ECON 433
Midterm: Market Power, Price Discrimination, and Oligopoly

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This exam will start at 10:30 and finish at 12:10.

Answer all questions in the space provided. Points will only be given for work shown.

1. *(9 points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test.

   Name and Surname: _

   Student ID: _

   Signature: _

2. *(8 points)* List two of the four reasons for natural market power discussed in class, explain each one and give an example of an industry where it is an important factor. You may use the same industry more than once.

   Solution 1 *The four reasons for natural market power are:*

   (a) *Small Markets: In a few industries the fixed costs of production are so high that the minimum efficient scale (when $S = \frac{AC}{MC} \leq 1$) requires at best a few firms. An example from the book is the market for Aspartame, another example would be large airplanes—where the market is dominated by Boeing and Airbus.*

   (b) *Always Decreasing Average Costs or Economies of Scale: These are industries where $S = \frac{AC}{MC} > 1$ for all levels of outputs. Generally these are industries with high fixed costs (research and development) and very low marginal costs. The movie industry could be an example, as well as Drug manufacture, the quintessential example is any Software. Marginal cost is mostly bug fixing (which also has economies of scale). Advertisement would be a fixed cost.*

   (c) *Network Externalities: These are goods that, like a language, have a higher benefit of using if more people around you are using them. Movies are this type of good, but again Software is an extreme example. For example many journals these days accept submission in Microsoft Word since everyone uses it.*

   (d) *Economies of the Market: Customers want to go where there is the greatest variety of goods (the most sellers) and Sellers want to go where there are the most Customers. This positive feedback loop means there will be naturally one market. These days with online* *markets it is actually feasible to have only one market that sells everything. In the United States this market is Amazon (with fringe competition from online branches of Brick and Mortar stores). Turkey is currently at a fascinating point in its online business history because there are at least three local firms (Hepsiburada, Trendyol, n11) and two global firms (Amazon and Ali Baba).*

They may try to list economies of scope which is included in this section. I would give them some partial credit for that, but not much. Economies of scope is a factor in almost every business but as a reason for having a few firms in an industry? Not really.

3. *(12 points)* Name and give the formula for three distinct measures of Industry market concentration discussed in class. For each one discuss the problem we have with this as an indication of whether there is actually a problem with concentration in that market.

Solution 2 *The three measures are the Lerner Index*

$$LI = \frac{P - mc}{P} \in [0, 1]$$

Let the market share of firm i be

$$s_i = \frac{q_i}{Q} * 100$$

and order the market share so that $s_1 \geq s_2 \geq s_3 ...$ then the two other distinct measures are

$$CR_4 = \sum_{i=1}^{4} s_i \in [0, 100]$$

the four firm concentration ratio, and the Herfindahl–Hirschman index:

$$HHI = \sum_{i=1}^{n} (s_i)^2 \in [0, 10000]$$

As a statistic the Lerner Index fails because it requires that you estimate marginal cost in multiple industries simultaneously. The one time someone attempted to apply this they got some indices that were negative, and those with high LI were not necessarily the most concentrated intuitively or by other measures.

The four firm concentration ratio has a problem because of the censoring, why are only the top four important and if an industry has fewer then four firms the CR_4 will always be identical. Consider the following four firm distributions:

Industry	s_1	s_2	s_3	s_4
a	*80*	*10*	*5*	*2*
b	*25*	*25*	*25*	*22*

these both would have a CR_4 of 97, but the first looks like essentially a monopoly industry with fringe competitors (it is similar to the distribution of OS systems for desktop computers) while the latter is concentrated, but looks like a fairly competitive oligopoly.

The HHI improves on this, and for industries with a small number of significant firms can easily pick out those with dominant firms. But from an industrial organization standpoint does this make it a better measure? That is unclear. As well it requires data on every firm within the industry, which is arduous.

Finally a criticism of especially the latter two measures is "what is an industry." For example if you just consider OS there are three leading firms—Windows, Android, and Apple—but if you consider only desktop computers then Android disappears. For both CR_4 and HHI this can make a market seem less concentrated then it really is. With the Lerner Index it is not as problematic. As long as MC is measured correctly adding a few firms from a "similar" industry should not affect results that much.

a	b	χ_1	χ_2	χ_3	χ_4	$q_1(q_2)$	$q_2(q_1)$	q_1^*	q_2^*	$Q(2)$	$Q(3)\|_{q_1 * q_2 > 0}$
21	$\frac{1}{2}$	11	4	2	1	$10 - \frac{1}{2}q_2$	$17 - \frac{1}{2}q_1$	2	16	18	23
18	$\frac{1}{4}$	13	11	7	9	$10 - \frac{1}{2}q_2$	$14 - \frac{1}{2}q_1$	4	12	16	23
25	$\frac{1}{2}$	12	17	8	5	$13 - \frac{1}{2}q_2$	$8 - \frac{1}{2}q_1$	12	2	14	19
15	$\frac{1}{4}$	6	9	3	6	$18 - \frac{1}{2}q_2$	$12 - \frac{1}{2}q_1$	16	4	20	27

a	b	χ_1	χ_2	χ_3	χ_4	$q_i < 0$	*Test*	$Q(3)$	$Q(4)$	$q_{(2)}^*$	q_3^*	q_4^*
21	$\frac{1}{2}$	11	4	2	1	*1*	$-\frac{3}{2}$	24	28	6	10	12
18	$\frac{1}{4}$	13	11	7	9	*1*	$-\frac{3}{4}$	24	27	1	17	9
25	$\frac{1}{2}$	12	17	8	5	*2*	$-\frac{3}{2}$	20	25	1	9	15
15	$\frac{1}{4}$	6	9	3	6	*2*	$-\frac{3}{4}$	28	30	6	18	6

4. *(31 points total)* Consider an industry with two firms that compete by choosing quantity, the price is then set at the market clearing price (Cournot Competition). The inverse demand curve is $P = a - bQ$, where $Q = q_1 + q_2$, and $c_1(q) = \chi_1 q_1$, $c_2(q) = \chi_2 q_2$.

   (a) *(2 points)* Set up the objective function of both firms (to be clear, you need two *separate* objective functions.)

   Solution 3

$$\begin{aligned} \pi_1(q_1, q_2) &= (a - b(q_1 + q_2))\, q_1 - \chi_1 q_1 \\ \pi_2(q_1, q_2) &= (a - b(q_1 + q_2))\, q_2 - \chi_1 q_2 \end{aligned}$$

   (b) *(6 points)* Find the first order conditions and best responses for both firms.

Solution 4

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_1} &= (a - b(q_1 + q_2)) - bq_1 - \chi_1 = 0 \\
0 &= a - \chi_1 - 2bq_1 - bq_2 \\
q_1 &= \frac{1}{2}\frac{a-\chi_1}{b} - \frac{1}{2}q_2
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi_2}{\partial q_2} &= (a - b(q_1 + q_2)) - bq_2 - \chi_2 = 0 \\
0 &= a - \chi_2 - bq_1 - 2bq_2 \\
q_2 &= \frac{1}{2}\frac{a-\chi_2}{b} - \frac{1}{2}q_1
\end{aligned}$$

(c) *(6 points)* Find the total output and the output of each firm.

Solution 5

$$\begin{aligned}
q_1 &= \frac{1}{4}q_1 + \frac{1}{4}\frac{a}{b} - \frac{1}{2b}\chi_1 + \frac{1}{4b}\chi_2 \\
\frac{3}{4}q_1 &= \frac{1}{4b}(a - 2\chi_1 + \chi_2) \\
q_1 &= \frac{1}{3b}(a - 2\chi_1 + \chi_2) \\
q_2 &= \frac{1}{2}\frac{a-\chi_2}{b} - \frac{1}{2}\left(\frac{1}{3b}(a - 2\chi_1 + \chi_2)\right) \\
&= \frac{1}{3b}(a + \chi_1 - 2\chi_2) \\
Q &= \frac{1}{3b}(a - 2\chi_1 + \chi_2) + \frac{1}{3b}(a + \chi_1 - 2\chi_2) \\
&= \frac{1}{3b}(2a - \chi_1 - \chi_2)
\end{aligned}$$

Alternatively:

$$\begin{aligned}
0 &= (a - \chi_1 - 2bq_1 - bq_2) + (a - \chi_2 - bq_1 - 2bq_2) \\
0 &= 2a - \chi_1 - \chi_2 - 3bq_1 - 3bq_2 \\
0 &= 2a - \chi_1 - \chi_2 - 3bQ \\
Q &= \frac{1}{3b}(2a - \chi_1 - \chi_2)
\end{aligned}$$

Now a third firm is entering the industry, with $c_3(q) = \chi_3 q_3$ and now $Q = q_1 + q_2 + q_3$.

(d) *(2 points)* Set up the objective function of firm three and find it's first order condition.

Solution 6

$$\pi_3(q_1, q_2, q_3) = (a - bQ)\, q_3 - \chi_3 q_3$$

$$\frac{\partial \pi_3}{\partial q_3} = a - bQ - bq_3 - \chi_3 = 0$$

(e) *(5 points)* Find the total output produced assuming all three firms produce, and then verify that one of the firms does *not* produce output, which one is it? Finally, find the true total output produced when there are three firms in the market.

Solution 7

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_1} &= (a - bQ) - bq_1 - \chi_1 = 0 \\
\frac{\partial \pi_2}{\partial q_2} &= (a - bQ) - bq_2 - \chi_2 = 0 \\
\frac{\partial \pi_3}{\partial q_3} &= (a - bQ) - bq_3 - \chi_3 = 0
\end{aligned}$$

$$\begin{aligned}
3(a - bQ) - bq_1 - bq_2 - bq_3 - \chi_1 - \chi_2 - \chi_3 &= 0 \\
3(a - bQ) - bQ - \chi_1 - \chi_2 - \chi_3 &= 0
\end{aligned}$$

$$Q = \frac{1}{4b}(3a - \chi_1 - \chi_2 - \chi_3)$$

But one can readily establish that

$$\left(a - b\left(\frac{1}{4b}(3a - \chi_1 - \chi_2 - \chi_3)\right)\right) - b(0) - \max(\chi_1, \chi_2) < 0$$

thus the firm with the higher marginal cost between firms 1 and 2 will shut down. Let us assume it is firm one, then we know that:

$$\begin{aligned}
\frac{\partial \pi_2}{\partial q_2} &= (a - bQ) - bq_2 - \min(\chi_1, \chi_2) = 0 \\
\frac{\partial \pi_3}{\partial q_3} &= (a - bQ) - bq_3 - \chi_3 = 0
\end{aligned}$$

$$2(a - bQ) - bQ - \min(\chi_1, \chi_2) - \chi_3 = 0$$

$$Q = \frac{1}{3b}(2a - \min(\chi_1, \chi_2) - \chi_3)$$

Now a fourth firm is entering the industry, with $c_4(q) = \chi_4 q_4$ and now $Q = q_1 + q_2 + q_3 + q_4$.

(f) *(2 points)* Set up the objective function of firm four and find it's first order condition.

Solution 8

$$\pi_3(q_1, q_2, q_3, q_4) = (a - bQ)\, q_4 - \chi_4 q_4$$

$$\frac{\partial \pi_4}{\partial q_4} = a - bQ - bq_4 - \chi_4 = 0$$

(g) *(3 points)* Which firm's output can you be sure is zero? Why?

Solution 9 *The firm that shut down when there were only three firms in the market will stay shut down, because even if the new firm does not produce any output the other two firms will produce too much output for that firm to want to produce.*

(h) *(5 points)* Find the total output in the industry when there are four firms, and the output of each firm.

Solution 10 *We will still assume it is firm one, and use a cute trick to make it unimportant.*

$$\begin{aligned}
\frac{\partial \pi_2}{\partial q_2} &= (a - bQ) - bq_2 - \min(\chi_1, \chi_2) = 0 \\
\frac{\partial \pi_3}{\partial q_3} &= (a - bQ) - bq_3 - \chi_3 = 0 \\
\frac{\partial \pi_4}{\partial q_4} &= (a - bQ) - bq_4 - \chi_4 = 0
\end{aligned}$$

$$3(a - bQ) - bQ - \min(\chi_1, \chi_2) - \chi_3 - \chi_4 = 0$$

$$Q = \frac{1}{4b}\left(3a - \min(\chi_1, \chi_2) - \chi_3 - \chi_4\right)$$

$$\begin{aligned}
\frac{\partial \pi_2}{\partial q_2} &= \left(a - b\left(\frac{1}{4b}\left(3a - \min(\chi_1, \chi_2) - \chi_3 - \chi_4\right)\right)\right) - bq_2 - \min(\chi_1, \chi_2) \\
&= \frac{1}{4}a + \frac{1}{4}\chi_3 + \frac{1}{4}\chi_4 + \frac{1}{4}\min(\chi_1, \chi_2) - \min(\chi_1, \chi_2) - bq_2 \\
bq_2 &= \frac{1}{4}a + \frac{1}{4}\chi_3 + \frac{1}{4}\chi_4 - \frac{3}{4}\min(\chi_1, \chi_2) \\
q_2 &= \frac{1}{4b}\left(a + \chi_3 + \chi_4 - 3\min(\chi_1, \chi_2)\right)
\end{aligned}$$

$$\frac{\partial \pi_3}{\partial q_3} = \frac{1}{4}a + \frac{1}{4}\chi_3 + \frac{1}{4}\chi_4 + \frac{1}{4}\min(\chi_1, \chi_2) - bq_3 - \chi_3 = 0$$

$$q_3 = \frac{1}{4b}\left(a - 3\chi_3 + \chi_4 + \min(\chi_1, \chi_2)\right)$$

$$\frac{\partial \pi_4}{\partial q_4} = \frac{1}{4}a + \frac{1}{4}\chi_3 + \frac{1}{4}\chi_4 + \frac{1}{4}\min(\chi_1, \chi_2) - bq_4 - \chi_4 = 0$$

$$q_4 = \frac{1}{4b}\left(a + \chi_3 - 3\chi_4 + \min\left(\chi_1, \chi_2\right)\right)$$

and since all of these are positive we have our solution.

a	b	α	β	χ	q_h^f	q_l^f	p^f	q_l^s	q_h^s
80	1	70	1	2	20	10	60	4	24
11	$\frac{1}{2}$	9	$\frac{1}{2}$	$\frac{1}{4}$	12	8	5	5	13
24	$\frac{1}{2}$	21	$\frac{1}{2}$	$\frac{1}{2}$	18	12	15	8	20
78	2	66	2	2	15	9	48	5	17

Remark 11 *In the problems I gave I assumed $\beta = b$ for simplicity. Thus I have changed the answers below to reflect this.*

5. *(40 points total)* Consider a monopolist who has two customers. Type h has a benefit function of $B_h(q) = \left(a - \frac{\beta}{2}q\right)q$ and type l has a benefit function of $B_l(q) = \left(\alpha - \frac{\beta}{2}q\right)q$. The monopolist has a cost function of $c(Q) = \frac{\chi}{2}Q^2$, where $Q = q_h + q_l$ is the total amount produced.

   (a) *(20 points total)* First Degree Price Discrimination: In this model the monopolist knows who is type h and who is type l. We can think of the solution as a (F_h, p_h, F_l, p_l) where the customer has to pay F_x if they want to buy any, and then can buy as a much as they want at the price p_x (for $x \in \{l, h\}$).

   i. *(4 points)* Set up the objective function of both types of consumers, what does this tell us about (F_h, F_l)?

   Solution 12 *The type h consumer will solve the problem:*

$$\max_q B_h(q) - p_h q - F_h$$

   subject to the constraint $B_h(q) - p_h q - F_h \geq 0$. The type l consumer will solve:

$$\max_q B_l(q) - p_l q - F_l$$

   subject to the constraint $B_l(q) - p_l q - F_l \geq 0$.

   ii. *(6 points)* Set up and simplify the monopolist's objective function.

   Solution 13 *The primitive solution is:*

$$\max_{(p_l, p_h, F_l, F_h)} F_h + F_l + p_h q_h + p_l q_l - \frac{\chi}{2}\left(q_h + q_l\right)^2$$

   But from the consumers problem we see that $F_h = B_h(q_h) - p_h q_h$ and $F_l = B_l(q_l) - p_l q_l$ thus:

$$\begin{aligned} F_h + F_l + p_h q_h + p_l q_l - \frac{\chi}{2}\left(q_h + q_l\right)^2 &= B_h(q_h) - p_h q_h + B_l(q_l) - p_l q_l \\ &\quad + p_h q_h + p_l q_l - \frac{\chi}{2}\left(q_h + q_l\right)^2 \\ &= B_h(q_h) + B_l(q_l) - \frac{\chi}{2}\left(q_h + q_l\right)^2 \end{aligned}$$

since we have lost all prices to maximize over we have the problem of basically choosing quantity.

$$\max_{q_h,q_l} B_h(q_h) + B_l(q_l) - \frac{\chi}{2}(q_h+q_l)^2$$
$$\max_{q_h,q_l} \left(a - \frac{\beta}{2}q_h\right) q_h + \left(\alpha - \frac{\beta}{2}q_l\right) q_l - \frac{\chi}{2}(q_h+q_l)^2$$

iii. *(6 points)* Find the optimal levels for q_h and q_l.

Solution 14 *the first order conditions are:*

$$\begin{aligned}\frac{\partial \Pi}{\partial q_h} &= \left(a - \frac{\beta}{2}q_h\right) - \frac{\beta}{2}q_h - 2\left(\frac{\chi}{2}(q_h+q_l)\right) = 0 \\ \frac{\partial \Pi}{\partial q_l} &= \left(\alpha - \frac{\beta}{2}q_l\right) - \frac{\beta}{2}q_l - 2\left(\frac{\chi}{2}(q_h+q_l)\right) = 0\end{aligned}$$

$$\begin{aligned}a - \beta q_h - \chi q_h - \chi q_l &= 0 \\ \alpha - \chi q_h - \beta q_l - \chi q_l &= 0\end{aligned}$$

and you can solve these however you want. From the first one $q_h = \frac{1}{\beta+\chi}(a - \chi q_l)$*, plugging this into the second one gives us:*

$$\alpha - \chi\left(\frac{1}{\beta+\chi}(a-\chi q_l)\right) - \beta q_l - \chi q_l = 0$$

$$\begin{aligned}q_l &= \frac{1}{\beta^2 + 2\chi\beta}(\alpha\beta - a\chi + \alpha\chi) \\ q_h &= \frac{1}{\beta+\chi}\left(a - \chi\left(\frac{\beta\alpha - a\chi + \alpha\chi}{\beta\beta + \beta\chi + \beta\chi}\right)\right) = \frac{1}{\beta^2+2\chi\beta}(a\beta + a\chi - \alpha\chi)\end{aligned}$$

In the table above all solutions to part ***a*** *are labeled with an f superscript.*

iv. *(4 points)* Find the optimal (p_h, p_l).

Solution 15 *First of all we can find* p_h *from*

$$MB_h = a - \beta q_h = a - \beta\left(\frac{1}{\beta^2+2\chi\beta}(a\beta + a\chi - \alpha\chi)\right) = \chi\frac{a+\alpha}{\beta+2\chi}$$

likewise

$$MB_l = \alpha - \beta q_l = \alpha - \beta\left(\frac{\beta\alpha - a\chi + \alpha\chi}{\beta\beta+\beta\chi+\beta\chi}\right) = \chi\frac{a+\alpha}{\beta+2\chi} = p_h$$

which should come as no surprise.

(b) *(17 points)* Second Degree Price Discrimination: In this model the monopolist knows there is a high demand and low demand customer, but does not know which is which. We can think of the solution as a (F_h, q_h, F_l, q_l) where the customer has to pay F_x to buy q_x (for $x \in \{l, h\}$).

I give you that the binding constraint for the high type will be that they do not want to buy the low type's bundle:

$$B_h(q_h) - F_h = B_h(q_l) - F_l$$

and that for the low type it is that they are just indifferent between buying anything and not.

$$B_l(q_l) - F_l = 0$$

i. *(3 points)* What are the optimal values for F_h and F_l as a function of $B_h(\cdot)$ and $B_l(\cdot)$?

Solution 16 *This is given immediately by the constraints above:*

$$\begin{aligned} F_l &= B_l(q_l) \\ F_h &= B_h(q_h) - B_h(q_l) + F_l \\ &= B_h(q_h) - B_h(q_l) + B_l(q_l) \end{aligned}$$

ii. *(6 points)* Set up and simplify the monopolist's objective function.

Solution 17 *In its primitive form it is:*

$$\max_{(F_h, F_l, q_h, q_l)} F_h + F_l - \frac{\chi}{2}(q_h + q_l)^2$$

but given the findings above:

$$\max_{(q_h, q_l)} B_h(q_h) - B_h(q_l) + B_l(q_l) + B_l(q_l) - \frac{\chi}{2}(q_h + q_l)^2$$

$$\max_{(q_h, q_l)} B_h(q_h) - B_h(q_l) + 2B_l(q_l) - \frac{\chi}{2}(q_h + q_l)^2$$

$$\max_{(q_h, q_l)} \left(a - \frac{\beta}{2}q_h\right) q_h - \left(a - \frac{\beta}{2}q_l\right) q_l + 2\left(\alpha - \frac{\beta}{2}q_l\right) q_l - \frac{\chi}{2}(q_h + q_l)^2$$

$$\max_{(q_h, q_l)} aq_h - aq_l + 2\alpha q_l - \frac{1}{2}\beta q_h^2 + \frac{1}{2}\beta q_l^2 - \frac{1}{2}\chi q_h^2 - \beta q_l^2 - \frac{1}{2}\chi q_l^2 - \chi q_h q_l$$

iii. *(6 points)* Find the optimal levels for q_h and q_l.

Solution 18 *the first order conditions are:*

$$\begin{aligned} \frac{\partial \Pi}{\partial q_h} &= a - \beta q_h - \chi q_h - \chi q_l = 0 \\ \frac{\partial \Pi}{\partial q_l} &= -a + 2\alpha + \beta q_l - 2\beta q_l - \chi q_l - \chi q_h = 0 \end{aligned}$$

And by order of simplicity I first find: $q_h = \frac{1}{\beta+\chi}\left(a - \chi q_l\right)$

$$-a + 2\alpha + \beta q_l - 2\beta q_l - \chi q_l - \chi\left(\frac{1}{\beta+\chi}\left(a - \chi q_l\right)\right) = 0$$

$$q_l = -\frac{1}{\beta^2 + 2\chi\beta}\left(a\beta + 2a\chi - 2\alpha\beta - 2\alpha\chi\right)$$

$$\begin{aligned} q_h &= \frac{1}{\beta+\chi}\left(a - \chi\left(-\frac{1}{\beta^2+2\chi\beta}\left(a\beta + 2a\chi - 2\alpha\beta - 2\alpha\chi\right)\right)\right) \\ &= \frac{1}{\beta^2 + 2\chi\beta}\left(a\beta + 2a\chi - 2\alpha\chi\right) \end{aligned}$$

(c) *(3 points)* Compare the value of q_h you found in part a and in part b. Why should they be the same? Why are they not the same?

Solution 19 *They should be the same because in both cases the only terms that involve* q_h *are* $B_h(q_h) - C(q_h + q_l)$*, thus to the first order they should be the same. However in this problem* $C(q_h + q_l) = \frac{\chi}{2}(q_h + q_l)^2$ *thus* q_h *will be affected by* q_l*. In all cases above* $q_l^f > q_l^s$ *thus* $q_h^f < q_h^s$*—in words the high type gets more because the low type gets less in second degree price discrimination.*