

Practice Questions

Monopoly
ECON 433
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These questions are to help you prepare for the exams only. Do not turn them in. Questions with lower numbers were asked in more recent years.

1. Consider a market with one consumer and a monopolist who has absolute control over all fees in this market. The consumer's benefit is $B(q) = q(20 - q)$ and the monopolist's cost are $c(q) = 8q$.
 - (a) Assume that the monopolist can only set a per-unit price, P , so that the monopolist's revenue are $R(q) = Pq$.
 - i. Show that the Consumer's inverse demand curve in this market is $P = 20 - 2q$.
 - ii. Set up the monopolist's objective function.
 - iii. Find the price and quantity the monopolist will charge.
 - iv. Find the profit of the monopolist (π^m) and the consumer surplus of the consumer (CS^m).
 - (b) Now assume that the monopolist can charge a per-unit price of P but now can charge the consumer a fixed fee for purchasing, F , so that now the monopolist's revenue is $R(q) = F + Pq$ (if $q > 0$, if $q = 0$ then the monopolist get's no revenue.) Assume the consumer will buy from the monopolist even when she is indifferent between buying and not.
 - i. Set up the consumer's objective function and find the quantity they will buy for each (F, P) .
 - ii. Set up the monopolist's objective function and find the optimal amount of (F, P) , and also the quantity the consumer will buy.
 - iii. Find the profit of the monopolist (π^1) and the consumer surplus of the consumer (CS^1).
 - (c) Is either a per-unit price or a per-unit price plus a fixed fee Pareto Dominant in this market? Why or why not?
 - (d) We define Welfare in these markets as the sum of Consumer's surplus and profits. Is either a per-unit price or a per-unit price plus a fixed fee Welfare Dominant in this market? Why or why not?
2. About one reason for market power.
 - (a) Define a *network good* and give an example, being clear about why this good is a network good.

- (b) A *word-of-mouth good* is similar to a *network good*. Define a word of mouth good making clear the difference between this and a network good and give an example, being clear about why this good is a word of mouth good but not a network good.
 - (c) Explain why with network or word-of-mouth goods we should expect to see firms have market power.
3. Throughout this question assume the monopolist has constant marginal cost, there are two customers (whose type may or may not be observable), each customer maximizes $B_i(q) - p_i q_i$ if they decide to purchase (or their personal Consumer Surplus), and that the customers can not trade the good between themselves after they purchase. Without loss of generality you can assume the consumer's demand curves are linear.
- (a) First Degree Price Discrimination:
 - i. Define First Degree Price Discrimination in this model.
 - ii. Give a real world example of First Degree Price Discrimination. If this is not possible explain why not.
 - iii. Explain why the Pareto Efficient quantity will be produced, but that consumers would still prefer the situation where the monopolist can not price discriminate.
 - (b) Second Degree Price Discrimination:
 - i. Define Second Degree Price Discrimination in this model.
 - ii. Give a real world example of Second Degree Price Discrimination. If this is not possible explain why not.
 - iii. In this model explain when First Degree Price discrimination will Pareto Dominate Second Degree Price discrimination. Be sure to go through the incentives of each party.
 - (c) Third Degree Price Discrimination:
 - i. Define Third Degree Price Discrimination in this model.
 - ii. Give a real world example of Third Degree Price Discrimination. If this is not possible explain why not.
 - iii. State precisely when it will be Pareto Improving to allow Third Degree Price Discrimination instead of not allowing any price discrimination. Be sure to go through the incentives of each party.
4. If customer's maximize $B_i(q) - p_i q_i$ if they decide to purchase (or their personal Consumer Surplus) explain why a monopolist using first degree price discrimination will always produce the Pareto Efficient quantity. Explain why consumers will still prefer the market where the output is produced by competitive firms.

5. Show that $MR = P \left(1 + \frac{1}{\varepsilon}\right)$ where P is the price this firm is charging, and ε is the elasticity of this firm's demand. If you want to you can assume this firm is a monopolist.
6. Consider a monopolist who has two buyers (for simplicity think of them as two individuals) with the following inverse demand curves:

$$\begin{aligned} P_1 &= 49 - 6Q_1 \\ P_2 &= 25 - 6Q_2 \end{aligned}$$

these are also the marginal benefit curves of these consumers. The monopolist has the costs of $c(Q) = Q$, where $Q = Q_1 + Q_2$.

- (a) Assume that the monopolist can charge a fee $F_1(Q_1)$ to consumer 1 and $F_2(Q_2)$ to consumer 2, and that the consumers can not trade output between themselves.
 - i. What kind of price discrimination is this? Give an example (if possible) of a real world application.
 - ii. Show that the total benefit of the two consumers from purchasing this good is:

$$\begin{aligned} B_1(Q_1) &= 49Q_1 - 3Q_1^2 \\ B_2(Q_2) &= 25Q_2 - 3Q_2^2. \end{aligned}$$

You may find it helpful to use a graph to get the answer.

- iii. What will be the relationship between $F_1(Q_1)$ and $B_1(Q_1)$? Why is this?
 - iv. Find the profit maximizing amount to supply to both parties, compare it to the efficient quantity and explain this result.
- (b) Assume that now the monopolist is restricted to charging a per-unit price of P_1 to person 1 and P_2 to person 2. (So the total revenue of the monopolist is $R = P_1Q_1 + P_2Q_2$).
 - i. What kind of price discrimination is this? Give an example (if possible) of a real world application.
 - ii. Find the optimal price and quantity for the monopolist to sell to both people.
 - iii. Find the profits of the monopolist and the consumer surplus of both consumers in this equilibrium. *Hint: Very little calculation is required to find the consumer surplus.*
- (c) Assume now that the monopolist is restricted to charging a per-unit price of P to both people. (So the total revenue of the monopolist is $R = PQ_1 + PQ_2$).
 - i. Show that the aggregate inverse demand curve is $P(Q) = 37 - 3Q$ when both people buy.

- ii. When will both people buy? If one person does not buy who will not? Find the aggregate inverse demand curve when only one person buys.
 - iii. Find the profit maximizing price and the quantities both people buy in equilibrium. Be sure to check whether both consumers are buying at your equilibrium.
 - iv. Find the profits of the monopolist and the consumer surplus of both consumers in this equilibrium. *Hint: Very little calculation is required to find the consumer surplus.*
 - v. Compare the profit and consumer surpluses of both consumers in parts *b* and *c* of this question. For each party state which one they prefer. Is one of them Pareto Dominant? Why or why not?
7. Define each of the three types of price discrimination. Which one is never applied in the real world? Why? For the other two give an example of a good where this type of price discrimination is applied. Be sure to explain your answer.
8. A monopolist has two different type of demanders (high and low, high have a subscript h on their variables, low have a subscript l on their variables.)

$$P_h = 96 - Q_h$$

$$P_l = 36 - Q_l$$

and the cost function $C(q) = 4q$.

- (a) If they can price discriminate:
 - i. Set up the firm's objective function.
 - ii. Find it's first derivatives.
 - iii. Find the price they would charge high and low demand customers.
 - (b) If they can not price discriminate what is their aggregate demand curve?
 - (c) If they can not price discriminate:
 - i. Set up the firm's objective function.
 - ii. Find it's first derivative.
 - iii. Find the price they would charge.
 - (d) Is it Pareto improving to allow price discrimination in this market? Why or why not? Be sure to consider all three different parties, high demand consumers, low demand consumers, and the monopolist.
9. In first degree or perfect price discrimination:

- (a) What is the consumer surplus?
 - (b) How does the profit maximizing level of output compare to the efficient level of output?
 - (c) Explain why this happens and how this can be considered a Pareto Efficient outcome.
10. Consider a monopolist who sells concert tickets to Students and the General Public. The demand curve of the general public is $q_g = 72 - 3P_g$ and of students is $q_s = 8 - P_s$; the costs of the monopolist are $c(Q) = 4Q$.
- (a) What is the aggregate demand if both groups are charged the same price?
 - (b) Assuming that the monopolist must charge the same price to both groups and both groups buy what is the aggregate quantity and price they will sell at?
 - (c) Find the quantity students will buy and the quantity the general public will buy at the price you found in part b.
 - (d) If the monopolist only sells tickets to the general public what price will they charge? Will the students want to buy at this price?
 - (e) What is the profit maximizing price and quantity if the monopolist must charge the same price to both groups? Explain your answer.
 - (f) Now assume that all students have ID's and the monopolist can charge a different price to students. What price will it charge and quantity will it sell to students and the general public?
Define welfare as $W = CS_g + CS_s + \Pi_m$, where CS_g is the consumer surplus of the general public, CS_s is the consumer surplus of the students, and Π_m is the profit of the monopolist.
 - (g) Without doing any calculations, did welfare increase or decrease when the monopolist used third degree price discrimination? Is it a Pareto improvement to allow the monopolist to use third degree price discrimination in this market?
11. Assume that a monopolist faces a demand curve of $Q(P) = P^{-e}$ and has a cost function of $c(Q) = cQ$.
- (a) Prove that $MR = P \left(1 + \frac{1}{e_d(P)}\right)$ where $e_d(P)$ is the elasticity of demand with regards to price. The proof should be true for general demand curves, not just the demand curve above.
 - (b) Show that for the demand curve above $e_d(P) = -e$. What do we know about e since a monopoly supplies this market?
 - (c) Find a formula that determines the market price as a function of e and c without setting up the objective function or finding the first order condition.

- (d) Show that if there is a per unit tax of t in this market the price the buyer pays increases more than the tax, or that $\frac{\partial P_b}{\partial t} > 1$.
12. Assume that the demand for Cell Phones is given by $Q = 120 - 2P$. In this question we will consider two methods of supplying this market: Monopoly and Perfect competition.
- (a) In this part you should consider only one Monopoly provider. This firm has a cost function of $c(Q) = 2Q$.
- Find the Inverse Demand Curve.
 - Set up the Monopolists objective function, be sure that is a function only of Quantity.
 - Find the Monopolists First order condition.
 - Solve for the optimal Quantity and Price in this market.
- (b) In this part you should consider only 12 perfectly competitive suppliers. These suppliers all have the same cost function $c(q) = 6q^2 + 54$. For simplicity all fixed costs are fixed start up costs, these costs do not have to be paid if no output is produced.
- Find one of these firm's marginal cost and average variable cost.
 - Find the lowest quantity at which these firms will not produce. What is their marginal cost at this quantity?
 - Find one firm's supply curve.
 - Find the industry's short run supply curve.
 - Find the Price and Quantity that will be produced in this market.
- (c) In this part you will compare the two ways of supplying this market.
- Which way produces more output? (Monopoly in part a or Perfect Competition in part b).
 - If the government allowed both the Monopolist and the Perfect Competitors to supply this market, would the Perfect Competitors produce any output?
 - Which method is Pareto Dominant in this market, Monopoly or Perfect Competition? Is there a way to make Perfect Competition Pareto Dominant?
 - Assume that the Monopolist has lower costs because they did Research and Development and invented a radically new way to produce cell phones. What type of goods are inventions? Why might the government not want to force the Monopolist to share her technology with other firms? *The answer to this question requires information from the chapter on Public Goods and Externalities.*
13. Since the Turkish government needs to raise revenue it wants to make the maximum possible profits off of the millions of historical sights in Turkey.

It faces two different demand curves, one for foreigners and one for Turks (or residents of Turkey :-)). The inverse demand curves for foreign visitors is $P_f = 120 - q_f$ —where q_f is the number of foreign visitors and P_f is the price they pay; and for Turks is $P_t = 60 - 3q_t$ —here q_t is the number of Turks and P_t is the price. The government has a cost of 6 per visitor, or their Total costs are $6Q$, $Q = q_f + q_t$.

- (a) First consider the case where they have to charge the same price to Turks and foreigners.
 - i. Find the aggregate demand curve assuming both types pay the same price.
 - ii. Find the price and quantity of tickets they will sell.
 - iii. Find the consumer surplus and profits.
 - (b) Now consider the real situation, where they charge a separate price to Turks and foreigners
 - i. Find a relationship between the quantities of tickets they sell to foreigners and the quantity they sell to Turks that is independent of costs.
 - ii. Explain how the relationship above is an immediate implication of profit maximization, or the choices the monopolists has.
 - iii. Find the quantity that the monopolist will sell in each market, and the price.
 - iv. Find the total consumer surplus and the profits.
 - (c) Which situation increases total consumer surplus, one price or two?
 - (d) Why must the monopolist do better when they can charge two prices?
14. Giggles the Great Ape Grape Grower owns the only farm around that can grow decent grapes. Since this makes her a monopolist the local high mucky-mucks want you to tell them how to regulate her. The demand curve for grapes is

$$Q = 40 - 10P$$

And her total cost is

$$TC = 30$$

- (a) What is her Marginal Revenue? What is her Marginal Cost?
- (b) What would be the price and quantity she would produce if she were not regulated?
- (c) What is the Pareto efficient price ceiling to enforce? If you force her to sell at this price, how much will she produce?
- (d) If the mucky mucks make it very clear to you that they will not subsidize Giggles, what price should you force her to sell at?

- (e) If you also take the idea of Pareto efficiency seriously and only want to implement Pareto improving policies, what price should you force her to sell at? How does your answer change if the mucky mucks are willing to subsidize Giggles?
 - (f) If the mucky mucks make it very clear to you that they will not subsidize Giggles, what price should you force her to sell at?
15. (This is an oligopoly question, but I am putting it here for convenience.) In the following question, assume the mucky mucks have not asked you to regulate Giggles yet. Yukon Jack the Mountain Goat has just figured out how to terrace the mountain sides and grow great grapes there. His total cost is:

$$TC = q_j$$

- (a) How much output will he produce given the production of Giggles (q_g)? (Or what is his reaction function.)
 - (b) If Giggles expects him to enter, how much will she choose to commit to producing before he does? How much will Yukon Jack produce once he enters?
 - (c) If Giggles can not commit to any level of production before Yukon Jack decides on how much to produce, how much should she produce? How much will Yukon Jack Produce?
 - (d) How much would she pay him for not entering in both scenarios? Would he be willing to accept it?
 - (e) If the high mucky mucks asked you to regulate the industry (and will do anything you ask for), how much output would you have Yukon Jack produce assuming that you do have Giggles produce some output? Explain your answer.
16. The Houston Museum of Natural Science faces two different demand curves. Children really want to go, but don't have any money. Their demand curve is

$$Q_c = 15 - 3P$$

Adults, while they have a lot of money, don't really want to go¹ and their demand curve is:

$$Q_a = 7 - P$$

The Museum's total cost is:

$$TC = Q + 21$$

where Q is the total number of visitors, $Q = Q_c + Q_a$. They have asked you to tell them whether they should use third degree price discrimination or not. (Assume that they act as profit maximizers.)

¹Or, should I say, are not willing to say that they want to go.

- (a) If they do third degree price discriminate, what price would they charge to the children? The adults?
- (b) What would be the dead weight loss? The Profit?
- (c) If they only sell one ticket to whomever comes up what price would they charge?
- (d) What is the dead weight loss? The Profit?
- (e) What would you recommend to them, why? Compare dead weight loss and profit of each option in your answer.