equilibrium will be. This is actually easier than doing what we did above.

The trick to finding the equilibrium is realizing that the marginal consumer has to be willing to buy the average car, and the marginal supplier has to be willing to sell it. So this means

\[(1.9 - .009Q) \frac{P}{2} = P\]

\[1.9 - 2 = .009Q\]

\[Q = -11.111\]

and unfortunately quantity cannot be negative. What does this mean? No one will buy a car, and the market will collapse. In this model in general what we need is:

\[P = \left( \frac{\bar{v} - \frac{\bar{v} - v}{100} Q}{\bar{v} - \bar{w}} \right) \frac{P}{2}\]

\[Q = \frac{100 \bar{v} - 2}{\bar{v} - \bar{w}}\]

\[P = \frac{\bar{v}}{100} Q = \frac{\bar{v}}{100} \left( \frac{100 \bar{v} - 2}{\bar{v} - \bar{w}} \right) = \frac{\bar{v} - 2}{\bar{v} - \bar{w}}\]

Notice that one thing we can be sure of is that \(\bar{v} > 2\) is necessary. We can see this by looking at one person’s incentives:

\[v_i E [w|w \leq P] \geq P\]
and this means if we have a uniform distribution that:

\[
\frac{v_i}{2} P \geq P \\
\frac{v_i}{2} \geq 2.
\]

Hah, so now you know why I made the consumer with the highest utility only value the good at \(1.9 \times E[w]\). The disturbing fact is that in this market trade is only possible if the consumer values the car twice as much as the seller.

Does that bother you? It bothers me. Of course with other distributions of quality is different then this result wouldn’t be true. What is always true is that there is some \(v_i^* > 1\) that is necessary for trade. Why do we care? Because whenever \(v_i > 1\) it is Pareto improving to trade, so there will always be some people with values \(v_i^* > v_i > 1\) who should be trading and won’t be.

1.2 Signalling and the Spence Job Market Model.

So what should you do in this situation? Well one option that some people follow is they try to find a trustworthy mechanic to certify that their car has no mechanical problems. This is a solution, but it is economically wasteful. After all they would not seek the certification if they wouldn’t pass the inspection, so the mere fact they want to be certified indicates they do not need it. This is a signal, an action which is economically wasteful but overcomes some asymmetric information problem.

Let me introduce you to another model that captures the impact of signalling in it’s most dramatic light. How much do you think your education will help you in your future job? For example, are you sure you’re going to use the foreign language that you have (or will) learn? Are you sure that you will even use English? So what is the point of you learning all of these languages when you aren’t going to use them? And what about your history classes? In the US this problem is even worse because people do not choose their major before they go to college. I’m thankful for that—otherwise I would be a biologist right now—but it means that you take a lot of general classes before you decide on your major. How much do you think I use my class on the sociological impact of journalism? Sigh. So why do we take these classes? Why are you in college? Well, that’s a good question.

Spence realized that part of the answer is signalling. It’s not that what you learn at Bilkent will be that useful, but the fact that you got into Bilkent and graduated with a good GPA that is useful information. I.e. it’s not what you learn but the fact that you CAN learn. That is a skill that many employers value, and with good reason. That is probably the skill that you will retain from all of this education until you die. What we teach you is not as important as the fact that you learn whatever we teach.

Let me illustrate this in a model that strips away all of the excess baggage and gets right to the heart of the problem. Let’s assume that education is completely worthless. (Don’t start cheering, this is just a model. I am sure (for example) that my class will be utterly invaluable when you are trying to figure
out why the new dye in your cloth dyeing factory keeps on running.) We’ll also assume that there are two types of workers, high quality and low quality.

Thus a firm values you at the level of $\pi_q$ where $q$ is your quality level, $q \in \{l, h\}$ and $\pi_h > \pi_l$. Now high quality workers also have a lower cost of being educated, let this cost be $c_q e$—where $e$ is the years of education and again $q \in \{l, h\}$ with $c_h < c_l$. We will assume that firms (due to competition for high quality workers) will pay a worker his value, or the wage is equal to the value, $w_q = \pi_q$.

A worker’s utility function is then:

$$w_q - c_q e_q$$

where $e_q$ is the education level that this type of worker receives. Now we can graphically illustrate a person’s indifference curves in a \{education, wage\} space. Let’s assume that $c_l = 2$ and $c_h = 1$, $\pi_h = 8$, $\pi_l = 2$, then if someone offers a wage of 6 for one year of education the indifference curves are:

![Indifference Curves Graph](image)

a person’s utility is increasing as we go towards the upper left hand corner, the horizontal lines are the productiveness of the two workers, and the sloped lines are the indifference curves. The one that is steeper is the indifference curve of the low quality worker, the one that is flatter is the indifference curve of the high quality worker.

Now looking at this picture notice that the high quality worker would prefer to get more education if they could get the high quality worker’s wage. For example they would be willing to go to school for an extra year—or even two—to get the high wage. The low quality workers, on the other hand, would not be willing to do this. If the high quality workers did this then this would be a \textit{signal}. They are getting education that is worthless in order to separate themselves from the low quality workers.

Now what are the equilibria in this market? Well there are two possibilities. First the firms—recognizing that education is worthless—could assign no value to education. In this case they will not be able to differentiate between the workers and will have to pay them their expected wage. Or:

$$\bar{w} = \rho_l \pi_l + \rho_h \pi_h$$
where $\rho_l$ is the fraction of low quality workers in the economy and $\rho_h$ is the fraction of high quality workers. Let’s assume that $\rho_l = \frac{2}{3}$, then $\bar{w} = \frac{2}{3} \times 2 + \frac{1}{3} \times 8 = 4$, and no one will go to school. Graphically this situation is:

![Graph 1](image1)

and this is an equilibrium. Since the pool of workers is not differentiated we call this a **pooling equilibrium**. Notice that if firms paid attention to the signalling value of education the high quality workers would be willing to go to school for 2 to 4 years in order to convince the firms that they are high quality, but in this type of equilibrium the firms don’t value education and no one goes to school.

There is also an equilibrium where high quality workers do signal, we call this type of equilibrium a **separating equilibrium** since the pool of workers is separated into types. In this equilibrium clearly the low quality workers will not go to school and will get paid their marginal product, or $w_l = \pi_l = 2$, and the high quality workers have to go to school for long enough to differentiate themselves from this type. To see how long they would be willing to go to school let’s look at the graph when everyone can get the wage of 2 if no one goes to school.

![Graph 2](image2)

Now the high quality workers have to go to school for at least 3 years in order to signal their quality, and they could go for up to 6 years (do you want to get a
Master’s??)). In fact it doesn’t matter what education level they get. As long as firms expect them to get a certain amount they will, assuming they don’t have to go for more than six years. The current equilibrium is 4 years, and this is sustained in my example. Workers have a choice between two \{\text{education, wage}\} pairs. If they don’t go to school they get $w_l = 2$, if they go to school for 4 years they get $w_h = \pi_h = 8$. And this is an equilibrium:

\begin{center}
\begin{tikzpicture}
\begin{axis}[
    title = {Equilibrium Graph},
    xlabel = {Education Years},
    ylabel = {Wage},
    xmin = 0, xmax = 6, 
    ymin = 0, ymax = 12,
    xtick = {0,1,2,3,4,5,6},
    ytick = {0,2,4,6,8,10,12},
]
\addplot[mark=none,thick]{x*2};
\addplot[mark=none,thick]{x*2+8};
\addplot[mark=none,thick]{x*2+10};
\end{axis}
\end{tikzpicture}
\end{center}

Notice that the indifference curve for the high types passes above the point \{0, 2\} in the graph, this means they prefer to be educated. As well the indifference curves for the low types is higher than the point \{4, 8\}, or they prefer not to be educated. This shows us this is an equilibrium.

So how do we feel about this? Well, economically speaking let me point out that the pooling equilibrium is not worse than the separating equilibrium. In the pooling equilibrium low quality workers do better, so separation is not a Pareto improvement. But separation is the way the world seems to work, and when we look at this case we notice a couple of disturbing facts. The first one is only mildly disturbing, high quality workers are getting wasteful education. But this is necessary for them to get their true value as workers, and so is only mildly disturbing. A more disturbing point is they are getting too much education, they only need to go to school for three years and yet they go for four. Why? Because firms expect it. College dropouts are judged to be no better than people with a high school degree. This is common in signalling models.

Now how does this relate to the real world? Obviously education does benefit people, everyone gets some education, but the result that does go through is that higher quality workers are generally getting too much education. Not because it increases their productivity but because they use it to signal their quality level. As well it’s quite possible that they are going for longer than they need to in order to signal. This is a tragedy of employer’s expectations. The marginal education might have no value even as a signal, but since firms have these expectations you do better to fulfill them.

As well notice that signalling is hardly restricted to this market. It is a common phenomena in everyday life. Asymmetric information is all around us,
and in order to get our true value we take worthless actions that separate us from others. I am constantly thinking about the signalling value of my actions and other’s actions, and not does this only clarify my motives but often times it helps me figure out what the right course of action is. I hope the marginal productivity of this lesson is higher than it’s marginal cost.