Practice Questions—Chapters 14.

Monopoly
ECON 204
Kevin Hasker

These questions are to help you prepare for the exams only. Do not turn them in. Note that not all questions can be completely answered using the material in the chapter in which they are asked. These are all old exam questions and often the answers will require material from more than one chapter. Questions with lower numbers were asked in more recent years. The order of the chapters corresponds to the chapter numbers in the 10^{th} edition of Snyder and Nicholson.

1 Chp 14—Monopoly

1. Throughout this question assume the monopolist has constant marginal cost, their are two customers (whose type may or may not be observable), each customer maximize $B_i(q) - p_i q_i$ if they decide to purchase (or their personal Consumer Surplus), and that the customers can not trade the good between themselves after they purchase. Without loss of generality you can assume the consumer's demand curves are linear.

   (a) First Degree Price Discrimination:

      i. Define First Degree Price Discrimination in this model.
      First degree price discrimination is charging each person a personalized price, $F_i(q_i)$, it is sufficient to make $F_i(q_i) = F_i + pq_i$, or to charge them a fixed fee that depends on the person and a constant price for each unit they purchase.

      ii. Give a real world example of First Degree Price Discrimination. If this is not possible explain why not.
      There are no real world examples of first degree price discrimination because it requires too much knowledge on the part of the monopolist.

      iii. Explain why the Pareto Efficient quantity will be produced, but that consumers would still prefer the situation where the monopolist can not price discriminate.
      Since the monopolist can charge an individual specific fixed fee they will get all of the consumer's surplus from this market. This means that their objective function is the same as welfare maximization and they will choose a per-unit price that will maximize welfare, or $p = mc$. Consumers would still prefer the situation where the monopolist can not price discriminate because then they will get consumer surplus (or at least some of them will) and this is strictly better for them than this equilibrium.

(b) Second Degree Price Discrimination:

   i. Define Second Degree Price Discrimination in this model.
   Second degree price discrimination is when they charge an $F(q)$*, where they can not observe who you are but they can charge you a different per-unit price. In this case (since there are two consumers)* $F(q) = F + pq$

   ii. Give a real world example of Second Degree Price Discrimination. If this is not possible explain why not.
   There are so many examples its hard to choose. Usually laundry soap has a lower price per kilo in larger bags. A student (Burak) pointed out that sometimes olive oil is more expensive per kilo in larger containers. Value menus means, essentially, that they give you fries and a drink for much less than buying the goods separately. The list goes on and on.

   iii. In this model explain when First Degree Price discrimination will Pareto Dominate Second Degree Price discrimination. Be sure to go through the incentives of each party.
   The monopolist will obviously prefer First degree, but in First degree consumers get zero consumer surplus, so we need to make sure that no consumer wants Second degree. The only way this could happen is if only the highest CS type was sold to in second degree, in this case consumers don't care because all of them are getting zero consumer surplus.

(c) Third Degree Price Discrimination:

   i. Define Third Degree Price Discrimination in this model.
   It is a different constant per-unit price that depends on type, P_1 *and* P_2.

   ii. Give a real world example of Third Degree Price Discrimination. If this is not possible explain why not.
   Again we are beggared for examples. Movie theatres often have a discount for students. Many attractions also have discounts for senior citizens and etceteras.

   iii. State precisely when it will be Pareto Improving to allow Third Degree Price Discrimination instead of not allowing any price discrimination. Be sure to go through the incentives of each party.
   Again the monopolist always wants to price discriminate. However we have to make sure that consumers either don't care or want price discrimination. If the price without discrimination is too high for one type to buy then they will want price discrimination. And in this case since the monopolist is only selling to the high types the high types will face the same price whether there is price discrimination or not. So they will not care, and everyone wants price discrimination.

2. One of the technical barriers to entry is described as *small markets*, define this technical barrier to entry and explain why it is not that important as an explanation of Monopoly in the modern world, but it may be important as an explanation of Monopolistic Competition.

   Small markets merely means that if more than one firm enters a market then their profits will be negative. Given the low barriers to trade between countries this is no longer very relevant. For example historically in the US the big five domestic producers were responsible for almost all car sales in the US, now their share is less than 50%. On the other hand free entry guarantees that price will be at average cost, or that if there is more entry then price will be below average cost. This is the model of Monopolistic competition, which is a good explanation of competition in many markets today.

3. If customer's maximize $B_i(q) - p_i q_i$ if they decide to purchase (or their personal Consumer Surplus) explain why a monopolist using first degree price discrimination will always produce the Pareto Efficient quantity. Explain why consumers will still prefer the market where the output is produced by competitive firms.

   The monopolist will set the fixed fee for the consumer so that they get all of the consumer surplus, given this they will optimally set the per unit fee at the marginal cost, and thus produce the Pareto Efficient amount. However notice this means that the Monopolist gets all of the Consumer Surplus, so obviously consumers would prefer the market where the good is provided by perfectly competitive firms, who would price at marginal cost and not charge a fixed fee.

4. Show that $MR = P\left(1 + \frac{1}{\varepsilon}\right)$ where P is the price this firm is charging, and ε is the elasticity of this firm's demand. If you want to you can assume this firm is a monopolist.

   Let this firm's inverse demand function be $P_i(Q_i)$. Then $R(Q) = P_i(Q_i)Q_i$

$$
\begin{aligned}
MR &= P_i(Q_i) + \frac{\partial P_i}{\partial Q_i} Q_i \\
\varepsilon_i &= \frac{\partial Q_i}{\partial P_i}\frac{P_i}{Q_i}, \ \frac{\partial P_i}{\partial Q_i} = \frac{1}{\varepsilon_i}\frac{P_i}{Q_i} \\
MR &= P_i + \left(\frac{1}{\varepsilon_i}\frac{P_i}{Q_i}\right) Q_i = P_i + \left(\frac{1}{\varepsilon_i} P_i\right) = P_i\left(1 + \frac{1}{\varepsilon_i}\right)
\end{aligned}
$$

5. Consider a monopolist who has two buyers (for simplicity think of them as two individuals) with the following inverse demand curves:

$$
\begin{aligned}
P_1 &= 49 - 6Q_1 \\
P_2 &= 25 - 6Q_2
\end{aligned}
$$

   these are also the marginal benefit curves of these consumers. The monopolist has the costs of $c(Q) = Q$, where $Q = Q_1 + Q_2$.

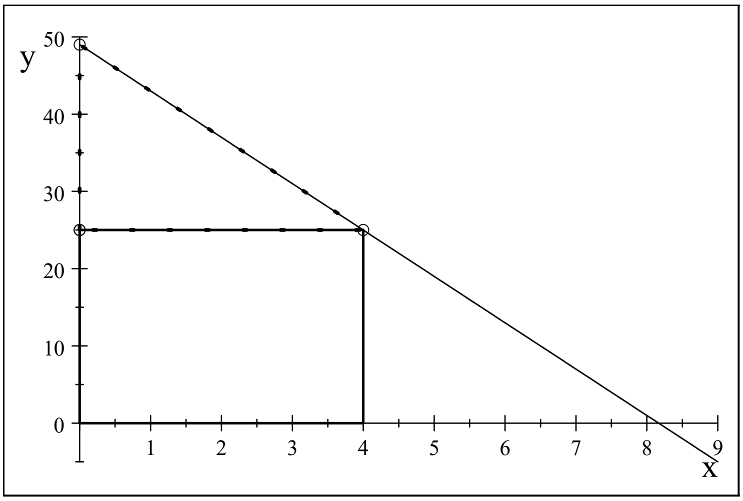
(a) Assume that the monopolist can charge a fee $F_1(Q_1)$ to consumer 1 and $F_2(Q_2)$ to consumer 2, and that the consumers can not trade output between themselves.

i. What kind of price discrimination is this? Give an example (if possible) of a real world application.
This is first degree or perfect price discrimination. It does not occur in the real world because it requires the firm to know exactly how much each person wants to buy.

ii. Show that the total benefit of the two consumers from purchasing this good is:

$$\begin{aligned} B_1(Q_1) &= 49Q_1 - 3Q_1^2 \\ B_2(Q_2) &= 25Q_2 - 3Q_2^2\ . \end{aligned}$$

You may find it helpful to use a graph to get the answer.
First I will graph $49 - 6Q_1$, and use it to show how I calculate the answer for arbitrary $a_i - b_iQ_i$. $B_i(Q_i)$ is the area of the upper triangle and the lower square:



$$\begin{aligned} triangle &= \frac{1}{2}bh,\ b = Q_i,\ h = a_i - (a_i - b_iQ_i) = Q_ib_i \\ triangle &= \frac{1}{2}Q_i^2b_i \end{aligned}$$

$$\begin{aligned} square &= bh,\ b = Q_i,\ h = a_i - b_iQ_i \\ square &= (a_i - b_iQ_i)\,Q_i \end{aligned}$$

$$
\begin{aligned}
B_1(Q_1) &= \frac{1}{2}Q_1^2 b_1 + (49 - 6Q_1) Q_1 \\
&= Q_1 (49 - 3Q_1)
\end{aligned}
$$

$$
\begin{aligned}
B_2(Q_2) &= \frac{1}{2}Q_2^2 b_2 + (25 - 6Q_2) Q_2 \\
&= Q_2 (25 - 3Q_2)
\end{aligned}
$$

iii. What will be the relationship between $F_1(Q_1)$ and $B_1(Q_1)$? Why is this?
$F_1(Q_1) = B_1(Q_1)$ because if the consumer does not buy anything they get $B_1(0) = 0$, if they buy Q_1 they will get $B_1(Q_1) - F_1(Q_1)$, they will buy if

$$
\begin{aligned}
B_1(Q_1) - F_1(Q_1) &\geq 0 \\
B_1(Q_1) &\geq F_1(Q_1)
\end{aligned}
$$

and obviously it is not profit maximizing to have a strict inequality.

iv. Find the profit maximizing amount to supply to both parties, compare it to the efficient quantity and explain this result.

$$
(49 - 3Q_1) Q_1 + (25 - 3Q_2) Q_2 - (Q_1 + Q_2)
$$

$$
\begin{aligned}
49 - 6Q_1 - 1 &= 0 \\
Q_1 &= -\frac{1}{6}(1 - 49) = 8 \\
25 - 6Q_2 - 1 &= 0 \\
Q_2 &= -\frac{1}{6}(1 - 25) = 4
\end{aligned}
$$

these are the efficient quantities, to see this notice that in an efficient equilibrium $P_1 = P_2 = 1$, and then can easily show that these quantities are as given above. The reason for this result is that in first degree price discrimination the consumer's benefit is actually the producer's. So when they maximize profits they are effectively maximizing benefit minus cost, thus they will obviously make the optimal decision.

(b) Assume that now the monopolist is restricted to charging a per-unit price of P_1 to person 1 and P_2 to person 2. (So the total revenue of the monopolist is $R = P_1Q_1 + P_2Q_2$).

i. What kind of price discrimination is this? Give an example (if possible) of a real world application.
This is third degree price discrimination, a simple example is student prices at the movies.

ii. Find the optimal price and quantity for the monopolist to sell to both people.

$$(49-6Q_1)\,Q_1+(25-6Q_2)\,Q_2-(Q_1+Q_2)$$

$$\begin{aligned}
49-12Q_1-1 &= 0\\
Q_1 &= -\frac{1}{12}(1-49)=4\\
P_1 &= 49-6Q_1=49-6\left(\frac{1}{2}\frac{49}{6}-\frac{1}{2}\frac{1}{6}\right)=25\\
25-2(6)Q_2-1 &= 0\\
Q_2 &= 2\\
P_2 &= 25-6Q_2=25-6\left(\frac{1}{2}\frac{25}{6}-\frac{1}{2}\frac{1}{6}\right)=13
\end{aligned}$$

iii. Find the profits of the monopolist and the consumer surplus of both consumers in this equilibrium. *Hint: Very little calculation is required to find the consumer surplus.*

$$\begin{aligned}
CS\,(Q) &= B\,(Q)-PQ\\
CS_1\,(Q_1) &= \left(49-\frac{1}{2}(6)Q_1\right)Q_1-(49-6Q_1)\,Q_1\\
&= \frac{1}{2}\,(Q_1)^2\,6=\frac{1}{2}\left(\frac{1}{2}\frac{49}{6}-\frac{1}{2}\frac{1}{6}\right)^2 6=\frac{1}{(8)(6)}\,(1-49)^2=48\\
CS_2\,(Q_2) &= \left(25-\frac{1}{2}(6)Q_2\right)Q_2-(25-6Q_2)\,Q_2=\frac{1}{2}\,(Q_2)^2\,6=\frac{1}{8(6)}\,(1-25)^2=12\\
\Pi &= P_1Q_1+P_2Q_2-1Q_1-1Q_2=(P_1-1)\,Q_1+(P_2-1)\,Q_2\\
&= \left(\frac{1}{2}+\frac{1}{2}(49)-1\right)\left(\frac{1}{2}\frac{49}{6}-\frac{1}{2}\frac{1}{6}\right)+\left(\frac{1}{2}+\frac{1}{2}25-1\right)\left(-\frac{1}{2(6)}\,(1-25)\right)\\
&= \frac{1}{4(6)}\,(1-49)^2+\frac{1}{4(6)}\,(1-25)^2=120
\end{aligned}$$

(c) Assume now that the monopolist is restricted to charging a per-unit price of P to both people. (So the total revenue of the monopolist is $R=PQ_1+PQ_2$).

i. Show that the aggregate inverse demand curve is $P\,(Q)=37-3Q$ when both people buy.

$$
\begin{aligned}
P &= 49 - 6Q_1,\ Q_1 = -\frac{1}{6}(P - 49) \\
P &= 25 - 6Q_2,\ Q_2 = -\frac{1}{6}(P - 25) \\
Q &= Q_1 + Q_2 = -\frac{1}{6}(P - 49) - \frac{1}{6}(P - 25) = \frac{1}{6}(49 - 2P + 25) \\
P &= \frac{1}{2}(25) + \frac{1}{2}(49) - \frac{1}{2}Q(6) = 37 - 3Q
\end{aligned}
$$

ii. When will both people buy? If one person does not buy who will not? Find the aggregate inverse demand curve when only one person buys.
Since $49 > 25$ *person 2 will not buy if anyone does not. She will not buy if* $P > 25$, *if this is true then the aggregate inverse demand curve is* $49 - 6Q_1$, *or the inverse demand of consumer 1.*

iii. Find the profit maximizing price and the quantities both people buy in equilibrium. Be sure to check whether both consumers are buying at your equilibrium.

$$
\left(\frac{1}{2}(49) + \frac{1}{2}(25) - \frac{1}{2}Q(6)\right)Q - Q
$$

$$
\begin{aligned}
\frac{1}{2}(49) + \frac{1}{2}(25) - Q(6) - 1 &= 0 \\
Q &= \frac{1}{2(6)}(49 - 2 + 25) = 6 \\
P &= \frac{1}{2}(49) + \frac{1}{2}(25) - \frac{1}{2}Q(6) = \frac{1}{2} + \frac{1}{4}(49) + \frac{1}{4}(25) = 19
\end{aligned}
$$

$$
\begin{aligned}
Q_1(P) &= \frac{49}{6} - \frac{1}{6}\left(\frac{1}{2} + \frac{1}{4}(49) + \frac{1}{4}(25)\right) = \frac{3}{4(6)}(49) - \frac{1}{2(6)} - \frac{1}{4(6)}(25) = 5 > 0 \\
Q_2(P) &= \frac{25}{6} - \frac{1}{6}\left(\frac{1}{2} + \frac{1}{4}(49) + \frac{1}{4}(25)\right) = \frac{3}{4(6)}(25) - \frac{1}{4(6)}(49) - \frac{1}{2(6)} = 1 > 0
\end{aligned}
$$

since both consumers are buying a strictly positive amount this is the equilibrium.

iv. Find the profits of the monopolist and the consumer surplus of both consumers in this equilibrium. *Hint: Very little calculation is required to find the consumer surplus.*

$$
\begin{aligned}
CS_1(Q_1) &= \frac{1}{2}(Q_1)^2(6) = \frac{1}{2}\left(\left(-\frac{1}{4(6)}(2-3(49)+25)\right)\right)^2 6 = 75 \\
CS_2(Q_2) &= \frac{1}{2}(Q_2)^2 6 = \frac{1}{2}\left(-\frac{1}{4(6)}(2+49-3(25))\right)^2(6) = 3 \\
\Pi &= PQ - Q \\
&= (P-1)Q = \left(\frac{1}{2}+\frac{1}{4}(49)+\frac{1}{4}(25)-1\right)\left(\frac{1}{2(6)}(49)-\frac{1}{6}+\frac{1}{2(6)}(25)\right) = 108
\end{aligned}
$$

v. Compare the profit and consumer surpluses of both consumers in parts *b* and *c* of this question. For each party state which one they prefer. Is one of them Pareto Dominant? Why or why not?

For the monopolist and type 2 consumers the case where the monopolist uses third degree price discrimination is better. The monopolist because they could always have the prices be the same if they wanted to, the consumers because if they are grouped with type 1 consumers they have to pay a higher price. For type 1 consumers the case without price discrimination is better, thus neither market structure is Pareto Dominant, some people want price discrimination and some do not.

6. Define each of the three types of price discrimination. Which one is never applied in the real world? Why? For the other two give an example of a good where this type of price discrimination is applied. Be sure to explain your answer.

- *First degree or Perfect Price discrimination—this is the type of price discrimination where each person is given an individual and personal fee for purchasing the good. This one is never applied in the real world because it requires too much information.*
- *Second Degree or Quantity based Price Discrimination—here the price per unit changes with the number of units. It is commonly applied, examples are value meal prices at fast food restaurants, but most goods seem to follow this pattern. For example the price per kilo for laundry detergent generally falls by the kilo. From personal experience I know that Kosla Vanish uses this type of price discrimination.*
- *Third Degree or Characteristic based Price Discrimination—Here the price is based on some observable characteristic like age, student status, or nationality. Some examples are student prices at movies, special Turk prices at historic sites (which I believe is no longer done). Other examples are family discounts at resorts and amusement parks, etceteras.*

7. A monopolist has two different type of demanders (high and low, high have a subscript h on their variables, low have a subscript l on their variables.)

$$\begin{aligned} P_h &= 96 - Q_h \\ P_l &= 36 - Q_l \end{aligned}$$

and the cost function $C(q) = 4q$.

(a) If they can price discriminate:

i. Set up the firm's objective function.

$$\max_{Q_h, Q_l} ((96) - Q_h) Q_h + ((36) - Q_l) Q_l - (4)(Q_h + Q_l)$$

ii. Find it's first derivatives.

$$\begin{aligned} (96) - 2Q_h - (4) &= 0 \\ (36) - 2Q_l - (4) &= 0 \end{aligned}$$

iii. Find the price they would charge high and low demand customers.

$$\begin{aligned} Q_h &= -\frac{1}{2}((4) - (96)) \\ P_h &= (96) - Q_h = (96) - \left(-\frac{1}{2}((4) - (96))\right) = \frac{1}{2}(4) + \frac{1}{2}(96) \\ Q_l &= -\frac{1}{2}((4) - (36)) \\ P_l &= (36) - Q_l = (36) - \left(-\frac{1}{2}((4) - (36))\right) = \frac{1}{2}(4) + \frac{1}{2}(36) \end{aligned}$$

(b) If they can not price discriminate what is their aggregate demand curve?

$$\begin{aligned} P &= (96) - Q_h \\ Q_h &= -1(P - (96)) \\ P &= (36) - Q_l \\ Q_l &= -1(P - (36)) \\ Q &= Q_h + Q_l = -1(P - (96)) - 1(P - (36)) \\ Q &= -1(P - (96)) - 1(P - (36)) \\ P &= \frac{1}{2}(96) + \frac{1}{2}(36) - \frac{1}{2}Q \end{aligned}$$

(c) If they can not price discriminate:

i. Set up the firm's objective function.

$$\max_Q \left(\frac{1}{2}(96) + \frac{1}{2}(36) - \frac{1}{2}Q\right) Q - (4)Q$$

ii. Find it's first derivative.

$$\frac{1}{2}(96) + \frac{1}{2}(36) - Q - (4) = 0$$

iii. Find the price they would charge.

$$\begin{aligned}
Q &= \frac{1}{2}\left((96) - 2(4) + (36)\right) \\
P &= \frac{1}{2}(96) + \frac{1}{2}(36) - \frac{1}{2}Q \\
&= \frac{1}{2}(96) + \frac{1}{2}(36) - \frac{1}{2}Q = \frac{1}{2}(96) + \frac{1}{2}(36) - \frac{1}{2}\left(\frac{1}{2}\left((96) - 2(4) + (36)\right)\right) \\
&= \frac{1}{2}(4) + \frac{1}{4}(96) + \frac{1}{4}(36)
\end{aligned}$$

(d) Is it Pareto improving to allow price discrimination in this market? Why or why not? Be sure to consider all three different parties, high demand consumers, low demand consumers, and the monopolist.

Clearly the monopolist is doing better because the prices he would like to charge to the low and high types are different, so he must be making more money by not having them constrained to be the same.

The low types are also better off with price discrimination because $P > P_l$. *The high types are doing worse though because* $P_h > P$. *So price discriminating is not Pareto Improving.*

8. Which two technical barriers to entry imply that software is generally going to be a monopolistic industry? Explain your answer.

The two barriers to entry are **always decreasing average cost** *and* **network externalities**.

- *Always decreasing average costs is very common in software industries because most of the cost of their products is in research and development, which is a fixed cost. After producing the software their only costs are advertisement, cost of printing CD's, and support costs. While these may have increasing marginal cost they are such an order of magnitude lower than research and development costs that average costs are generally decreasing.*
- *Network externalities is the way if one person consumes a good it increases the benefits to others who consume the same good. With software this primarily due to learning costs, if someone you knows uses the same software then it is substantially easier for you to learn*

how to use the software. This is also true for (local) product support, if you have a problem and other people use the same software you can just ask them for the solution

9. In first degree or perfect price discrimination:

   (a) What is the consumer surplus?

   zero, nothing

   (b) How does the profit maximizing level of output compare to the efficient level of output?

   they are the same.

   (c) Explain why this happens and how this can be considered a Pareto Efficient outcome.

   This happens because the fee the monopolist charges is essentially the entire benefit the consumer derives from the good, thus they are maximizing the benefit of the good minus the costs of the good, which obviously results in the Pareto Efficient output.

   It is Pareto Efficient because we don't care about how the surplus is divided (between the consumer and the firm) only about whether the right amount is supplied.

10. Consider a monopolist who sells concert tickets to Students and the General Public. The demand curve of the general public is $q_g = 72 - 3P_g$ and of students is $q_s = 8 - P_s$; the costs of the monopolist are $c(Q) = 4Q$.

    (a) What is the aggregate demand if both groups are charged the same price?

$$Q = q_s + q_g = \begin{cases} 8 - P + 72 - 3P = 80 - 4P & \text{if} \quad P \leq 8 \\ 72 - 3P & \text{if} \quad P \leq 24 \\ 0 & \text{else} \end{cases}$$

    (b) Assuming that the monopolist must charge the same price to both groups and both groups buy what is the aggregate quantity and price they will sell at?

$$
\begin{aligned}
\text{if } P &\leq 8 \\
P &= 20 - \frac{1}{4}Q \\
\Pi &= \left(20 - \frac{1}{4}Q\right) Q - 4Q \\
&= 16Q - \frac{1}{4}Q^2 \\
0 &= 16 - \frac{1}{2}Q \\
Q &= 32 \\
P &= 24 - \frac{1}{4}(32) = 16 > 8
\end{aligned}
$$

Thus this is not a correct solution, the low types will not buy and the aggregate inverse demand curve is:

$$P = 24 - \frac{1}{3}Q$$

$$
\begin{aligned}
\Pi &= \left(24 - \frac{1}{3}Q\right) Q - 4Q \\
&= 20Q - \frac{1}{3}Q^2 \\
0 &= 20 - \frac{2}{3}Q \\
Q &= 30 \\
P &= 24 - \frac{1}{3}(30) = 14 < 24
\end{aligned}
$$

so this is the solution.

(c) Find the quantity students will buy and the quantity the general public will buy at the price you found in part *b*.

$q_s = 0$, $q_g = 72 - 3(14) = 30$

(d) If the monopolist only sells tickets to the general public what price will they charge? Will the students want to buy at this price?

This is the problem we solved above, and the solution was the price was 14, so no, students will not want to buy.

(e) What is the profit maximizing price and quantity if the monopolist must charge the same price to both groups? Explain your answer.

As I explained above the optimal price is 14 and quantity is 30, and they only sell to the high types.

(f) Now assume that all students have ID's and the monopolist can charge a different price to students. What price will it charge and quantity will it sell to students and the general public?

Define welfare as $W = CS_g + CS_s + \Pi_m$, where CS_g is the consumer surplus of the general public, CS_s is the consumer surplus of the students, and Π_m is the profit of the monopolist.

The price and quantity to the general public is above, the price and quantity to the students is found by:

$$\max_{Q_s} (8 - Q_s) Q_s - 4Q_s = 4Q_s - Q_s^2$$

$$\begin{aligned} 4 - 2Q_s &= 0 \\ Q_s &= 2 \\ P_s &= 8 - 2 = 6 \end{aligned}$$

notice that we can solve the two problems independently since we have constant marginal cost.

(g) Without doing any calculations, did welfare increase or decrease when the monopolist used third degree price discrimination? Is it a Pareto improvement to allow the monopolist to use third degree price discrimination in this market.

The profit must have increased because the firm has two prices. The consumer surplus of non-students has not changed because the price is the same in both scenarios for non-students. Students however now have some Consumer Surplus because they are buying tickets. Thus with price discrimination the students and the firm do strictly better, and the general public does not care. It is a Pareto Improvement.

11. Assume that a monopolist faces a demand curve of $Q(P) = P^{-e}$ and has a cost function of $c(Q) = cQ$.

(a) Prove that $MR = P\left(1 + \frac{1}{e_d(P)}\right)$ where $e_d(P)$ is the elasticity of demand with regards to price. The proof should be true for general demand curves, not just the demand curve above.

$$\begin{aligned} MR &= P'Q + P \\ &= P\left(P'\frac{Q}{P} + 1\right) \\ &= P\left(\frac{1}{\frac{\partial Q}{\partial P}\frac{P}{Q}} + 1\right) \\ &= P\left(\frac{1}{e_d(P)} + 1\right) \end{aligned}$$

(b) Show that for the demand curve above $e_d(P) = -e$. What do we know about e since a monopoly supplies this market?

$$e_d(P) = \frac{\partial Q}{\partial P}\frac{P}{Q} = -eP^{-e-1}\frac{P}{P^{-e}} = -e$$

We must have $-e < -1$ so that the demand curve is elastic.

(c) Find a formula that determines the market price as a function of e and c without setting up the objective function or finding the first order condition.

$$\begin{aligned} MR &= MC \\ P\left(\frac{1}{-f}+1\right) &= c \\ P &= c\frac{e}{e-1} \end{aligned}$$

(d) Show that if there is a per unit tax of t in this market the price the buyer pays increases more than the tax, or that $\frac{\partial P_b}{\partial t} > 1$.

$$\begin{aligned} MC_n &= c+t \\ P_b &= (c+t)\frac{e}{e-1} \\ \frac{\partial P_b}{\partial t} &= \frac{e}{e-1} > 1 \end{aligned}$$

12. Assume that the demand for Cell Phones is given by $Q = 120 - 2P$. In this question we will consider two methods of supplying this market: Monopoly and Perfect competition.

(a) In this part you should consider only one Monopoly provider. This firm has a cost function of $c(Q) = 2Q$.

i. Find the Inverse Demand Curve.
$\frac{120}{2} - \frac{1}{2}Q = 60 - \frac{1}{2}Q$

ii. Set up the Monopolists objective function, be sure that is a function only of Quantity.

$$\max_{Q}\left(\frac{120}{2} - \frac{1}{2}Q\right)Q - 2Q$$

iii. Find the Monopolists First order condition.

$$\left(\frac{120}{2} - \frac{1}{2}Q\right) - \frac{1}{2}Q - 2 = 0$$

iv. Solve for the optimal Quantity and Price in this market.

$$\begin{aligned} Q^m &= \frac{1}{2}120 - \frac{1}{2}2 * 2 = 60 - 2 = 58 \\ P^m &= \frac{120}{2} - \frac{1}{2}Q^m = 60 - 29 = 31 \end{aligned}$$

(b) In this part you should consider only 12 perfectly competitive suppliers. These suppliers all have the same cost function $c(q) = 6q^2 + 54$. For simplicity all fixed costs are fixed start up costs, these costs do not have to be paid if no output is produced.

i. Find one of these firm's marginal cost and average variable cost.

$$\begin{aligned} MC &= 2 * 6 * q = 12q \\ AVC &= \frac{6q^2 + 54}{q} \end{aligned}$$

ii. Find the lowest quantity at which these firms will not produce. What is their marginal cost at this quantity?

$$\begin{aligned} 12q &\geq \frac{6q^2 + 54}{q} \\ q &\geq \sqrt{\frac{54}{6}} = 3 \\ MC &= 12q = 2\sqrt{54 * 6} = 36 \end{aligned}$$

iii. Find one firm's supply curve.

$$\begin{aligned} P &= MC = 12q \\ q &= \frac{P}{12} \end{aligned}$$

$$s(P) = \begin{cases} \frac{P}{12} & P \geq 36 \\ 0 & P \leq 36 \end{cases}$$

iv. Find the industry's short run supply curve.

$$S(P) = 12s(P) = \begin{cases} \frac{12P}{12} & P \geq 36 \\ 0 & P \leq 36 \end{cases}$$

v. Find the Price and Quantity that will be produced in this market.

$$\begin{aligned}
\frac{12P}{12} &= 120 - 2P \\
P^* &= \frac{2 * 120 * 6}{12 + (2 * 2 * 6)} = 40 \\
Q^* &= 120 - 2P^* \\
&= 40
\end{aligned}$$

(c) In this part you will compare the two ways of supplying this market.

i. Which way produces more output? (Monopoly in part a or Perfect Competition in part b).
Monopoly because

$$\begin{aligned}
Q^M &> Q^* \\
58 &> 40
\end{aligned}$$

ii. If the government allowed both the Monopolist and the Perfect Competitors to supply this market, would the Perfect Competitors produce any output?
No because

$$\begin{aligned}
P^m &< MC\left(q^{sd}\right) \\
31 &< 36
\end{aligned}$$

iii. Which method is Pareto Dominant in this market, Monopoly or Perfect Competition? Is there a way to make Perfect Competition Pareto Dominant?
Without government intervention Monopoly is. This is because Monopoly produces more quantity and the competitive firms would not produce at the Monopolist's price.
The government could pay the Monopolist his profit and force him to share his technology, in this case $Q = 120 - 2P$

iv. Assume that the Monopolist has lower costs because they did Research and Development and invented a radically new way to produce cell phones. What type of goods are inventions? Why might the government not want to force the Monopolist to share her technology with other firms? *The answer to this question requires information from the chapter on Public Goods and Externalities.*
Inventions are club goods, they are non-rival but excludable. The government often allows firms to have monopolies over new technologies so they can recapture the expense of their research and development

13. Since the Turkish government needs to raise revenue it wants to make the maximum possible profits off of the millions of historical sights in Turkey.

It faces two different demand curves, one for foreigners and one for Turks (or residents of Turkey :-)). The inverse demand curves for foreign visitors is $P_f = 120 - q_f$—where q_f is the number of foreign visitors and P_f is the price they pay; and for Turks is $P_t = 60 - 3q_t$—here q_t is the number of Turks and P_t is the price. The government has a cost of 6 per visitor, or their Total costs are $6Q$, $Q = q_f + q_t$.

(a) First consider the case where they have to charge the same price to Turks and foreigners.

i. Find the aggregate demand curve assuming both types pay the same price.

$$\begin{aligned} q_f &= 120 - P \\ q_t &= 20 - \frac{1}{3}P \\ Q &= q_f + q_t \\ &= 120 - P + 20 - \frac{1}{3}P \\ &= 140 - \frac{4}{3}P \\ P &= 105 - \frac{3}{4}Q \end{aligned}$$

ii. Find the price and quantity of tickets they will sell.

$$\max_Q \left(105 - \frac{3}{4}Q\right)Q - 6Q$$

$$\begin{aligned} -\frac{3}{4}Q + \left(105 - \frac{3}{4}Q\right) - (6) &= 0 \\ Q &= 66 \\ P &= 105 - \frac{3}{4}(66) \\ &= \frac{111}{2} \\ &= \frac{3}{8}(120) + \frac{1}{8}(60) + \frac{1}{2}(6) \end{aligned}$$

iii. Find the consumer surplus and profits.

$$
\begin{aligned}
CS &= Q^* \left(P\left(0\right) - P^*\right) \frac{1}{2} \\
&= 66 \left(105 - \frac{111}{2}\right) \frac{1}{2} \\
&= \frac{3267}{2} \\
\Pi &= \left(P^* - 6\right) Q^* \\
&= \left(\frac{111}{2} - 6\right) 66 \\
&= 3267
\end{aligned}
$$

(b) Now consider the real situation, where they charge a separate price to Turks and foreigners

i. Find a relationship between the quantities of tickets they sell to foreigners and the quantity they sell to Turks that is independent of costs.

$$
\begin{aligned}
MR_t &= MR_f \\
120 - 2q_f &= 60 - 6q_t \\
q_f &= 30 + 3q_t
\end{aligned}
$$

ii. Explain how the relationship above is an immediate implication of profit maximization, or the choices the monopolists has.
We can think of the monopolist, once they have produced the output, as choosing which market to sell the good in. Since they can make this choice their marginal benefit (or marginal revenue) *in each market should be equal.*

iii. Find the quantity that the monopolist will sell in each market, and the price.

$$
\max_{q_f, q_t} \left(120 - q_f\right) q_f + \left(60 - 3q_t\right) q_t - 6\left(q_f + q_t\right)
$$

$$
\begin{aligned}
120 - q_f - q_f - (6) &= 0 \\
q_f &= 57 \\
p_f &= 63 \\
(60 - 3q_t) - 3q_t - (6) &= 0 \\
q_t &= 9 \\
p_t &= 33
\end{aligned}
$$

iv. Find the total consumer surplus and the profits.

$$\begin{aligned} CS &= \left(p_f(0) - p_f\right) q_f \frac{1}{2} + \left(p_t(0) - p_t\right) q_t \frac{1}{2} \\ &= (120 - 63)(57)\frac{1}{2} + (60 - 33)(9)\frac{1}{2} \\ &= 1746 \\ \Pi &= 3492 \end{aligned}$$

(c) Which situation increases total consumer surplus, one price or two?

$$\begin{aligned} CS_{1\text{ prices}} &< CS_{2\text{ prices}} \\ \frac{3267}{2} &= 1633.5 < 1746 \end{aligned}$$

(d) Why must the monopolist do better when they can charge two prices?
Because they can always choose to charge the same price in both markets. If they do not then they must be making higher profits. (And yes, technically speaking, the question should have asked "at least as well." So sue me!)

14. Giggles the Great Ape Grape Grower owns the only farm around that can grow decent grapes. Since this makes her a monopolist the local high mucky-mucks want you to tell them how to regulate her. The demand curve for grapes is

$$Q = 40 - 10P$$

And her total cost is

$$TC = 30$$

(a) What is her Marginal Revenue? What is her Marginal Cost?

$$TR = P * Q = (4 - \frac{1}{10}Q)Q$$

$$MR = \frac{dTR}{dQ} = 4 - \frac{2}{10}Q$$

$$MC = \frac{dTC}{dQ} = 0$$

(b) What would be the price and quantity she would produce if she were not regulated?
She will produce up to point where $MR = MC$,

$$4 - \frac{2}{10}Q^* = 0 \Rightarrow Q^* = 20 \Rightarrow P^* = \frac{40 - Q^*}{10} = 2$$

(c) What is the Pareto efficient price ceiling to enforce? If you force her to sell at this price, how much will she produce?

The monopoly price is not Pareto efficient. There are consumers that are willing to pay more than MC*, and monopolist would benefit from selling at a lower price to those customers ONLY (i.e. if he doesn't have to lower the price for all others). So we could make everyone better off if we could sell to those left out at a lower price, as long as this 'discount' price is not lower than* MC*. So the Pareto efficient outcome when price discrimination is allowed, is* $P = MC = 0$.

But if monopolist is not allowed to price discriminate, and has to charge a uniform price, no Pareto improvement is possible. If monopolist wants to lower the price, she must lower the price for EVERYONE, and that would decrease her profit.

If price is regulated to be zero (and must be the same for all consumers), then the monopolist will not produce, because she will make a loss. Let's check where the demand and average cost intersect:

$AC = P$

$\frac{30}{Q} = 4 - \frac{1}{10}Q$

$Q^a = 30, \qquad P^a = 1$

Whenever price is lower than 1, monopolist will not cover her average cost and will make a loss.

(d) If the mucky mucks make it very clear to you that they will not subsidize Giggles, what price should you force her to sell at?

If you can't subsidize, you can't make the monopoly charge $P = 0$. *You will want to make them sell at the lowest price at which they're not making any losses, i.e. charge* $P = AC$ *(average cost pricing). From part c) we know that* $P^a = 1$. *At this price monopolist makes 0 profit, consumers are better off than under monopoly pricing, but inefficiency still exists since* $P > MC-$ *we are choosing the 'second best' option (marginal cost pricing is first-best)*

(e) If you also take the idea of Pareto efficiency seriously and only want to implement Pareto improving policies, what price should you force her to sell at? How does your answer change if the mucky mucks are willing to subsidize Giggles?

Without subsidies any price that we make her charge will result in a lower profit than the monopoly profit, so we can't regulate her at all.

When the subsidy is allowed, we can keep monopoly in business at $P^* = 0$*, by subsidizing it by the amount equal to the losses. Since she makes no revenue at this price, we must reimburse all the profits she would have made before, i.e.* $S = P^*Q^* = 2 * 20 = 40$.

(f) If the mucky mucks make it very clear to you that they will not subsidize Giggles, what price should you force her to sell at?

If $P^a = 1$*, then at this point* $MR = 1$*, i.e. it is constant (flat line) because we regulate the monopoly, but only up to the quantity* *demanded. At $P^a = 1$ quantity demanded is 30. $MR = 0$ (flat line) for any output greater than 30 - we cannot sell more than 30 at $P^a = 1$.*

15. (This is an oligopoly question, but I am putting it here for convenience.) In the following question, assume the mucky mucks have not asked you to regulate Giggles yet. Yukon Jack the Mountain Goat has just figured out how to terrace the mountain sides and grow great grapes there. His total cost is:

$$TC = q_j$$

(a) How much output will he produce given the production of Giggles (q_g)? (Or what is his reaction function.)

total demand: $q_j + q_g = 40 - 10P$ $\Longrightarrow$inverse demand function: $P = \frac{1}{10}(40 - q_j - q_g)$

$$\max_{q_j} \pi_j = \left(4 - \frac{1}{10}q_j - \frac{1}{10}q_g\right) q_j - q_j$$

F.O.C.:

$$4 - \frac{2}{10}q_j - \frac{1}{10}q_g - 1 = 0$$

$$q_j = 15 - \frac{1}{2}q_g$$

Note that Yukon Jacks production level depends on how much is produced by Giggles.

(b) If Giggles expects him to enter, how much will she choose to commit to producing before he does? How much will Yukon Jack produce once he enters?

Giggles (leader) and Yukon (follower) will decide the level of their production sequentially. Leader chooses his output level thinking how the follower responds to his choice. We are therefore solving for a Stackelberg equilibrium

$$\max_{q_g} \pi_g = (4 - \frac{1}{10}q_j\,(q_g) - \frac{1}{10}q_g)q_g - 30$$

substituting the reaction function of Yukon Jack:

$$\begin{aligned}
\max_{q_g} \pi_g &= \left(4 - \frac{1}{10}\left(15 - \frac{1}{2}q_g\right) - \frac{1}{10}q_g\right) q_g - 30 \\
\max_{q_g} \pi_g &= \left(\frac{5}{2} - \frac{1}{20}q_g\right) q_g - 30
\end{aligned}$$

F.O.C.:

$$4 - \frac{3}{2} + \frac{1}{10}q_g - \frac{2}{10}q_g = 0$$

$$q_g = 25$$

So Jack produces:

$$q_j = 15 - \frac{1}{2}(25) = 2.5$$

(c) If Giggles can not commit to any level of production before Yukon Jack decides on how much to produce, how much should she produce? How much will Yukon Jack Produce?

This time we will assume Giggles and Jack decide on the quantity they produce simultaneously, hence we will solve for the Cournot equilibrium.

We have already found the reaction function of Yukon Jack in part a of this question. The reaction function of Giggles is:

$\max \pi_j = (4 - \frac{1}{10}q_j - \frac{1}{10}q_g)q_g - 30$

F.O.C.: $4 - \frac{1}{10}q_j - \frac{2}{10}q_g = 0$

$q_g = 20 - \frac{1}{2}q_j$

Solving the reaction functions together, we get the equilibrium quantities:

$q_j = 15 - \frac{1}{2}(20 - \frac{1}{2}q_j) \Longrightarrow q_j = 6\frac{2}{3}$

$q_g = 20 - \frac{1}{2}(6\frac{2}{3}) = 16\frac{2}{3}$

(d) How much would she pay him for not entering in both scenarios? Would he be willing to accept it?

We must first find her profits, and we will need prices for that:

Stackelberg:

$P = \frac{1}{10}(40 - 2.5 - 20) = 1.25$

$\pi_g = 1.25 * 25 - 30 = 1.25$

When Giggles was a monopolist, she was making a profit of 10. When Yukon does not enter, she would still be making 10. To keep Yukon away from the market she would pay up to $(10 - 1.25) = 8.75$. *Let us see whether Yukon accepts it or not. He will make a profit of:*

$\pi_j = 1.25 * 2.5 - 2.5 = 0.625$

So whenever he receives something greater than 0.625 from Giggles, he will accept it and stay out.

Cournot:

$P = \frac{1}{10}(40 - 6\frac{2}{3} - 16\frac{2}{3}) = 1\frac{2}{3}$

$\pi_g = 1\frac{2}{3} * 16\frac{2}{3} - 30 = -2.22$

$\pi_j = 1\frac{2}{3} * 6\frac{2}{3} - 6\frac{2}{3} = 4.44$

Since Giggles profits are negative under this scenario, she will be driven out of the market. Jack will become a monopoly and choose the following quantity:

$\max \pi_j^m = (4 - \frac{1}{10}Q)Q - Q$
F.O.C.: $4 - \frac{2}{10}Q - 1 = 0$
$Q = 15 \qquad P = 4 - \frac{1}{10} * 15 = \frac{5}{2}$
$\pi_j^m = 15(\frac{5}{2}) - 15 = 22.5$
Since Giggles is only willing to pay 10 to keep him off the market, Jack will reject the offer, enter the market and become a new monopolist.

(e) If the high mucky mucks asked you to regulate the industry (and will do anything you ask for), how much output would you have Yukon Jack produce assuming that you do have Giggles produce some output? Explain your answer.

High mucky mucks want to regulate the industry and achieve the solution you had in question 2. You are allowed to do anything, and then you can pay subsidies.

First case: set price equal to 0. At this price neither Jack nor Giggles will produce. But we can give them subsidies. We know that if we want Giggles to produce, we must give her a subsidy of 30 no matter how much she produces. So there is no reason to keep Jack in business and pay him any extra subsidy. The cheapest subsidy scheme is to let Giggles produce all 40 units of output and pay her 30 in subsidy (this is under the assumption that Giggles does produce some output).

16. The Houston Museum of Natural Science faces two different demand curves. Children really want to go, but don't have any money. Their demand curve is

$$Q_c = 15 - 3P$$

Adults, while they have a lot of money, don't really want to go[1] and their demand curve is:

$$Q_a = 7 - P$$

The Museum's total cost is:

$$TC = Q + 21$$

where Q is the total number of visitors, $Q = Q_c + Q_a$. They have asked you to tell them whether they should use third degree price discrimination or not. (Assume that they act as profit maximizers.)

(a) If they do third degree price discriminate, what price would they charge to the children? The adults?

$\max_{Q_a, Q_c} \Pi(Q_a, Q_c) = P_a Q_a + P_c Q_c - Q_a - Q_c - 21$
The inverse demand functions are:
$P_a = 7 - Q_a$
$P_c = 5 - \frac{1}{3}Q_c$

[1] Or, should I say, are not willing to say that they want to go.

Plugging these into maximization problem:

$$\max_{Q_a, Q_c} \Pi(Q_a, Q_c) = (7 - Q_a)Q_a + (5 - \tfrac{1}{3}Q_c)Q_c - Q_a - Q_c - 21$$

F.O.C.:

$$\frac{\partial \Pi(Q_a, Q_c)}{\partial Q_a} = 7 - 2Q_a - 1 = 0$$

$$\frac{\partial \Pi(Q_a, Q_c)}{\partial Q_c} = 5 - \tfrac{2}{3}Q_c - 1 = 0 \Longrightarrow \qquad Q_a = 3, \qquad Q_c = 6$$

Plugging the equilibrium quantities into respective demand equations:

$P_a = 7 - Q_a = 7 - 3 = 4$

$P_c = 5 - \frac{1}{3}Q_c = 5 - 2 = 3$

we can find actual prices for adults and children.

(b) What would be the dead weight loss? The Profit?

To figure out DWL, we need to know the competitive price and quantity in each market

$P_a^e = MC = 1$

$P_c^e = MC = 1$

since in efficient markets, price is equal to marginal cost, this is easy.

$Q_a^e = 7 - 1 = 6$

$Q_c^e = 15 - 3 * 1 = 12$

And the dead weight loss is:

$$DWL = \tfrac{1}{2}(P_a^m - P_a^e)(Q_a^m - Q_a^e) + \tfrac{1}{2}(P_c^m - P_c^e)(Q_c^m - Q_c^e)$$

$$DWL = \tfrac{1}{2}(4 - 1)(6 - 3) + \tfrac{1}{2}(3 - 1)(12 - 6) = 10.5$$

And the profit can be found from the profit function:

$$\Pi(Q_a, Q_c) = P_a^m Q_a^m + P_c^m Q_c^m - Q_a^m - Q_c^m - 21 =$$

$$= 4 * 3 + 3 * 6 - 3 - 6 - 21 = 0$$

(c) If they only sell one ticket to whomever comes up what price would they charge?

In this case they face one demand curve:

$Q = Q_a + Q_c$

$$Q = \begin{cases} 7 - P & P > 5 \\ 22 - 4P & 0 < P \leq 5 \end{cases}$$

Notice that it has a kink in it. You can safely assume that I wouldn't be so evil as to make it relevant, but it's still important to note.

$$\max_Q \Pi(Q) = PQ - Q - 21$$

$$\max_Q \Pi(Q) = (\tfrac{11}{2} - \tfrac{1}{4}Q)Q - Q - 21$$

$$\frac{d\Pi(Q)}{dQ} = \frac{11}{2} - \frac{1}{2}Q - 1 = 0$$

$Q = 9$

$P = \frac{11}{2} - \frac{1}{4} * 9 = \frac{13}{4} = 3.25 < 5$

*Since price is less than 5, we are indeed on the flat part of the demand curve (important)*they only sell one ticket to whomever comes up what price would they charge?

(d) What is the dead weight loss? The Profit?

First we need to calculate the number of children and adults that will come at price 3.25.

$Q_a^m = 7 - P = 3.75$

$Q_c^m = 15 - 3 * P = 5.25$

Notice: we have to cut a kid in quarters to get the number right (just kidding).

$DWL = \frac{1}{2}(P_a^m - P_a^e)(Q_a^m - Q_a^e) + \frac{1}{2}(P_c^m - P_c^e)(Q_c^m - Q_c^e)$

$DWL = \frac{1}{2}(3.25 - 1)(6 - 3.75) + \frac{1}{2}(3.25 - 1)(12 - 5.25) = 10.125$

$\Pi = P^m Q^m - Q^m - 21 =$

$= 3.25 * 9 - 9 - 21 = -.75$

(e) What would you recommend to them, why? Compare dead weight loss and profit of each option in your answer.

Well, I seem to have discovered that the Houston Museum of National Science is a greedy profit maximizer. Since DWL is lower when they charge one ticket price, no price discrimination is more efficient (in terms of welfare) and they must be responding to profit incentives since they are third-degree price discriminators. I always knew we couldn't trust them.

What should we recommend? well, we have found that profits of the HMNS also decrease when price discrimination is dropped. Furthermore the group that is paying a lower price under price discrimination also prefers this to the one-price scheme. Thus switching to one-price scheme is not a Pareto improvement.