

Practice Questions

Oligopoly

ECON 433

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These questions are to help you prepare for the exams only. Do not turn them in. These are all old exam questions. Questions with lower numbers were asked in more recent years.

1 From ECON204 (Micro Theory II)

1. Consider a Cournot oligopoly with J firms where the inverse demand curve is $P = 49 - 3Q$ and all firms have the same cost function, $c(q) = 13q$.
 - (a) If $J = 2$:
 - i. Set up the objective function of firm 1.
 - ii. Find the best response of firm 1.
 - iii. Why do you not need to find the best response of firm 2?
 - iv. Find the Nash equilibrium output of each firm, and the market price.
 - (b) If $J = 11$:
 - i. Set up the objective function of firm 1 using $Q_{-1} = \sum_{j=2}^{11} q_j$ for the total output of all other firms.
 - ii. Find the best response of firm 1 as a function of Q_{-1} .
 - iii. Find the Nash equilibrium output of each firm, and the market price.
2. Consider a Hotelling location model. There are two firms (a and b) and price is fixed and strictly above the constant marginal cost of either firm. There are $L = \{1, 2, 3, \dots, 6\}$ locations and both firms choose a location, $l_a \in L$ and $l_b \in L$. Consumers are at a location and buy from the store that is closest to them, or consumer $i \in I$ at location $l_i \in L$ goes to a if $(l_a - l_i)^2 < (l_b - l_i)^2$, to b if $(l_a - l_i)^2 > (l_b - l_i)^2$ and visit either store with probability $\frac{1}{2}$ if $(l_a - l_i)^2 = (l_b - l_i)^2$. $D_a(l_a, l_b)$ is the total number of customers (on average) that buy from firm a when the location of firm a is l_a and the location of firm b is l_b . The number of consumers at each location are:

1	2	3	4	5	6	Total
10	8	4	8	10	2	42

- (a) Find the best response of firm a to each location of firm b . For location 3 find the demand of firm one ($D_a(l_a, l_b)$) for each location

she can choose, for the rest of the locations you only need to fill in the demand enough to convince me you have found the best response.

if $l_b =$	1	2	3	4	5	6
$BR_a(l_b) =$						
$D_a(1, l_b) =$						
$D_a(2, l_b) =$						
$D_a(3, l_b) =$						
$D_a(4, l_b) =$						
$D_a(5, l_b) =$						
$D_a(6, l_b) =$	_____	_____	_____	_____	_____	_____

- (b) Explain why for a general Hotelling model $BR_a(l_b) \in \{l_b - 1, l_b, l_b + 1\}$.
- (c) Explain why you do not need to find the best responses for firm b .
- (d) Find the Nash equilibrium locations of firms a and b .
3. Sami's Superb Strawberries are famous throughout all of Turkey. No one buys strawberries that aren't grown by Sami the Snake. He owns all of the lowlands used in Turkey for growing strawberries, and has the cost function

$$c_s(q_s, k_s) = \begin{cases} 2q_s & \text{if } q_s \leq k_s \\ \infty & \text{if } q_s > k_s \end{cases}.$$

Now Yoruk the Yalya Yilan (OK, he's still a snake, but he's a highland meadow snake) has realized that mountain strawberries are more delicious than lowland strawberries and decided to enter the market. His costs, of course, are higher, they are:

$$c_y(q_y, k_y) = \begin{cases} 18q_y + 12 & \text{if } q_y \leq k_y \\ \infty & \text{if } q_y > k_y \end{cases}$$

Even though Yoruk may be right about mountain strawberries being more delicious Turks still treat them as perfect substitutes for Sami's strawberries, resulting in the inverse demand curve of:

$$P = 58 - 2Q$$

where $Q = q_s + q_y$. Throughout this problem we assume that firm(s) choose their capacity first, and then choose their price as Bertrand competitors given their fixed capacity. (*Note: You may assume that in every variation of the problem below both Sami and Yoruk produce a positive amount of output.*)

- (a) What theoretic model tells us that we can treat this as a Cournot or Stackelberg model where firms just compete by choosing capacities? Explain the intuition of this model. **You may just assume this throughout the rest of the question even if you can not answer this question.**

- (b) Now assume that both of these firms choose their capacities simultaneously, or they act as Cournot competitors.
- Set up Sami's Objective Function and find his first order conditions.
 - Find Sami's Best Response or Reaction Function.
 - Set up Yoruk's Objective Function and find his first order conditions.
 - Find Yoruk's Best Response or Reaction Function.
 - Find the equilibrium quantity both of them produce and the market price.
- (c) Now assume that Sami chooses his capacity, and then Yoruk chooses his capacity, or these firms act as Stackelberg Competitors with Sami being the Stackelberg Leader.
- Why don't I have to ask you to set up Yoruk's Objective function or find his best response?
 - Set up Sami's Objective Function and find his first order conditions.
 - Find the equilibrium quantity Sami will produce.
 - Find Yoruk's equilibrium quantity and the market price.
- (d) Now assume that Yoruk chooses his capacity, and then Sami chooses his capacity, or these firms act as Stackelberg Competitors with Yoruk being the Stackelberg Leader.
- Why don't I have to ask you to set up Sami's Objective function or find his best response?
 - Set up Yoruk's Objective Function and find his first order conditions.
 - Find the equilibrium quantity Yoruk will produce.
 - Find Sami's equilibrium quantity and the market price.
- (e) It is natural to assume that Sami gets to choose his capacity first, so imagine that Yoruk has appealed to the Turkish Government to enforce either a level playing field (they both choose their capacities simultaneously, part *b*) or to force Sami to choose his capacity second (part *d*). Assuming that the Turkish government only cares about Consumer Surplus, will they agree to his request? Also tell me how they would order the three possible equilibria, the equilibrium in part *b*, *c*, and *d*. *Hint: No further calculation is required for this part. Indeed the answer could be based on a-priori logic.*
4. Consider a standard Hotelling model. There are a finite number of locations $l \in (1, 2, 3, 4, 5)$, and a fixed number of customers at each location given by this table:

1	2	3	4	5
8	6	2	16	2

There are two firms, firm A has location l_A and firm B has location l_B . (We allow $l_A = l_B$). For fixed locations (l_A, l_B) customers always buy from the firm that is the closest, and split their business if both firms are equally close. Firms choose their location to maximize the number of customers they have.

- (a) For each location of firm B , $l_B \in (1, 2, 3, 4, 5)$ find the best response of firm A . You can use the table below to help you, and at least for one location you should figure out the profit of each location for firm A .

$l_B =$	1	2	3	4	5
$BR_A(l_B) =$					
$\Pi_A(1, l_B) =$					
$\Pi_A(2, l_B) =$					
$\Pi_A(3, l_B) =$					
$\Pi_A(4, l_B) =$					
$\Pi_A(5, l_B) =$					

- (b) Why can you assume that the best responses of firm B are the same as the best responses of firm A ?
- (c) Find the Nash equilibrium of this game.
5. Giggles the Great Ape Grape Grower owns all the lowlands in Tanzania, the inverse demand curve for grapes is:

$$P = 16 - \frac{1}{2}Q$$

and his costs are $c_g(Q) = 6Q$.

- (a) At first she is a monopolist.
- Find Giggles's objective function.
 - Find her first order condition.
 - Find the quantity she will produce, the price she sells grapes for, and her profits. (For clarity you should assume she can only charge a per-unit price.)
- (b) Now Henry the Highland Goat has gotten interested in growing grapes. Unfortunately he has to grow them in the high hills, thus he has higher costs and a large fixed cost of setting up production. His cost function is $c_h(q_h) = 8q_h + 1$. At this point Giggles can not commit to any fixed level of production, so if Henry produces Henry and Giggles act as Cournot Competitors.
- Set up the objective function of both firms.
 - Find their first order conditions.
 - Find their best responses or reaction functions.

- iv. Find the equilibrium quantities they will produce, the market price, and the profits of both firms.
- (c) Now Giggles realizes she can commit to a fixed level of production, so she can effectively choose her level of output before Henry chooses his. In other words she can act as a Stackleberg leader.
- Explain the general principle that says this commitment will always have some value to Giggles. Your answer should apply to all oligopoly games, not just this particular game.
 - Explain why Henry's best response or reaction function will be the same in this game as it was in the Cournot game.
 - Find Giggles's new objective function.
 - Find her first order condition.
 - Find the quantity Giggles will produce, the quantity Henry will produce, and the market price.
 - Find the profits of both firms in this game, use this to explain the value of commitment to Giggles.
- (d) Now assume that Henry's fixed costs are 6 instead of 1. Explain why you do not need to recalculate Henry's best responses when you make this change. Given this find the equilibrium of both games you just analyzed (the Cournot and Stackleberg), and use the profits in equilibrium to express Giggles's **new** value to being able to commit.
6. Consider a standard Hotelling model. There are a finite number of locations $l \in (1, 2, 3, 4, 5)$, and a fixed number of customers at each location given by this table:

1	2	3	4	5
6	12	8	4	4

There are two firms, firm A has location l_A and firm B has location l_B . (We allow $l_A = l_B$). For fixed locations (l_A, l_B) customers always buy from the firm that is the closest, and split their business if both firms are equally close. Firms choose their location to maximize the number of customers they have.

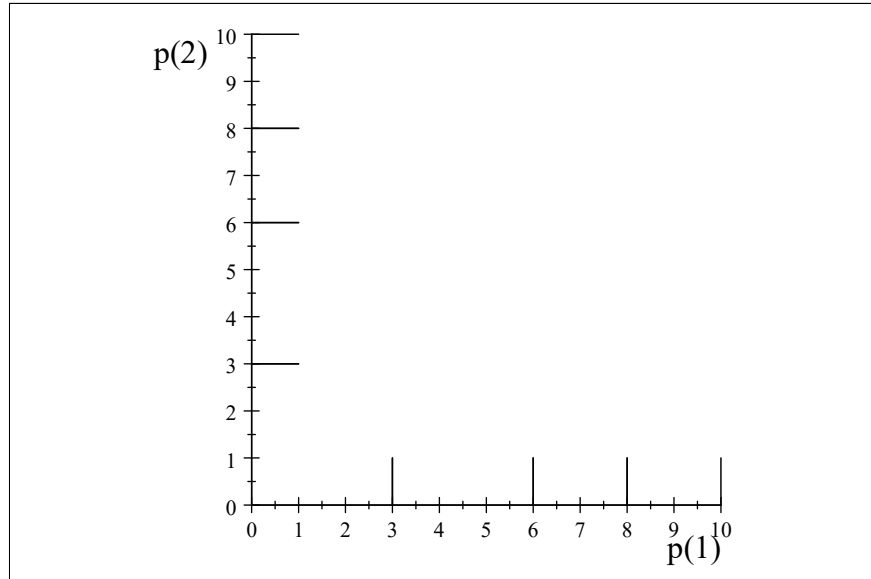
- (a) For each location of firm B , $l_B \in (1, 2, 3, 4, 5)$ find the best response of firm A . You can use the table below to help you, and at least for one location you should figure out the profit of each location for firm A .

$l_B =$	1	2	3	4	5
$BR_A(l_B) =$					
$\Pi_A(1, l_B) =$					
$\Pi_A(2, l_B) =$					
$\Pi_A(3, l_B) =$					
$\Pi_A(4, l_B) =$					
$\Pi_A(5, l_B) =$					

- (b) Why can you assume that the best responses of firm B are the same as the best responses of firm A ?
 - (c) Find the Nash equilibrium of this game.
 - (d) Why can't the Nash equilibrium be welfare optimal if your welfare function decreases in the distance customers have to travel?
7. Consider two firms that are competing in a market by choosing their quantity, the price is then set to clear the market. The costs of both firms are $c_1(q_1) = 12q_1$ and $c_2(q_2) = 12q_2$ and the market demand curve is $P = 24 - \frac{1}{2}Q$, where $Q = q_1 + q_2$.
- (a) Assume that these firms choose quantity simultaneously, or act as Cournot Competitors.
 - i. Set up the objective function of one of the firms.
 - ii. Find that firm's reaction function.
 - iii. Why didn't I ask you to set up the objective function of both firms? How can this help you solve the rest of this problem?
 - iv. Find the equilibrium quantity produced by both firms and the equilibrium price.
 - v. Find the profit of both firms in equilibrium.
 - (b) Now assume that firm 1 is the incumbent, and assume that means they get to choose their output first, then firm 2 gets to choose their output after they have seen the output of firm 1.
 - i. Find the reaction function of firm 2. *Hint: No calculation is required to do this.*
 - ii. Set up firm 1's objective function.
 - iii. Find the equilibrium quantity produced by both firms and the equilibrium price.
 - iv. Find the profit of both firms in equilibrium.
 - (c) Now assume that there is a fixed start up cost of $F = 60$, briefly explain how the equilibrium in both models would change.
8. Consider the standard Bertrand model, where firms split demand if they charge the same price and get all of demand if they charge a strictly lower price than their competitors. Assume that all firms have the same constant marginal cost.
- (a) What will be the equilibrium price?
 - (b) Explain why this will be the equilibrium.
 - (c) What is the underlying assumption of the model that makes this the equilibrium? Is this assumption reasonable? Why or why not?

9. Assume that there are two firms who act by choosing quantities, price is then set so that the market clears. The demand curve in their market is $P = 48 - Q$ and the costs for firm one are $c_1(q_1) = 8q_1$ and for firm 2 are $c_2(q_2) = 16q_2$.
- (a) If they act as Cournot Competitors:
 - i. Find the objective functions for both firms.
 - ii. Find the first order conditions for both firms.
 - iii. Find the reaction functions for both firms.
 - iv. Find the quantity each firm produces in equilibrium, the market quantity, the price, and both firm's profits.
 - (b) If they act as Stackelberg competitors, firm 1 choosing quantity first.
 - i. Find firm 2's reaction function.
 - ii. Find firm 1's objective function.
 - iii. Find firm 1's first order condition.
 - iv. Find the quantity firms 1 and 2 produce in equilibrium, the market price, and the profit of both firms.
 - v. Without any mathematical analysis guess at whether the consumers would prefer the high cost or the low cost firm choosing quantity first in this model. Explain your reasoning.
 - (c) Is the Stackelberg model a Pareto Improvement over the Cournot Model? Who is better off with the equilibrium in the Stackelberg model? Who is worse off? Explain your reasoning. (Please consider all three parties, the two firms and the consumers.)
10. Consider a model of Differentiated Bertrand, where both firms choose the price that maximizes their profits and meet the quantity demanded at that price. The demand of firm one is $q_1 = 44 - 11p_1 + 11p_2$ and of firm two is $q_2 = 44 - 11p_2 + 11p_1$ and the cost function is $c(q) = 2q$.
- (a) Set up firm 2's profit maximization problem and find his best response. Use symmetry to find the best response of firm 1. Graph

their best responses in the graph below.



- (b) Find the equilibrium price both firms charge, the quantity they sell, and the profits of both firms.
Now consider a variation on the model where firm 1 sets its price first, and then firm 2 sets its price after observing firm 1's price.
 - (c) Argue that firm 2's best response will be the same as in the first part of this problem.
 - (d) Using this set up firm 1's problem and find the price it will charge and the quantity it will sell. Also do this for firm 2 and find the profits of both firms.
 - (e) Would both firms be willing to sign a legally binding document making sure that firm 1 chooses its price first and will not change its price after firm 2 sets its price? How much would firm 2 be willing to pay to be the second firm to set their price? Is there any way that these firms could implicitly come to such an agreement?
11. Assume there are two Cournot Competitors in a market with a demand curve of $Q = 144 - 3P$. Firm one has the cost function of $C_1(q_1) = 2q_1$, firm 2 has a cost function of $C_2(q_2) = q_2$.
- (a) Set up both firms' objective functions.
 - (b) Find the first order conditions and their reaction functions.
 - (c) Find the quantity produced and the price in the Cournot equilibrium.
 - (d) What is the Pareto Efficient price? Which firm will not produce if the two firms produce efficiently?

- (e) Discuss the two types of inefficiencies in a Cournot Equilibrium, using this question to illustrate both of them.
12. Assume that a monopolist faces the demand curve $Q = 32 - 2P$, and that their cost function is $c(q) = 30 + 4q$. (Note that all fixed costs are start up costs.)
- (a) Find the monopoly quantity and price.
- What is their marginal revenue (Notice this should be in terms of Q , not P).
 - What Quantity will they produce?
 - What price will they sell at?
 - What Profits will they make?
- (b) Assume there is a new firm with a new technology that enters the market, they have a cost function $c(q) = 4q$. Assuming that the incumbent knows about this entry (and therefore will choose their quantity first) find the quantities produced by the entrant and the incumbent and the market price.
- Find what quantity the entrant will produce as a function of the quantity the incumbent produces.
 - Find the entrant's marginal revenue.
 - Find the entrant's reaction function.
 - Find what quantity the incumbent will produce.
 - Set up the incumbent's objective function.
 - Solve the first order condition for the quantity they will produce.
 - What will be the price in this market?
 - What will be the profits of the incumbent and the entrant?
- (c) Assume that the incumbent's fixed start up costs are higher, specifically that they are 70 instead of 30. What will happen when the entrant tries to enter? I do not want to know what the price and quantity will be, just explain what will occur. Answering this question does not require an answer to part b, a good guess well defended by an argument will be sufficient.
13. Tekel used to be the monopoly producer of Raki in Turkey. Assume that the demand curve is $Q = 60 - P$ and that Tekel's costs are $c_t(q_t) = 8q_t$.
- (a) When Tekel is a Monopolist:
- Find Tekel's objective function and their first order condition.
 - Find the optimal quantity for Tekel to produce, and the price it will sell Raki at.

- (b) Now assume that the Turkish government has decided to allow one more firm into the market for Raki, Efe, and that Efe's cost is $c_e(q_e) = 4q_e$. Assume that these firms are Cournot Competitors.
- Set up the objective function of Tekel and Efe.
 - Find their first order conditions and their reaction functions.
 - Find the Cournot-Nash Equilibrium quantities.
- (c) Now assume that Tekel, as the incumbent firm, chooses their production level first and that Efe chooses their production level second. Or that these firms are in a Stackleberg game, with Tekel as the first mover.
- Set up the objective function of Efe.
 - Find Efe's first order conditions and its reaction function.
 - Set up Tekel's objective function incorporating the reaction function of Efe.
 - Find Tekel's first order condition and how much Tekel will produce.
 - Find out how much Efe will produce.
 - If Efe was the incumbent firm do you think that more or less output would be produced? Why is assuming that Tekel has higher marginal costs than Efe reasonable?
- (d) Assume that Tekel's profits are used to fund government expenditures and that the government cannot tax Efe. Why might having both Tekel and Efe produce not be Pareto Superior to giving Tekel a monopoly?

14. Assume there are three firms that face a demand curve for Baklava of:

$$Q = 280 - P$$

each firm has a cost function of $c(q) = 2q^2$.

- (a) (Competition) Assuming that the firms act as if they are perfect competitors:
- Solve for their short run supply curve. BE SURE TO FIND THE SHUTDOWN POINT! (The answer is a little surprising but you need to show how to do it.)
 - Solve for the aggregate supply curve.
 - Solve for the equilibrium price and quantity.
- (b) (Cartel) Assuming that the firms collude to produce the monopoly output:
- Using *only* their cost functions solve for a relationship between the firm's outputs. Is this relationship the same as when they acted like perfect competitors?

- ii. Solve for the monopoly output.
- iii. Solve for the price they will sell at.
- (c) (Cournot) Assuming each firm chooses the quantity that maximizes their profit given the other firm's outputs:
 - i. How is the concept of *symmetry* useful in solving the following problem? Give a general definition of this concept.
 - ii. Solve for one firm's reaction function.
 - iii. Find the Cournot-Nash equilibrium outputs.
- 15. In the market for Cranberries there are two producers, market demand is given by:

$$Q = 126 - 6P$$

- (a) Assume that firm one has a total cost of $c(q_1) = 0$ and that firm two has a total cost of $c(q_2) = 6q_2$, what is the Cournot-Nash equilibrium? What is the price that goods are sold at?
- (b) What is elasticity of demand at the Cournot-Nash equilibrium?
- (c) Assume that you are a consultant for the prosecution in a federal anti-trust case. The firm under question points out that demand is inelastic, and argues that therefore they have no market power. Given this analysis, how would you respond?

2 From ECON439 (Game Theory)

- 1. Consider a Cournot Oligopoly where the inverse demand curve is given by $P = 17 - Q$ and the costs of a type a firm is $c_a(q^a, q^b) = q^a$ and the costs of a type b firm is $c_b(q^a, q^b) = 3q^b$.
 - (a) In this part of the question assume that there is one firm of both types.
 - i. Set up the objective function of both firms.
 - ii. Find the best responses of both firms.
 - iii. Find the Nash equilibrium quantities.
 - iv. Find the profit of both firms in the Nash equilibrium.
 - (b) Now assume that there are two firms of type b , firm 1 and firm 2, firm 1 produces q_1^b and firm 2 produces q_2^b .
 - i. Set up the objective function of both types of firms.
 - ii. Find the best responses of both types of firms.
 - iii. Why can you assume that $q_1^b = q_2^b$ in equilibrium?
 - iv. Using the insight in the last part of the question, find the Nash equilibrium quantities.

- v. Find the profit of both types of firms in the Nash equilibrium.
- (c) Now assume that the costs of firms of type b is $c_b(q^a, q^b) = 3q^b + F$, where F is a fixed cost. Further consider a *free entry equilibrium* where as many firms of type b can enter as want. (Note that only one firm of type a will be in the market, and that the costs of that type of firm do not change.) This means that for firms of type b , $\pi_i^b = 0$ in equilibrium.
 - i. If $F = 9$, what will be the equilibrium number of firms of type b ? Why?
 - ii. If $F = 20$, what will be the equilibrium number of firms of type b ? What will be total quantity produced by firms in the market?