Kevin Hasker This exam will start at 10:30 and finish at 12:10.

Answer all questions in the space provided. Points will only be given for work shown.

1. (13 points) Please read and sign the following statement:

	I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test.
	Name and Surname: Student ID: Signature:
2.	(8 points) List two of the four reasons for natural market power discussed
	in class, explain each one and give an example of an industry where it is
	an important factor. You may use the same industry more than once.

- 4. (31 points total) Consider an industry with two firms that compete by choosing quantity, the price is then set at the market clearing price (Cournot Competition). The inverse demand curve is $P = 21 \frac{1}{2}Q$, where $Q = q_1 + q_2$, and $c_1(q) = 11q_1$, $c_2(q) = 4q_2$.
 - (a) (2 points) Set up the objective function of both firms (to be clear, you need two separate objective functions.)
 - (b) (6 points) Find the first order conditions and best responses for both firms.

- Now a third firm is entering the industry, with $c_3\left(q\right)=2q_3$ and now $Q=q_1+q_2+q_3$.
- (d) (2 points) Set up the objective function of firm three and find it's first order condition.
- (e) (5 points) Find the total output produced assuming all three firms produce, and then verify that one of the firms does not produce output, which one is it? Finally, find the true total output produced when there are three firms in the market.

- Now a fourth firm is entering the industry, with $c_4(q) = q_4$ and now $Q = q_1 + q_2 + q_3 + q_4$.
- (f) (2 points) Set up the objective function of firm four and find it's first order condition.
- (g) (3 points) Which firm's output can you be sure is zero? Why?

- 5. (36 points total) Consider a monopolist who has two customers. Type h has a benefit function of $B_h(q) = \left(24 \frac{1}{4}q\right)q$ and type l has a benefit function of $B_l(q) = \left(21 \frac{1}{4}q\right)q$. The monopolist has a cost function of $c(Q) = \frac{1}{4}Q^2$, where $Q = q_h + q_l$ is the total amount produced.
 - (a) (18 points total) First Degree Price Discrimination: In this model the monopolist knows who is type h and who is type l. We can think of the solution as a (F_h, p_h, F_l, p_l) where the customer has to pay F_x if they want to buy any, and then can buy as a much as they want at the price p_x (for $x \in \{l, h\}$).
 - i. (4 points) Set up the objective function of both types of consumers, what does this tell us about (F_h, F_l) ?

iii. (6 points) Find the optimal levels for q_h and q_l .

- iv. (2 points) Find the optimal (p_h, p_l) .
- (b) (15 points total) Second Degree Price Discrimination: In this model the monopolist knows there is a high demand and low demand customer, but does not know which is which. We can think of the solution as a (F_h, q_h, F_l, q_l) where the customer has to pay F_x to buy q_x (for $x \in \{l, h\}$).

I give you that the binding constraint for the high type will be that they do not want to buy the low type's bundle:

$$B_h(q_h) - F_h = B_h(q_l) - F_l$$

$$B_l\left(q_l\right) - F_l = 0$$

- i. (3 points) What are the optimal values for F_h and F_l as a function of $B_h\left(\cdot\right)$ and $B_l\left(\cdot\right)$?
- ii. $(6 \ points)$ Set up and simplify the monopolist's objective function.
- iii. (6 points) Find the optimal levels for q_h and q_l .

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	in class, explain each one and give an example of an industry where it is
	an important factor. You may use the same industry more than once.

- 4. (31 points total) Consider an industry with two firms that compete by choosing quantity, the price is then set at the market clearing price (Cournot Competition). The inverse demand curve is $P=18-\frac{1}{4}Q$, where $Q=q_1+q_2$, and $c_1(q)=13q_1$, $c_2(q)=11q_2$.
 - (a) (2 points) Set up the objective function of both firms (to be clear, you need two separate objective functions.)
 - (b) $(6 \ points)$ Find the first order conditions and best responses for both firms.

- Now a third firm is entering the industry, with $c_3\left(q\right)=7q_3$ and now $Q=q_1+q_2+q_3$.
- (d) (2 points) Set up the objective function of firm three and find it's first order condition.
- (e) (5 points) Find the total output produced assuming all three firms produce, and then verify that one of the firms does not produce output, which one is it? Finally, find the true total output produced when there are three firms in the market.

- Now a fourth firm is entering the industry, with $c_4\left(q\right)=9q_4$ and now $Q=q_1+q_2+q_3+q_4$.
- (f) $(2\ points)$ Set up the objective function of firm four and find it's first order condition.
- (g) (3 points) Which firm's output can you be sure is zero? Why?

- 5. (36 points total) Consider a monopolist who has two customers. Type h has a benefit function of $B_h(q) = (78 q)q$ and type l has a benefit function of $B_l(q) = (66 q)q$. The monopolist has a cost function of $c(Q) = Q^2$, where $Q = q_h + q_l$ is the total amount produced.
 - (a) (18 points total) First Degree Price Discrimination: In this model the monopolist knows who is type h and who is type l. We can think of the solution as a (F_h, p_h, F_l, p_l) where the customer has to pay F_x if they want to buy any, and then can buy as a much as they want at the price p_x (for $x \in \{l, h\}$).
 - i. (4 points) Set up the objective function of both types of consumers, what does this tell us about (F_h, F_l) ?

iii. (6 points) Find the optimal levels for q_h and q_l .

- iv. (2 points) Find the optimal (p_h, p_l) .
- (b) (15 points total) Second Degree Price Discrimination: In this model the monopolist knows there is a high demand and low demand customer, but does not know which is which. We can think of the solution as a (F_h, q_h, F_l, q_l) where the customer has to pay F_x to buy q_x (for $x \in \{l, h\}$).

I give you that the binding constraint for the high type will be that they do not want to buy the low type's bundle:

$$B_h(q_h) - F_h = B_h(q_l) - F_l$$

$$B_l\left(q_l\right) - F_l = 0$$

- i. (3 points) What are the optimal values for F_h and F_l as a function of $B_h\left(\cdot\right)$ and $B_l\left(\cdot\right)$?
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- 4. (31 points total) Consider an industry with two firms that compete by choosing quantity, the price is then set at the market clearing price (Cournot Competition). The inverse demand curve is $P=25-\frac{1}{2}Q$, where $Q=q_1+q_2$, and $c_1(q)=12q_1$, $c_2(q)=17q_2$.
 - (a) (2 points) Set up the objective function of both firms (to be clear, you need two separate objective functions.)
 - (b) $(6 \ points)$ Find the first order conditions and best responses for both firms.

- Now a third firm is entering the industry, with $c_3(q) = 8q_3$ and now $Q = q_1 + q_2 + q_3$.
- (d) (2 points) Set up the objective function of firm three and find it's first order condition.
- (e) (5 points) Find the total output produced assuming all three firms produce, and then verify that one of the firms does not produce output, which one is it? Finally, find the true total output produced when there are three firms in the market.

- Now a fourth firm is entering the industry, with $c_4\left(q\right)=5q_4$ and now $Q=q_1+q_2+q_3+q_4$.
- (f) (2 points) Set up the objective function of firm four and find it's first order condition.
- (g) (3 points) Which firm's output can you be sure is zero? Why?

- 5. (36 points total) Consider a monopolist who has two customers. Type h has a benefit function of $B_h(q) = \left(80 \frac{1}{2}q\right)q$ and type l has a benefit function of $B_l(q) = \left(70 \frac{1}{2}q\right)q$. The monopolist has a cost function of $c(Q) = Q^2$, where $Q = q_h + q_l$ is the total amount produced.
 - (a) (18 points total) First Degree Price Discrimination: In this model the monopolist knows who is type h and who is type l. We can think of the solution as a (F_h, p_h, F_l, p_l) where the customer has to pay F_x if they want to buy any, and then can buy as a much as they want at the price p_x (for $x \in \{l, h\}$).
 - i. (4 points) Set up the objective function of both types of consumers, what does this tell us about (F_h, F_l) ?

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$$B_h(q_h) - F_h = B_h(q_l) - F_l$$

$$B_l\left(q_l\right) - F_l = 0$$

- i. (3 points) What are the optimal values for F_h and F_l as a function of $B_h\left(\cdot\right)$ and $B_l\left(\cdot\right)$?
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- 4. (31 points total) Consider an industry with two firms that compete by choosing quantity, the price is then set at the market clearing price (Cournot Competition). The inverse demand curve is $P=15-\frac{1}{4}Q$, where $Q=q_1+q_2$, and $c_1(q)=6q_1$, $c_2(q)=9q_2$.
 - (a) (2 points) Set up the objective function of both firms (to be clear, you need two separate objective functions.)
 - (b) (6 points) Find the first order conditions and best responses for both firms.

- Now a third firm is entering the industry, with $c_3\left(q\right)=3q_3$ and now $Q=q_1+q_2+q_3$.
- (d) (2 points) Set up the objective function of firm three and find it's first order condition.
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- Now a fourth firm is entering the industry, with $c_4\left(q\right)=6q_4$ and now $Q=q_1+q_2+q_3+q_4$.
- (f) (2 points) Set up the objective function of firm four and find it's first order condition.
- (g) (3 points) Which firm's output can you be sure is zero? Why?

- 5. (36 points total) Consider a monopolist who has two customers. Type h has a benefit function of $B_h(q) = \left(11 \frac{1}{4}q\right)q$ and type l has a benefit function of $B_l(q) = \left(9 \frac{1}{4}q\right)q$. The monopolist has a cost function of $c(Q) = \frac{1}{8}Q^2$, where $Q = q_h + q_l$ is the total amount produced.
 - (a) (18 points total) First Degree Price Discrimination: In this model the monopolist knows who is type h and who is type l. We can think of the solution as a (F_h, p_h, F_l, p_l) where the customer has to pay F_x if they want to buy any, and then can buy as a much as they want at the price p_x (for $x \in \{l, h\}$).
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