ECON 433
Midterm Exam
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This exam will last 100 minutes.

Answer all questions in the space provided. Points will only be given for work shown.

1. *(18 points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I further promise to not assist others nor use any electronic device during the exam, specifically I know that calculators are not allowed.

   Name and Surname: $_____$

   Student ID: $_____$

   Signature:

   $_____$

2. *(10 points total)* Consider a Cournot oligopoly where there are n firms, the costs of firm i are $c_i(q) = c_i q_i$ and the inverse demand curve is $P = 16 - Q$. Where $Q = \sum_{i=1}^{n} q_i = q_1 + q_2 + q_3... + q_n$. Let $Q_{-i} = Q - q$. Firms objective is to maximize profit over quantity, $q_i \geq 0$.

   (a) *(4 points)* Find firm i's best response.

   $$\max_{q_i} \pi_i(q_i, Q_{-i}) = (16 - q_i - Q_{-i}) q_i - c_i q_i$$

   $$(16 - q_i - Q_{-i}) - q_i - c_i = 0$$

   $\vdots$

   $$q_i = 8 - \frac{1}{2}Q_{-i} - \frac{1}{2}c_i$$

   (b) *(3 points)* Find the equilibrium output for an arbitrary distribution of c_i's. You may assume all firms produce.

   There are two ways to do this, the easiest is to sum up the first order conditions:

   $$\Sigma_i\left[(16 - Q) - q_i - c_i\right] = 0$$
   $$n16 - nQ - \Sigma_i q_i - \Sigma_i c_i = 0$$

   Now $\Sigma_i q_i = Q$, and it is elegant to write $\Sigma_i c_i = n\bar{c}$ where $\bar{c} = \frac{1}{n}\Sigma_i c_i$ thus the condition can be written as:

   $$n16 - (n+1)Q - n\bar{c} = 0$$

   $$Q = \frac{n}{n+1}(16 - \bar{c})$$

The second way to do this is to sum the best responses:

$$\Sigma_i q_i = \Sigma_i 8 - \frac{1}{2}\Sigma_i Q_{-i} - \frac{1}{2}\Sigma_i c_i$$

$$Q = n8 - \frac{1}{2}n\bar{c} - \frac{1}{2}\Sigma_i Q_{-i}$$

Now $Q_{-i} = Q - q_i$

$$\Sigma_i Q_{-i} = nQ - Q$$

$$Q = n8 - \frac{1}{2}n\bar{c} - \frac{1}{2}(nQ - Q)$$

$$Q = \frac{n}{n+1}(16 - \bar{c})$$

(c) *(4 points)* Assume $n = 11$ and that there are three firms with $c_i(q) = 8q_i$ and eight firms with $c_j(q) = 28q_j$. Find the equilibrium output of each firm, the equilibrium price and (importantly) the profit of each firm in that equilibrium.

Again there are two approaches to doing this, the first is to notice that $28 > 16$ and proceed with this information, or that 8 of the firms will never produce. From above this tells us that

$$Q = \frac{3}{3+1}(16 - 8) = 6$$

$$P = 16 - 6 = 10$$

and we have confirmed our result. On the other hand we might not notice this and then:

$$Q = \frac{11}{11+1}\left(16 - \frac{3}{11}(8) - \frac{8}{11}(28)\right) = -6 < 0$$

and thus this cannot be a solution. I had no intention of making the second type of firm so extreme, but since it made the question easier I went with it.

Given this, we need to find the output of each firm, which is $q = \frac{Q}{n} = 2$ then their profits are:

$$\pi = (10 - 8)\,2 = 4$$

3. *(16 points)* About price discrimination, consider a monopolist who has (for simplicity) two consumers. The good type has a benefit of consuming of $B_g(q) = -q(q - 43)$ and thus the demand curve $q_g = \frac{43}{2} - \frac{1}{2}P$. The bad type has a benefit of consuming of of $B_b(q) = -q(q - 27)$ and thus the demand curve $q_b = \frac{27}{2} - \frac{1}{2}P$. The monopolist's objective is to maximize their profits given their costs of $c(Q) = 7Q$, where $Q = q_g + q_b$. Consumers can not resell the good to the other consumer.

In the simple model the monopolist chooses a per unit price, P, and customers have to buy what they want at that price.

(a) *(3 points)* Set up the monopolists objective function—note that you might find it simpler to maximize over price. It will not affect analysis.

I will do it both ways, with constant marginal cost

$$\pi = (P - c)\,Q$$

and if we set it up to maximize over price we notice that $Q = \frac{43}{2} - \frac{1}{2}P + \frac{27}{2} - \frac{1}{2}P = 35 - P$ then:

$$\pi = (P - 7)\,(35 - P)$$

if instead we decided to set it up over quantity then since $Q = 35 - P$ we know that $P = 35 - Q$ and our objective function is:

$$\pi = (35 - Q - 7)\,Q$$

(b) *(5 points)* Find the first order condition of the objective function you are working with, and then the profit maximizing per unit price (P).

If we price maximize:

$$
\begin{aligned}
(P - 7) - (35 - P) &= 0 \\
P &= 21
\end{aligned}
$$

If we quantity maximize:

$$
\begin{aligned}
(35 - Q - 7) - Q &= 0 \\
Q &= 14 \\
P &= 35 - (14) = 21
\end{aligned}
$$

Now we let consumers choose whether to consume or not, but the monopolist knows each consumers identity and value of the good and can use a two part tariff: (F_i, P). Where consumers pay F_i if they choose to consume.

(c) *(5 points)* What do we know about the optimal values of F_i and P? Will this outcome be socially optimal (or Pareto efficient)? If so, will it be fair? (NOTE: the last term has not been defined.)

This is first degree price discrimination, so $F_i = CS_i(q_i) = B_i(q_i) - Pq_i$ and price is set at marginal cost, or $P = 7$. It will be Pareto efficient because the correct amount of output will be produced, but it will not be "fair" because the monopolist gets all of the social surplus.

Finally now the monopolist does knows they have one type g and one type b consumer, but does not know which is which.

(d) *(3 points)* What do we now know about the optimal values of F_g and F_b? (You may assume the good customer will pay F_g and the bad F_b).

This is second degree price discrimination so we know that $F_b = CS_b(q_b) = B_b(q_b) - Pq_b$ and that F_g will be set so that the good type will be indifferent between paying F_g for q_g and paying F_b for q_b or:

$$B_q(q_q) - F_g = B_q(q_b) - F_b$$

$$F_g = B_q(q_q) - B_q(q_b) + F_b$$

4. *(39 points total)* It is undeniable that Microsoft has massive market power—between their operating system and popular office suite of software probably about 90% of the worlds work is done using their programs.

   (a) *(8 points)* List and discuss the two reasons to expect that it is a natural monopoly. Assume you work for the current Turkish government and your boss has asked for a quick summary of the issues.

   The two reasons are:

      i. Economies of scale or always decreasing average cost—*in most computer software, almost all of the costs are for research and development, thus we expect that these firms have always decreasing average cost.*

      ii. Network externalities—*for goods that are like languages, the more others consume these goods the higher benefit you will have. For example the reason most of us use Microsoft Windows is because if there is a problem with it we can quickly find someone to fix it.*

   (b) *(5 points)* Is there any other reason to think that the internet is giving rise to monopolies? If so give the definition and an example of an industry in which it applies.

   This is **Economies of the marketplace***, or the way consumers want to buy where there are the most sellers and vice-versa. This applies most strongly in online retail. Amazon and Ali baba are currently fighting to see which will be the general retailer for the world.*

   (c) *(10 points)* Both the EU and the United States have discussed how to regulate Microsoft. The two major ideas that have been made have been either breaking up Microsoft (making it sell each product—like excel and MS paint—as separate goods) or requiring them to make their source code open—so that anyone can replicate their software for free. Discuss both ideas, why are they good and why are they bad? For your information neither has been implemented.

   Breaking up Microsoft: The good in this approach is that the main way we can control Microsoft's monopoly is to break up the different goods they make. We cannot overcome their natural monoopoly, but we can stop them from expanding into every software niche. Why, indeed, should MS Office and Windows be sold by the same company?

   On the other hand the counter argument is that this limits the merging of computer programs, consumers like it when all their services

are included in the basic computer, and so these programs (which in the future might be considered necessary to be merged) should be allowed to merge with the main program now. As a trivial example of this, I used to get irfanview for pictures downloaded each time I got a new computer. Now everyone expects an image viewing program to be part of their basic computer package. As a non-trivial example, about 10 years ago it was widely understood that one of the risks of using Windows was that almost all computer viruses targetted Windows. Now Windows has embedded (and constantly updates) an anti-virus program into their software. Of course this has the negative that the original producer's market is drying up, but clearly this is a significant improvement for us.

Open source code: The reason for this is that Microsoft has a clear advantage when developing their software in that they actually know the exact code of the operating system. This allows them to use "back doors" that others cannot access. But again the negative is what I just mentioned, computer viruses. If they did this (and updated it whenever they updated their software) it would make it trival for hackers to figure out how to break Windows.

(d) *(16 points)* After hearing your explanation, your boss still doesn't understand. She asks what she, as a Turkish minister, should do about this. You need to provide two answers, one in which she imagines the Turkish government can control Microsoft and another where it recognizes it cannot.

There is one solution that does not require government control because it is a Pareto improvement. First you require them to set price at marginal cost (nearly zero in this case) and then you pay them the profits they would have made from standard monopoly pricing. This is, unfortunately, ridiculous, especially with a firm that probably third degree price discriminates. "So, how much were you planning on charging this year? And you sold how many units?"

A more feasible plan is average cost pricing, though this is not easy it at least guarentees the firm will make zero profit. Notice that this would severely discourage investment in software production.

Given this is not feasible because Microsoft could just abandon Turkey a simple and elegant solution would be to require that they invest most of their profits into software reasarch and development in Turkey. This would encourage research and development, and as long as the profit share is not too draconian will be accepted by Microsoft. Of course like average cost pricing it would discourage innovation in the long run, but not as much. Notice this does not address the deadweight loss at all.

5. *(17 points)* Consider a duopoly where one firm maximizes over price and the other over quantity. To be precise each has constant marginal costs

and the same symmetric demand curve $q_i = \alpha - \beta p_i + \mu\beta p_j$ where $i \in \{1, 2\}$ is one of the firms and $j \neq i \in \{1, 2\}$ is the other and $\alpha > 0$, $\beta > 0$ and $0 < \mu < 1$.

Let firm one choose quantity, then one can show that in equilibrium $\pi_1^* > \pi_2^*$, where π_i^* is the equilibrium profit of firm $i \in \{1, 2\}$.

(a) *(8 points)* Imagine an extensive game, first firms choose simultaneously whether to maximize over price or quantity then after that they simultaneoulsy choose either the optimal quantity or price. To be clear, firm 2 can choose quantity and firm one could choose price if they wished.

What will be the equilibrium of this game? How do you know?

To be precise I have not given you enough information let π_{xy} be the profit when firm i chooses x and the other chooses y.

The truth $\pi_{pp} < \pi_{qp} < \pi_{qq}$

In this case q is the unique dominant strategy equilibrium. To see this consider choosing p when your opponent is choosing q. I have given you the information that $\pi_{qq} > \pi_{qp} > \pi_{pq}$ thus you will choose q.

$\pi_{pp} > \pi_{qp}$ and $\pi_{qp} < \pi_{qq}$

In this case you have two pure strategy equilibria, like above you would never choose p when your opponent chooses q.

$\pi_{pp} < \pi_{qp}$ and $\pi_{qp} > \pi_{qq}$

in this case you would have a mixed strategy equilibrium

(b) *(9 points)* Assume firm one maximizes over quantity and firm two maximizes over price, to be clear they have no choice on this matter. Now, however, they can choose to either declare their strategy first or second. You should answer this question based on intuition, no mathematical work will earn any points.

    i. *(3 points)* Which will the price maximizing firm prefer? Will they want to go first or last? Explain.

    The would want to go last, one point is that in the dynamic version of the price competition it is the second mover who makes the highest profit. Another is that they can take advantage of firm 1's over production by raising their price a lot.

    ii. *(3 points)* Which will the quantity maximizing firm prefer? Will they want to go first or last? Explain.

    They would want to go first, for one thing because in the Stakleberg game the second mover does worse than the simultaneous game. For another they can push the other firm into a corner by over producing.

    iii. *(3 points)* Is their an equilibrium in pure strategies of this game? If so tell us what it is, if not explain why you think there is not.

Since above I hypothesized firm the price setting firm would naturally want to go last and the quantity first, this means there will be a pure strategy equilibrium where the quantity setting firm goes first and the price setting firm goes last.