# ECON 433
# Quizzes on Dynamic Oligopoly, Patents, and Mergers

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1.

| $\chi_1 = \chi_2$ | $t$ | $p_a(p_b)$ | $p_b(p_a)$ | $p_a^s = p_b^s$ | $\pi_a^s = \pi_b^s$ | $p_a^d$ | $p_b^d$ | $\pi_a^d$ | $\pi_b^d$ |
|---|---|---|---|---|---|---|---|---|---|
| 10 | 4 | $\frac{1}{2}p_b + 7$ | $\frac{1}{2}p_a + 7$ | 14 | 2 | 16 | 15 | $\frac{9}{4}$ | $\frac{25}{8}$ |
| 2 | 8 | $\frac{1}{2}p_b + 5$ | $\frac{1}{2}p_a + 5$ | 10 | 4 | 14 | 12 | $\frac{9}{2}$ | $\frac{25}{4}$ |
| 4 | 12 | $\frac{1}{2}p_b + 8$ | $\frac{1}{2}p_a + 8$ | 16 | 6 | 22 | 19 | $\frac{27}{4}$ | $\frac{75}{8}$ |
| 4 | 8 | $\frac{1}{2}p_b + 6$ | $\frac{1}{2}p_a + 6$ | 12 | 4 | 16 | 14 | $\frac{9}{2}$ | $\frac{25}{4}$ |

2. *(18 points total)* Consider a linear city where the customers are distributed uniformly over the interval $[0,1]$. Firm $a$ is located at 0 and firm $b$ is located at 1. Their cost functions are $c_1(q) = \chi_1 q_a$ and $c_2(q) = \chi_2 q_b$. These firms choose prices, $p_a$ and $p_b$. A consumer at location $x \in [0,1]$ has the preferences:

$$u(p_a, p_b, x) = \begin{cases} V - p_a - tx^2 & \text{if they buy from firm } a \\ V - p_b - t(1-x)^2 & \text{if they buy from firm } b \\ 0 & \text{if they do not buy.} \end{cases}$$

You may assume that $V$ is high enough that they always buy from either firm $a$ or firm $b$.

(a) *(2 points)* Confirm that:

$$\begin{aligned} D_a &= \frac{1}{2t}(t + p_b - p_a) \\ D_b &= \frac{1}{2t}(t + p_a - p_b) \end{aligned}$$

you may use this from now on even if you can not derive it.

(b) *(2 points)* Set up the objective functions of firm $a$ and $b$.

$$\begin{aligned} \pi_a(p_a, p_b) &= (p_a - \chi_1)\frac{1}{2t}(t + p_b - p_a) \\ \pi_b(p_a, p_b) &= (p_b - \chi_2)\frac{1}{2t}(t + p_a - p_b) \end{aligned}$$

**Remark 1** *When I wrote the answers I was assuming $\chi_1 \neq \chi_2$, but I decided that was too complex. Thus if they argue "by symmetry" at any point they are correct and should receive full credit—unless explicitly ruled out. In other words in all cases except when they are finding the equilibrium of the static game.*

(c) *(4 points)* Find the best responses of both firms.

$$\frac{\partial \pi_a}{\partial p_a} = \frac{1}{2t}\left(t + p_b - p_a\right) - \frac{1}{2t}\left(p_a - \chi_1\right) = 0$$

$$p_a = \frac{1}{2}t + \frac{1}{2}\chi_1 + \frac{1}{2}p_b$$

$$\frac{\partial \pi_b}{\partial p_b} = \frac{1}{2t}\left(t + p_a - p_b\right) - \left(p_b - \chi_2\right)\frac{1}{2t} = 0$$

$$p_b = \frac{1}{2}t + \frac{1}{2}\chi_2 + \frac{1}{2}p_a$$

(d) *(3 points)* Find the equilibrium prices of both firms and their profits. **NOTE:** You may not assume the equilibrium prices are the same.

$$\begin{aligned} p_a &= \frac{1}{2}t + \frac{1}{2}\chi_1 + \frac{1}{2}\left(\frac{1}{2}t + \frac{1}{2}\chi_2 + \frac{1}{2}p_a\right) \\ p_a &= t + \frac{2}{3}\chi_1 + \frac{1}{3}\chi_2 \end{aligned}$$

$$\begin{aligned} p_b &= \frac{1}{2}t + \frac{1}{2}\chi_2 + \frac{1}{2}\left(t + \frac{2}{3}\chi_1 + \frac{1}{3}\chi_2\right) \\ &= t + \frac{1}{3}\chi_1 + \frac{2}{3}\chi_2 \end{aligned}$$

$$\begin{aligned} \pi_a &= \left(\left(t + \frac{2}{3}\chi_1 + \frac{1}{3}\chi_2\right) - \chi_1\right)\frac{1}{2t}\left(t + \left(t + \frac{1}{3}\chi_1 + \frac{2}{3}\chi_2\right) - \left(t + \frac{2}{3}\chi_1 + \frac{1}{3}\chi_2\right)\right) \\ &= \left(t - \frac{1}{3}\chi_1 + \frac{1}{3}\chi_2\right)\frac{1}{2t}\left(t - \frac{1}{3}\chi_1 + \frac{1}{3}\chi_2\right) \\ &= \frac{1}{18t}\left(3t - \chi_1 + \chi_2\right)^2 \end{aligned}$$

$$\begin{aligned} \pi_b &= \left(\left(t + \frac{1}{3}\chi_1 + \frac{2}{3}\chi_2\right) - \chi_2\right)\frac{1}{2t}\left(t + \left(t + \frac{2}{3}\chi_1 + \frac{1}{3}\chi_2\right) - \left(t + \frac{1}{3}\chi_1 + \frac{2}{3}\chi_2\right)\right) \\ &= \frac{1}{18t}\left(3t + \chi_1 - \chi_2\right)^2 \end{aligned}$$

Now consider a model where firm $a$ chooses their price and then firm $b$ chooses their price.

(e) *(1 point)* Why do you not need to derive the best response of firm $b$ again?

**Solution 2** *Because they will have the same best response, in the simultaneous game they answer the question "if firm a charges $p_a$" in the sequential game they answer the question "given firm a charges $p_a$"*

(f) *(2 points)* Set up the objective function of firm $a$ in this new environment.

$$\begin{aligned}\pi_a(p_a,p_b) &= (p_a-\chi_1)\frac{1}{2t}(t+p_b(p_a)-p_a)\\ &= (p_a-\chi_1)\frac{1}{2t}\left(t+\left(\frac{1}{2}t+\frac{1}{2}\chi_2+\frac{1}{2}p_a\right)-p_a\right)\\ &= (p_a-\chi_1)\frac{1}{4t}(3t+\chi_2-p_a)\end{aligned}$$

(g) *(4 points)* Find the optimal value for $p_a$, $p_b$, and both of their profits. How do their profits compare to those found when they chose price simultaneously?

$$\frac{\partial\pi_a}{\partial p_a}=\frac{1}{4t}(3t+\chi_2-p_a)-(p_a-\chi_1)\frac{1}{4t}=0$$

$$\begin{aligned}p_a &= \frac{3}{2}t+\frac{1}{2}\chi_1+\frac{1}{2}\chi_2\\ p_b &= \frac{1}{2}t+\frac{1}{2}\chi_2+\frac{1}{2}\left(\frac{3}{2}t+\frac{1}{2}\chi_1+\frac{1}{2}\chi_2\right)\\ &= \frac{5}{4}t+\frac{1}{4}\chi_1+\frac{3}{4}\chi_2\end{aligned}$$

$$\begin{aligned}\pi_a &= \left(\left(\frac{3}{2}t+\frac{1}{2}\chi_1+\frac{1}{2}\chi_2\right)-\chi_1\right)\frac{1}{4t}\left(3t+\chi_2-\left(\frac{3}{2}t+\frac{1}{2}\chi_1+\frac{1}{2}\chi_2\right)\right)\\ &= \frac{1}{16t}(3t-\chi_1+\chi_2)^2\end{aligned}$$

$$\begin{aligned}\pi_b &= \left(\left(\frac{5}{4}t+\frac{1}{4}\chi_1+\frac{3}{4}\chi_2\right)-\chi_2\right)\frac{1}{2t}\left(t+\left(\frac{3}{2}t+\frac{1}{2}\chi_1+\frac{1}{2}\chi_2\right)-\left(\frac{5}{4}t+\frac{1}{4}\chi_1+\frac{3}{4}\chi_2\right)\right)\\ &= \frac{1}{32t}(5t+\chi_1-\chi_2)^2\end{aligned}$$

Given that $\chi_1=\chi_2$ we can easily establish that $\pi^s=\frac{1}{2}t<\frac{9}{16}t=\pi_a^d<\frac{25}{32}t=\pi_b^d$ .

3. *(6 points)* Is price fixing common, rare, or somewhere in between? Is it a significant factor in the economy or not? Give an example of an important industry that has been found guilty of price fixing in this century.

**Solution 3** *It is not common, but I would say it is not as rare as we would like. (This can be a point of opinion, so they can disagree with my characterization and still be right.)*

*It is a significant factor in the economy, as evidenced by the billions of dollars and euros competition authorities have been paid by various industries in the last thirty years alone.*

*Probably the most significant industry that has had a price fixing case found against it is the LCD screen industry. The US and EU both found them guilty of price fixing (though in the US they "settled" the case) and each charged that industry over $1 billion (or 1 billion euros).*

*The table of alternatives is based on fines to a given company—fines for a case like this will usually be leveled on multiple companies (p 350 in the book). This for the United States, and the top and fourth are for LCD TVs. Number two is Vitamins, number 3 is Automobile parts. If they list dairy collusion in the UK give them at most one pity point.*

4. *(6 points)* List two of the four stylized facts about entry to and exit from industries. Which stylized fact indicates that entry and exit are not due to market conditions?

   **Solution 4** *The four stylized facts are:*

   (a) *entry is common, about 8% to 10% per year.*
   (b) *Entry is small-scale, the 8% to 10% new entrants generally have about 13.9% to 18.8% market share*
   (c) *Most entrants exit relatively quickly—or the survival rate is low—61.5% exit within 5 years, 79.6% exit within ten years.*
   (d) *Entry rates and exit rates are highly correlated, or industries with large entry also have large exit.*

   *This last fact is the most important because it shows that it is not market conditions per se that is driving this process. We expect when an industry is doing badly there will be exit, when it is doing great their will be entry. If we observe both at once it is clear both the exit and the entry are not driven by industry, rather we are observing a steady state.*

5. *(6 points)* What does Shumpeter say is the most important type of competition? What does this imply about the value of this class? Do you agree? (Any serious answer backed up with a logical argument is correct for the last part.)

   **Solution 5** *Whoa, am I gutsy or what? I am literally letting them say this class is worthless—if they can argue their case convincingly.*

   *Shumpeter says the most important type of competition is from new products, innovations wiping out old industries and replacing them with new.*

*This implies that this class, that primarily focuses on competition between firms in an industry, is trivial. Instead we should be studying new industries, the creation and destruction of industries. Unfortunately this is hard to model... so... I guess we should just do case studies???*

*While I certainly agree that in the long run this is correct in the long run we are all dead (to quote Keynes). Take, for example, LCD screens. Around 2011-2012 LCD screen producers were found guilty by competition authorities of price fixing. Since that time we have a seen a massive influx of cheaper LCD screens, improving life. Taken to its logical extreme, Shumpeter would say this was a waste of time [I do not think he would actually say that].*

*And on the back end, there are many industries where there is precious little innovation. Should we just ignore them? Again, obviously not. For example the scandal over price fixing of dairy products in the UK.*

*Shumpeter would not say this class is worthless, he would probably vehemently defend this class as an important one for a well rounded economist. However he is making a point, the way that the economy moves forward is through innovation. It might be hard to understand this process, but it is an important task for economists.*

*Did you know that a significant reason that shipping costs have fallen so much is because of computers? Their are a series of very complicated rules about what can be shipped next to what, etcetera. Not to mention what needs to be off-loaded at the first stop, what at the last. These days loading a cargo ship is not possible without a computer. In other words the computer revolution has now affected how much it costs for you to buy a Nazar Boyuncu (Evil eye for you foreigners, a traditional eastern Mediterranean decoration that is mostly made in China these days.)*

6. *(6 points)* Explain intuitively why a merger between firms who's strategy variables are strategic compliments will result in higher profits than before the merger. You may assume their strategic variable is price.

**Solution 6** *Let firms one and two merge, since $s_1$ and $s_2$ are strategic compliments this will cause both to rise—increasing $s_1$ results in higher profits for firm 2, and vice versa—increasing the profits of both firms. Furthermore since they are both strategic compliments for the other firm's variables all of those variables will rise as well. In this case of price this means that the profits of all firms in the industry will rise.*

| $\chi$ | $a$ | $b$ | $q_2(q_1)$ | $q_1^{sd}$ | $q_1 \vert q_2 > 0$ | $\pi_1 \vert q_2 > 0$ |
|---|---|---|---|---|---|---|
| 4 | 28 | 4 | $3 - \frac{1}{2}q_1$ | $6 - 12\phi$ | 3 | 18 |
| 8 | 28 | 2 | $5 - \frac{1}{2}q_1$ | $10 - 20\phi$ | 5 | 25 |
| 16 | 28 | 1 | $6 - \frac{1}{2}q_1$ | $12 - 24\phi$ | 6 | 18 |
| 8 | 24 | 1 | $8 - \frac{1}{2}q_1$ | $16 - 32\phi$ | 8 | 32 |

7. *(12 points total)* Consider a Stackelberg leader model. Firm 1 chooses their quantity first and has total cost $c_1(q) = \chi q_1$, firm 2 chooses their output second and has total cost $c_2(q) = \chi q_2 + \phi^2 \frac{(a-\chi)^2}{b}$. The inverse demand curve is $P = a - bQ$.

   (a) *(2 points)* Assuming firm 2 produces a positive output, what will be their best response (as a function of the quantity firm one produces)?

   **Solution 7**

$$\pi_2(q_1, q_2) = (a - b(q_1 + q_2))\, q_2 - \chi q_2 - \phi^2 \frac{(a-\chi)^2}{b}$$

$$(a - b(q_1 + q_2)) - bq_2 - \chi = 0$$

$$q_2 = \frac{1}{2b}(a - \chi) - \frac{1}{2}q_1$$

   *The reason this question has so few points is because this should be a simple exercise by now.*

   ***Remark 8*** *For this question if they write down the answer without work give them one of two points. Give them two points if they explicitly says "since this is a standard Cournot we know* $q_2 = \frac{1}{2b}(a - \chi) - \frac{1}{2}q_1$ *" and then plug in the values.*

   (b) *(4 points)* As a function of $\phi$, what output for firm one will make firm 2 shut down? (I.e. they would make a negative profit if they produced any output.

   **Solution 9**

$$\pi_2(q_1, q_2(q_1)) = \left(a - b\left(q_1 + \left(\frac{1}{2b}(a-\chi) - \frac{1}{2}q_1\right)\right)\right)\left(\frac{1}{2b}(a-\chi) - \frac{1}{2}q_1\right) - \chi\left(\frac{1}{2b}(a-\chi)\right.$$

$$\pi_2(q_1, q_2(q_1)) = \frac{1}{4b}(a - \chi - bq_1)^2 - \phi^2 \frac{(a-\chi)^2}{b}$$

$$\pi_2(q_1, q_2(q_1)) = \frac{1}{4b}(a - \chi - bq_1)^2 - \phi^2 \frac{(a-\chi)^2}{b} = 0$$

$$\begin{aligned} (a - \chi - bq_1)^2 &= 4\phi^2 (a-\chi)^2 \\ a - \chi - bq_1 &= 2\phi(a-\chi) \\ q_1^{sd} &= (1 - 2\phi)\frac{(a-\chi)}{b} \end{aligned}$$

   (c) *(4 points)* Assuming firm 2 produces a positive output, what quantity will firm 1 choose? What will their profits be?

**Solution 10**

$$\pi_1(q_1, q_2) = (a - b(q_1 + q_2))\, q_1 - \chi q_1$$

$$q_2 = \frac{1}{2b}(a - \chi) - \frac{1}{2}q_1$$

$$\begin{aligned}
\pi_1(q_1, q_2) &= \left(a - b\left(q_1 + \left(\frac{1}{2b}(a - \chi) - \frac{1}{2}q_1\right)\right)\right) q_1 - \chi q_1 \\
&= \frac{1}{2}q_1(a - \chi - bq_1)
\end{aligned}$$

$$\frac{1}{2}(a - \chi - bq_1) + \frac{1}{2}q_1(-b) = 0$$

$$q_1 = \frac{1}{2b}(a - \chi)$$

$$\begin{aligned}
\pi_1^* &= \frac{1}{2}\left(\frac{1}{2b}(a - \chi)\right)\left(a - \chi - b\left(\frac{1}{2b}(a - \chi)\right)\right) \\
&= \frac{1}{8b}(a - \chi)^2
\end{aligned}$$

(d) (*2 points)* As a function of $\phi$, when will making firm 2 shut down result in higher profits than letting firm 2 produce a strictly positive output? You do not need to explicitly solve for the range of $\phi$, you can find them as the solution to an equation.

**Solution 11** *From above we know that if* $q_1 = q_1^{sd} = (1 - 2\phi)\frac{(a-\chi)}{b}$ *then* $q_2 = 0$ *is optimal, Thus we know that*

$$\pi_1\left(q_1^{sd}, 0\right) = \left(a - b\left(q_1^{sd} + 0\right)\right) q_1^{sd} - \chi q_1^{sd}$$

$$\begin{aligned}
\pi_1\left(q_1^{sd}, 0\right) &= \left(a - b\left(\left((1 - 2\phi)\frac{(a - \chi)}{b}\right) + 0\right)\right)\left((1 - 2\phi)\frac{(a - \chi)}{b}\right) - \chi\left((1 - 2\phi)\frac{(a - \chi)}{b}\right. \\
&= \frac{2}{b}\phi(1 - 2\phi)(a - \chi)^2
\end{aligned}$$

*thus we want to know when:*

$$\begin{aligned}
\frac{2}{b}\phi(1 - 2\phi)(a - \chi)^2 &\geq \frac{1}{8b}(a - \chi)^2 \\
\frac{\frac{2}{b}\phi(1 - 2\phi)(a - \chi)^2}{\frac{2}{b}(a - \chi)^2} &\geq \frac{\frac{1}{8b}(a - \chi)^2}{\frac{2}{b}(a - \chi)^2} \\
\phi(1 - 2\phi) &\geq \frac{1}{16}
\end{aligned}$$

*the solution is* $\phi \in \left[-\frac{1}{8}\sqrt{2} + \frac{1}{4}, \frac{1}{8}\sqrt{2} + \frac{1}{4}\right]$ *so the numeric solution is* $\phi \geq -\frac{1}{8}\sqrt{2} + \frac{1}{4} = .073$.