ECON 433, 2021

Quizess before the Midterms, answer key

Dr. Kevin Hasker

1. **Remark 1** *There are four versions of the quiz. In the following table the values for key answers are given.*

a	b	χ	$BR_1(q_2)$	q^c	Q^c	P^c	P^m	$\frac{1}{8}(a-\chi)^2 - 1 > 0$	*Bertrand NE*
16	1	4	$6 - \frac{1}{2}q_2$	4	8	8	10	17	$\{4, 5\}$
18	1	6	$6 - \frac{1}{2}q_2$	4	8	10	12	17	$\{6, 7\}$
16	1	10	$3 - \frac{1}{2}q_2$	2	4	12	13	$\frac{7}{2}$	$\{10, 11\}$
26	1	8	$9 - \frac{1}{2}q_2$	6	12	14	17	$\frac{79}{2}$	$\{8, 9\}$

Consider a world where there are two firms (1 and 2) each of them who have $C(q) = \chi q_i$ for $i \in \{1, 2\}$. The inverse demand curve is given by $P = a - bQ$ where Q is the market quantity.

2. *(9 points total)* (Cournot Oligopoly) Each firm chooses q_i and $Q = q_1 + q_2$.

   (a) *(2 points)* Set up the objective function of firm 1.

$$\pi_1(q_1, q_2) = (a - b(q_1 + q_2))\, q_1 - \chi q_1$$

   (b) *(4 points)* Find the best response.

$$\frac{\partial \pi_1}{\partial q_1} = (a - b(q_1 + q_2)) - bq_1 - \chi = 0$$

$$\begin{aligned} a - \chi - 2bq_1 - bq_2 &= 0 \\ \frac{1}{2b}(a - \chi) - \frac{1}{2}q_2 &= q_1 \end{aligned}$$

   (c) *(3 points)* Find the equilibrium quantities of both firms, the total quantity, and market price. **NOTE:** I have told you three different ways to do this.

   Solution 2 *Using symmetry we can assume $q_1 = q_2 = q$ in which case:*

$$\begin{aligned} \frac{1}{2b}(a - \chi) - \frac{1}{2}q &= q \\ \frac{1}{2b}(a - \chi) &= \frac{3}{2}q \\ \frac{1}{3b}(a - \chi) &= q \end{aligned}$$

we can also intersect the best responses.

$$\begin{aligned}
\frac{1}{2b}(a-\chi)-\frac{1}{2}\left(\frac{1}{2b}(a-\chi)-\frac{1}{2}q_1\right) &= q_1\\
\frac{1}{4}q_1+\frac{1}{4b}(a-\chi) &= q\\
\frac{1}{4b}(a-\chi) &= \frac{3}{4}q\\
\frac{1}{3b}(a-\chi) &= q
\end{aligned}$$

And we can also sum the first order conditions:

$$\begin{aligned}
(a-\chi-2bq_1-bq_2)+(a-\chi-2bq_2-bq_1) &= 0\\
2a-2\chi-3b(q_1+q_2) &= 0\\
2a-2\chi-3bQ &= 0\\
\frac{2}{3b}(a-\chi) &= Q
\end{aligned}$$

$$\begin{aligned}
(a-bQ)-bq_1-\chi &= 0\\
\left(a-b\left(\frac{2}{3b}(a-\chi)\right)\right)-bq_1-\chi &= 0\\
\frac{1}{3b}(a-\chi) &= q_1
\end{aligned}$$

Using any of these three techniques

$$\begin{aligned}
Q &= q_1+q_2=\frac{2}{3b}(a-\chi)\\
P &= a-bQ=a-b\frac{2}{3b}(a-\chi)\\
&= \frac{1}{3}a+\frac{2}{3}\chi
\end{aligned}$$

3. *(9 points total)* (Bertrand Oligopoly) Now the two firms choose price, p_i, **which must be an integer**. The market price, P is the minimum of the two firm's prices. Customers always buy from the lower price firm, and if both firms charge the same price split their demand between the two firms.

   (a) *(6 points)* For all p_2, find the best response(s) for firm one. Be sure to be careful when $p_2=\chi$, $p_2<\chi$ and p_2 is very high.

   Remark 3 *Shouldn't this question be worth more than 6 points? No, because I didn't want it to be.*

   Be sure to be generous with partial credit on this question—though if everyone does excellently you can be harsher on those who did not.

   HOWEVER "no work no credit" means that writing down a best response in some case without justification gets nothing.

Solution 4 *Noticing that* $Q = \frac{a}{b} - \frac{1}{b}P$ *we first have to find the monopoly price:*

$$\begin{aligned}
&\max_P P\left(\frac{a}{b} - \frac{1}{b}P\right) - \chi\left(\frac{a}{b} - \frac{1}{b}P\right) \\
&\max_P (P - \chi)\left(\frac{a}{b} - \frac{1}{b}P\right)
\end{aligned}$$

$$\begin{aligned}
\frac{1}{b}(a - P) - \frac{1}{b}(P - \chi) &= 0 \\
\frac{1}{2}a + \frac{1}{2}\chi &= P^m
\end{aligned}$$

Thus if $p_2 > \frac{1}{2}a + \frac{1}{2}\chi$ *the best response is the monopoly price,* $p_1 = \frac{1}{2}a + \frac{1}{2}\chi$
If $p_2 = \frac{1}{2}a + \frac{1}{2}\chi$ $p_1 = \frac{1}{2}a + \frac{1}{2}\chi - 1$ *is optimal if:*

$$\begin{aligned}
\left(\frac{1}{2}a + \frac{1}{2}\chi - \chi\right)\frac{1}{2}\left(\frac{a}{b} - \frac{1}{b}\left(\frac{1}{2}a + \frac{1}{2}\chi\right)\right) &< \left(\frac{1}{2}a + \frac{1}{2}\chi - 1 - \chi\right)\left(\frac{a}{b} - \frac{1}{b}\left(\frac{1}{2}a + \frac{1}{2}\chi - 1\right)\right) \\
\frac{1}{8b}(a - \chi)^2 &< \frac{1}{4b}\left(a^2 - 2a\chi + \chi^2 - 4\right) \\
0 &< \frac{1}{4b}\left(a^2 - 2a\chi + \chi^2 - 4\right) - \frac{1}{8b}(a - \chi)^2 \\
1 &< \frac{1}{8}(a - \chi)^2
\end{aligned}$$

which is true in all versions of the quiz.
Now if $p_2 = \chi + 2$ *then* $\chi + 1$ *is optimal if:*

$$\begin{aligned}
(\chi + 2 - \chi)\frac{1}{2}\left(\frac{a}{b} - \frac{1}{b}(\chi + 2)\right) &< (\chi + 1 - \chi)\left(\frac{a}{b} - \frac{1}{b}(\chi + 1)\right) \\
-\frac{1}{b}(\chi - a + 2) &< -\frac{1}{b}(\chi - a + 1)
\end{aligned}$$

which is always true.
If $p_2 = \chi + 1$ *then the three options are* $p_1 \in \{\chi + 2, \chi + 1, \chi\}$ *if* $p_1 = \chi + 2$ *then demand is zero, of* $p_1 = \chi$ *then the profit per unit is zero, thus the unique best response is* $p_1 = \chi + 1$.
If $p_2 = \chi$ *then obviously they do not want to price below marginal cost but if they price at marginal cost or strictly higher then they always get zero profits. Thus the best responses are* $\{\chi, \chi + 1, \chi + 2, ...\}$
If $p_2 < \chi$ *then they obviously do not want to match firm 2's price or beat it, because they will make a negative profit per unit. Thus their best responses are to get zero demand, which will happen if* $p_1 \in \{p_2 + 1, p_2 + 2, p_2 + 3, ...\}$.

In summary:

$$BR_1(p_2) = \begin{cases} \frac{1}{2}a + \frac{1}{2}\chi & \text{if} \quad p_2 > \frac{1}{2}a + \frac{1}{2}\chi \\ p_2 - 1 & \text{if} \quad \chi + 2 \leq p_2 \leq \frac{1}{2}a + \frac{1}{2}\chi \\ p_2 & \text{if} \quad p_2 = \chi + 1 \\ \{p_2, p_2 + 1, p_2 + 2, ...\} & \text{if} \quad p_2 = \chi \\ \{p_2 + 1, p_2 + 2, p_2 + 3...\} & \text{if} \quad p_2 < \chi \end{cases}$$

(b) *(3 points)* Find the Nash equilibria of this game.

Solution 5 *Obviously the best responses of firm 2 are the same, and equally obviously* $p_1 = p_2 = \chi + 1$ *is a Nash equilibrium because there is a unique best response which is to charge the same price.*

There is also a Nash equilibrium where $p_1 = p_2 = \chi$. A *best response to* $p_2 = \chi$ *is* $p_1 = \chi$ *and vice versa. Thus it is a Nash equilibrium.*

Remark 6 *The points here are mostly for the explanation, maybe give 1 point if they do not explain their answers.*

4. *(8 points)* List two of the four reasons for natural market power discussed in class, explain each one and give an example of an industry where it is an important factor. You may use the same industry more than once.

Solution 7 *(1 point for each reason, two points for the explanation, and 1 point for the industry). The four reasons for natural market power are:*

(a) Small Markets: In a few industries the fixed costs of production are so high that the minimum efficient scale (when $S = \frac{AC}{MC} \leq 1$*) requires at best a few firms. An example from the book is the market for Aspertame, another example would be large airplanes—where the market is dominated by Boeing and Airbus.*

(b) Always Decreasing Average Costs or Economies of Scale: These are industries where $S = \frac{AC}{MC} > 1$ *for all levels of outputs. Generally these are industries with high fixed costs (research and development) and very low marginal costs. The movie industry could be an example, as well as Drug manufacture, the quintisential example is any Software. Marginal cost is mostly bug fixing (which also has economies of scale). Advertisement would be a fixed cost.*

(c) Network Externalities: These are goods that, like a language, have a higher benefit of using if more people around you are using them. Movies are this type of good, but again Software is an extreme example. For example many journals these days accept submission in Microsoft Word since everyone uses it.

(d) Economies of the Market: Customers want to go where there is the greatest variety of goods (the most sellers) and Sellers want to go where there are the most Customers. This positive feedback loop

means there will be naturally one market. These days with online markets it is actually feasible to have only one market that sells everything. In the United States this market is Amazon (with fringe competition from online branches of Brick and Mortar stores). Turkey is currently at a fascinating point in its online business history because there are at least three local firms (Hepsiburada, Trendyol, n11) and two global firms (Amazon and Ali Baba).

They may try to list economies of scope which is included in this section. I would give them some partial credit for that, but not much. Economies of scope is a factor in almost every business but as a reason for having a few firms in an industry? Not really.

5. *(10 points)* Derive the Lerner index for a Monopoly firm with the inverse demand curve $P(Q)$ (with $P' < 0$) and the cost function $C(Q)$ (with $C' > 0$). What are the bounds for the Lerner Index? Why do we not use this structural measure to assess market concentration? Name and give the formula for two distinct alternative measures that are commonly used instead.

Solution 8 *The derivation is worth three points:*

$$\max_{Q} P(Q)\,Q - C(Q)$$

$$\begin{aligned} P + P'Q - C' &= 0 \\ P - MC &= -P'Q \\ \frac{P - MC}{P} &= -P'\frac{Q}{P} \end{aligned}$$

the left hand side is the Lerner index, but the right hand side can be expressed more elegantly as:

$$e_d = \frac{dQ}{dP}\frac{P}{Q} = \left(P'\frac{Q}{P}\right)^{-1}$$

thus the final form is:

$$LI = \frac{P - MC}{P} = \left|\frac{1}{e_d}\right|$$

the Lerner Index is between zero and one (1 point). We do not use this measure because it requires an estimate of marginal cost, which is difficult when one wants to analyze many different industries at once (2 points).

Remark 9 *Just for your information, if firm i is one of n firms then for them the forumla would be:*

$$\frac{P - MC_i}{P} = -P'\frac{q_i}{P} = -P'\frac{Q}{P}\frac{q_i}{Q} = \left|\frac{1}{e_d}\right|\frac{s_i}{100}$$

we can then sum this across the industry to get the Industry Lerner Index:

$$\sum_{i=1}^{n} \frac{P - MC_i}{P} = \left| \frac{1}{e_d} \right|$$

just a fun fact, I don't expect them to know this.

Instead we use either the four firm concentration ratio (CR_4) or the Herfindahl-Hirschman Index (HHI or HI), both are based on $s_i = \frac{q_i}{Q} * 100$. We order the firms so that $s_1 \geq s_2 \geq ...$ then:

$$\begin{aligned} CR_4 &= s_1 + s_2 + s_3 + s_4 = \sum_{i=1}^{4} s_i \in [0, 100] \\ HHI &= \sum_{i=1}^{n} s_i^2 \in [0, 10,000] \end{aligned}$$

(Four points, two for the names and two for the forumlas. If they try to use both CR_4 and $CR_8 = \sum_{i=1}^{8} s_i$ give them a maximum of two points. If they give the abbreviations for the names instead of the full name give them full credit)

a	b	χ_1	χ_2	χ_3	$q_1(q_2)$	$q_2(q_1)$	q_1^*	q_2^*	Q^*	$Q(3)$	*i shuts down*	$\frac{\partial \pi_i}{\partial q_i}\vert_{q_i=0}$
30	1	21	15	14	$\frac{9}{2} - \frac{1}{2}q_2$	$\frac{15}{2} - \frac{1}{2}q_1$	1	7	8	10	*1*	-1
25	1	12	2	5	$\frac{13}{2} - \frac{1}{2}q_2$	$\frac{23}{2} - \frac{1}{2}q_1$	1	11	12	14	*1*	-1
27	$\frac{1}{2}$	16	20	15	$11 - \frac{1}{2}q_2$	$7 - \frac{1}{2}q_1$	10	2	12	15	*2*	$-\frac{1}{2}$
22	$\frac{1}{2}$	7	13	6	$15 - \frac{1}{2}q_2$	$9 - \frac{1}{2}q_1$	14	2	16	20	*2*	-1

6. *(19 points total)* Consider an industry with two firms that compete by choosing quantity, the price is then set at the market clearing price (Cournot Competition). The inverse demand curve is $P = a - bQ$, where $Q = q_1 + q_2$, and $c_1(q) = \chi_1 q_1$, $c_2(q) = \chi_2 q_2$.

   (a) *(2 points)* Set up the objective function of both firms (to be clear, you need two *separate* objective functions.)

   Solution 10

$$\begin{aligned} \pi_1(q_1, q_2) &= (a - b(q_1 + q_2)) q_1 - \chi_1 q_1 \\ \pi_2(q_1, q_2) &= (a - b(q_1 + q_2)) q_2 - \chi_2 q_2 \end{aligned}$$

   (b) *(6 points)* Find the first order conditions and best responses for both firms.

Solution 11 *(1 point for the FOC, 2 for the BR)*

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_1} &= a - b\left(q_1 + q_2\right) - bq_1 - \chi_1 = 0 \\
q_1 &= \frac{1}{2b}\left(a - \chi_1\right) - \frac{1}{2}q_2
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_2} &= a - b\left(q_1 + q_2\right) - bq_2 - \chi_2 = 0 \\
q_2 &= \frac{1}{2b}\left(a - \chi_2\right) - \frac{1}{2}q_1
\end{aligned}$$

(c) *(6 points)* Find the total output and the output of each firm two different ways.

Solution 12 *(Grading: 1 point for each of the three quantities, 2 points for the first method, 1 point for the second)*
Method 1: intersect best responses:

$$\begin{aligned}
q_1 &= \frac{1}{2b}\left(a - \chi_1\right) - \frac{1}{2}\left(\frac{1}{2b}\left(a - \chi_2\right) - \frac{1}{2}q_1\right) \\
q_1 &= \frac{1}{4}q_1 + \frac{1}{4}\frac{a}{b} - \frac{1}{2b}\chi_1 + \frac{1}{4b}\chi_2 \\
q_1 &= \frac{1}{3b}\left(a - 2\chi_1 + \chi_2\right) \\
q_2 &= \frac{1}{3b}\left(a + \chi_1 - 2\chi_2\right) \\
Q &= q_1 + q_2 = \frac{1}{3b}\left(a - 2\chi_1 + \chi_2\right) + \frac{1}{3b}\left(a + \chi_1 - 2\chi_2\right) \\
&= \frac{1}{3b}\left(2a - \chi_1 - \chi_2\right)
\end{aligned}$$

Method 2: Summing the first order conditions:

$$\begin{aligned}
a - b\left(q_1 + q_2\right) - bq_1 - \chi_1 &= 0 \\
a - b\left(q_1 + q_2\right) - bq_2 - \chi_2 &= 0
\end{aligned}$$

$$\begin{aligned}
a - b\left(q_1 + q_2\right) - bq_1 - \chi_1 + a - b\left(q_1 + q_2\right) - bq_2 - \chi_2 &= 0 \\
2a - \chi_1 - \chi_2 - 3b\left(q_1 + q_2\right) &= 0 \\
2a - \chi_1 - \chi_2 - 3bQ &= 0
\end{aligned}$$

$$Q = \frac{1}{3b}\left(2a - \chi_1 - \chi_2\right)$$

$$a - b\left(\frac{1}{3b}\left(2a - \chi_1 - \chi_2\right)\right) - bq_1 - \chi_1 = 0$$

$$q_1 = \frac{1}{3b}\left(a - 2\chi_1 + \chi_2\right)$$

$$a - b\left(\frac{1}{3b}\left(2a - \chi_1 - \chi_2\right)\right) - bq_2 - \chi_2 = 0$$

$$q_2 = \frac{1}{3b}\left(a + \chi_1 - 2\chi_2\right)$$

Now a third firm is entering the industry, with $c_3(q) = \chi_3 q_3$ and now $Q = q_1 + q_2 + q_3$.

(d) *(2 points)* Set up the objective function of firm three and find it's first order condition.

$$\pi_3(q_3, Q) = (a - bQ)\, q_3 - \chi_3 q_3$$

$$\frac{\partial \pi_3}{\partial q_3} = (a - bQ) - bq_3 - \chi_3 = 0$$

(e) *(3 points)* Find the total output produced assuming all three firms produce, and then verify that one of the firms does *not* produce output, which one is it?

Solution 13 *(Grading: 1 point for the total output, one for guessing which firm shuts down, and one for verification. I expect few will get even a point from this part, so be kind.)*

$$\begin{aligned} a - bQ - bq_1 - \chi_1 &= 0 \\ a - bQ - bq_2 - \chi_2 &= 0 \\ a - bQ - bq_3 - \chi_3 &= 0 \end{aligned}$$

$$\begin{aligned} (a - bQ - bq_1 - \chi_1) + (a - bQ - bq_2 - \chi_2) + (a - bQ - bq_3 - \chi_3) &= \\ 3a - \chi_1 - \chi_2 - \chi_3 - bq_1 - bq_2 - bq_3 - 3Qb &= 0 \\ 3a - \chi_1 - \chi_2 - \chi_3 - bQ - 3Qb &= 0 \end{aligned}$$

$$Q = \frac{1}{4b}\left(3a - \chi_1 - \chi_2 - \chi_3\right)$$

and then one will find that either:

$$a - b\left(\frac{1}{4b}\left(3a - \chi_1 - \chi_2 - \chi_3\right)\right) - bq_1 - \chi_1 < 0$$

for all $q_1 \geq 0$*, which means that*

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_1}|_{q_1=0} &= a - b\left(\frac{1}{4b}\left(3a - \chi_1 - \chi_2 - \chi_3\right)\right) - \chi_1 < 0 \\ \frac{\partial \pi_1}{\partial q_1}|_{q_1=0} &= \frac{1}{4}a - \frac{3}{4}\chi_1 + \frac{1}{4}\chi_2 + \frac{1}{4}\chi_3 < 0 \end{aligned}$$

if it is firm two then:

$$a - b\left(\frac{1}{4b}\left(3a - \chi_1 - \chi_2 - \chi_3\right)\right) - bq_2 - \chi_2 < 0$$

for all $q_2 \geq 0$*, which likewise means that:*

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_2}|_{q_2=0} &= a - b\left(\frac{1}{4b}\left(3a - \chi_1 - \chi_2 - \chi_3\right)\right) - \chi_2 < 0 \\
\frac{\partial \pi_1}{\partial q_2}|_{q_2=0} &= \frac{1}{4}a + \frac{1}{4}\chi_1 - \frac{3}{4}\chi_2 + \frac{1}{4}\chi_3 < 0
\end{aligned}$$