Practice Questions

Oligopoly
ECON 433
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These questions are to help you prepare for the exams only. Do not turn them in. These are all old exam questions. Questions with lower numbers were asked in more recent years.

1 From ECON204 (Micro Theory II)

1. Consider a Cournot oligopoly with J firms where the inverse demand curve is $P = 49 - 3Q$ and all firms have the same cost function, $c(q) = 13q$.

   (a) If $J = 2$:

      i. Set up the objective function of firm 1.

      $$\max_{q_1} (49 - 3(q_1 + q_2)) q_1 - 13q_1$$

      ii. Find the best response of firm 1.

      $$(49 - 3(q_1 + q_2)) - 3q_1 - 13 = 0$$

      $$q_1 = 6 - \frac{1}{2}q_2$$

      iii. Why do you not need to find the best response of firm 2?

      Solution 1 *Because of symmetry, they have the same objective function thus they have the same best response.*

      iv. Find the Nash equilibrium output of each firm, and the market price.

      $$\begin{aligned} q &= 6 - \frac{1}{2}q \\ q &= 4 \\ Q &= 8 \\ P &= 49 - 3 * 8 = 25 \end{aligned}$$

   (b) If $J = 11$:

      i. Set up the objective function of firm 1 using $Q_{-1} = \Sigma_{j=2}^{11} q_j$ for the total output of all other firms.

      $$\max_{q_1} (49 - 3(q_1 + Q_{-1})) q_1 - 13q_1$$

ii. Find the best response of firm 1 as a function of Q_{-1}.

$$(49 - 3(q_1 + Q_{-1})) - 3q_1 - 13 = 0$$

$$q_1 = 6 - \frac{1}{2}Q_{-1}$$

iii. Find the Nash equilibrium output of each firm, and the market price.

Solution 2 *Assuming they are symmetric* $Q_{-1} = (n-1)\,q_1 = 10q_1$ *and then:*

$$\begin{aligned} q_1 &= 6 - \frac{1}{2}(10q_1) \\ q_1 &= 1 \\ Q &= 11 \\ P &= 49 - 3 * 11 = 16 \end{aligned}$$

Without assuming a symmetric solution we can sum the first order conditions:

$$(49 - 3Q) - 3q_1 - 13 = 0$$

$$\begin{aligned} 11(49 - 3Q) - 3Q - 11(13) &= 0 \\ 396 - 36Q &= 0 \\ Q &= 11 \end{aligned}$$

and

$$\begin{aligned} (49 - 3(11)) - 3q_1 - 13 &= 0 \\ q_1 &= 1 > 0 \end{aligned}$$

so our assumption that each firm produced a postive output was correct.

2. Consider a Hotelling location model. There are two firms (a and b) and price is fixed and strictly above the constant marginal cost of either firm. There are $L = \{1, 2, 3, ..6\}$ locations and both firms choose a location, $l_a \in L$ and $l_b \in L$. Consumers are at a location and buy from the store that is closest to them, or consumer $i \in I$ at location $l_i \in L$ goes to a if $(l_a - l_i)^2 < (l_b - l_i)^2$, to b if $(l_a - l_i)^2 > (l_b - l_i)^2$ and visit either store with probability $\frac{1}{2}$ if $(l_a - l_i)^2 = (l_b - l_i)^2$. $D_a(l_a, l_b)$ is the total number of customers (on average) that buy from firm a when the location of firm a is l_a and the location of firm b is l_b. The number of consumers at each location are:

1	2	3	4	5	6	*Total*
10	8	4	8	10	2	42

(a) Find the best response of firm a to each location of firm b. For location 3 find the demand of firm one $(D_a(l_a, l_b))$ for each location she can choose, for the rest of the locations you only need to fill in the demand enough to convince me you have found the best response.

if $l_b =$	1	2	3	4	5	6
$BR_a(l_b) =$	2	3	3	3	4	5
$D_a(1, l_b) =$	21	10	14	18	20	22
$D_a(2, l_b) =$	$\underline{32}$	21	18	20	22	26
$D_a(3, l_b) =$	28	$\underline{24}$	$\underline{21}$	$\underline{22}$	26	30
$D_a(4, l_b) =$	24	22	20	21	$\underline{30}$	35
$D_a(5, l_b) =$	22	20	16	12	21	$\underline{40}$
$D_a(6, l_b) =$	20	16	12	7	2	21

(b) Explain why for a general Hotelling model $BR_a(l_b) \in \{l_b - 1, l_b, l_b + 1\}$.

Solution 3 *Because of* business stealing. *By choosing* $\{l_b - 1, l_b + 1\}$ *they can take all the customers either to the left or right, if they choose* l_b *then they can compete for every customer.*

(c) Explain why you do not need to find the best responses for firm b.

Solution 4 *Since they have the same objective function, they will have a symmetric best response.*

(d) Find the Nash equilibrium locations of firms a and b.

Solution 5 *By definition it is a* l *such that* $l = BR_a(BR_b(l))$*, the only point that satisfies this condition is* 3.

3. Sami's Superb Strawberries are famous throughout all of Turkey. No one buys strawberries that aren't grown by Sami the Snake. He owns all of the lowlands used in Turkey for growing strawberries, and has the cost function

$$c_s(q_s, k_s) = \begin{cases} 2q_s & \text{if} \quad q_s \leq k_s \\ \infty & \text{if} \quad q_s > k_s \end{cases}.$$

Now Yoruk the Yalya Yilan (OK, he's still a snake, but he's a highland meadow snake) has realized that mountain strawberries are more delicious than lowland strawberries and decided to enter the market. His costs, of course, are higher, they are:

$$c_y(q_y, k_y) = \begin{cases} 18q_y + 12 & \text{if} \quad q_y \leq k_y \\ \infty & \text{if} \quad q_y > k_y \end{cases}$$

Even though Yoruk may be right about mountain strawberries being more delicious Turks still treat them as perfect substitutes for Sami's strawberries, resulting in the inverse demand curve of:

$$P = 58 - 2Q$$

where $Q = q_s + q_y$. Throughout this problem we assume that firm(s) choose their capacity first, and then choose their price as Bertrand competitors given their fixed capacity. (*Note: You may assume that in every variation of the problem below both Sami and Yoruk produce a positive amount of output.)*

(a) What theoretic model tells us that we can treat this as a Cournot or Stackelberg model where firms just compete by choosing capacities? Explain the intuition of this model. **You may just assume this throughout the rest of the question even if you can not answer this question.**

The model is the Kreps-Shenkman Synthesis. In this model (like here) firms first choose their capacities and then are Bertrand competitors. In the second stage the equilibrium is that the price is set to clear market at the capacities. Thus in the first stage they will act as Cournot or Stackelberg competitors, with the difference that they will be choosing capacities, not outputs.

The reason that the price will exactly clear markets at the capacities is that if at the market price firms are below their capacities they will act like standard Bertrand competitors. If at the market price demand is strictly greater than the sum of capacities they can raise their price without loosing any customers—the customers can't go to the other firm anyway.

(b) Now assume that both of these firms choose their capacities simultaneously, or they act as Cournot competitors.

i. Set up Sami's Objective Function and find his first order conditions.

$$\max_{k_s} \left(58 - 2\left(k_s + k_y\right)\right) k_s - 2k_s$$

$$\left(58 - 2\left(k_s + k_y\right)\right) - 2k_s - 2 = 0$$

ii. Find Sami's Best Response or Reaction Function.

$$\begin{aligned} \left(58 - 2\left(k_s + k_y\right)\right) - 2k_s - 2 &= 0 \\ 14 - \frac{1}{2}k_y &= k_s \end{aligned}$$

iii. Set up Yoruk's Objective Function and find his first order conditions.

$$\max_{k_y} \left(58 - 2\left(k_y + k_s\right)\right) k_y - 18k_y - 12$$

$$\left(58 - 2\left(k_y + k_s\right)\right) - 2k_y - 18 = 0$$

iv. Find Yoruk's Best Response or Reaction Function.

$$\begin{aligned} \left(58 - 2\left(k_y + k_s\right)\right) - 2k_y - 18 &= 0 \\ 10 - \frac{1}{2}k_s &= k_y \end{aligned}$$

v. Find the equilibrium quantity both of them produce and the market price.

$$\begin{aligned}14-\frac{1}{2}\left(10-\frac{1}{2}k_s\right) &= k_s\\ \frac{1}{4}k_s+14-5 &= k_s\\ 12 &= k_s\end{aligned}$$

$$\begin{aligned}10-\frac{1}{2}(12) &= k_y\\ 4 &= k_y\end{aligned}$$

$$\begin{aligned}P &= 58-2(k_s+k_y)\\ &= 58-2(12+4)\\ &= 26\end{aligned}$$

(c) Now assume that Sami chooses his capacity, and then Yoruk chooses his capacity, or these firms act as Stackelberg Competitors with Sami being the Stackelberg Leader.

i. Why don't I have to ask you to set up Yoruk's Objective function or find his best response?

ii. Set up Sami's Objective Function and find his first order conditions.

$$\max_{k_s}\left(58-2\left(k_s+\left(\frac{1}{4}(58-18)-\frac{1}{2}k_s\right)\right)\right)k_s-2k_s$$

$$\max_{k_s}\frac{1}{2}k_s(58-4+18-2k_s)$$

$$\frac{1}{2}(58-4+18-2k_s)-k_s=0$$

iii. Find the equilibrium quantity Sami will produce.

$$\frac{1}{2}(58-4+18-2k_s)-k_s=0$$

$$k_s=18$$

iv. Find Yoruk's equilibrium quantity and the market price.

$$\begin{aligned}\frac{1}{4}(58-18)-\frac{1}{2}k_s &= k_y\\ 10-\frac{1}{2}(18) &= k_y\\ 1 &= k_y\end{aligned}$$

$$\begin{aligned} P &= 58 - 2(k_s + k_y) \\ &= 58 - 2(18 + 1) \\ &= 20 \end{aligned}$$

(d) Now assume that Yoruk chooses his capacity, and then Sami chooses his capacity, or these firms act as Stackelberg Competitors with Yoruk being the Stackelberg Leader.

i. Why don't I have to ask you to set up Sami's Objective function or find his best response?

ii. Set up Yoruk's Objective Function and find his first order conditions.

$$\max_{k_y} \left(58 - 2\left(k_y + \left(\frac{1}{4}(58-2) - \frac{1}{2}k_y\right)\right)\right) k_y - 18k_y - 12$$

$$\max_{k_y} \frac{1}{2}58k_y - 12 + k_y - 18k_y - k_y^2$$

$$\frac{1}{2}58 + \frac{1}{2}2 - 18 - 2k_y = 0$$

iii. Find the equilibrium quantity Yoruk will produce.

$$\frac{1}{2}58 + \frac{1}{2}2 - 18 - 2k_y = 0$$

$$6 = k_y$$

iv. Find Sami's equilibrium quantity and the market price.

$$\begin{aligned} \frac{1}{4}(58-2) - \frac{1}{2}k_y &= k_s \\ 14 - \frac{1}{2}(6) &= k_s \\ 11 &= k_s \end{aligned}$$

$$\begin{aligned} P &= 58 - 2(k_s + k_y) \\ &= 58 - 2(11 + 6) \\ &= 24 \end{aligned}$$

(e) It is natural to assume that Sami gets to choose his capacity first, so imagine that Yoruk has appealed to the Turkish Government to enforce either a level playing field (they both choose their capacities simultaneously, part *b*) or to force Sami to choose his capacity second (part *d*). Assuming that the Turkish government only cares about Consumer Surplus, will they agree to his request? Also tell me how they would order the three possible equilibria, the equilibrium in part *b*, *c*, and *d*. *Hint: No further calculation is required for this part. Indeed the answer could be based on a-priori logic.*

Since the Turkish Government wants to maximize Consumer Surplus all they really care about is the price under the alternative models. We know that when a firm is the Stackelberg leader they produce more output, and the follower cuts back his output by half the increase of the leader, so this tells us $P_d < P_b$ and $P_c < P_b$. Obviously it is better to have the lower cost firm produce more output, so $P_c < P_d$. This is the default state, so the Turkish government should tell Yoruk to just live with it.

4. Consider a standard Hoetelling model. There are a finite number of locations $l \in (1, 2, 3, 4, 5)$, and a fixed number of customers at each location given by this table:

1	2	3	4	5
8	6	2	16	2

There are two firms, firm A has location l_A and firm B has location l_B. (We allow $l_A = l_B$). For fixed locations (l_A, l_B) customers always buy from the firm that is the closest, and split their business if both firms are equally close. Firm's choose their location to maximize the number of customers they have.

(a) For each location of firm B, $l_B \in (1, 2, 3, 4, 5)$ find the best response of firm A. You can use the table below to help you, and at least for one location you should figure out the profit of each location for firm A.

$l_B =$	1	2	3	4	5
$BR_A(l_B) =$	2	3	4	4	4
$\Pi_A(1, l_B) =$	17	8	11	14	15
$\Pi_A(2, l_B) =$	26	17	14	15	16
$\Pi_A(3, l_B) =$	23	20	17	16	24
$\Pi_A(4, l_B) =$	20	19	18	17	32
$\Pi_A(5, l_B) =$	19	18	10	2	17

(b) Why can you assume that the best responses of firm B are the same as the best responses of firm A?

Because they have the same objective function, or the game is symmetric.

(c) Find the Nash equilibrium of this game.

It is always $l_A = l_B = l_M$ where l_M is the location of the median customer. This location is 4 in this case.

5. Giggles the Great Ape Grape Grower owns all the lowlands in Tanzia, the inverse demand curve for grapes is:

$$P = 16 - \frac{1}{2}Q$$

and his costs are $c_g(Q) = 6Q$.

(a) At first she is a monopolist.

i. Find Giggles's objective function.

$$\max_{Q} \left(16 - \frac{1}{2}Q\right) Q - 6Q$$

ii. Find her first order condition.

$$\left(16 - \frac{1}{2}Q\right) - \frac{1}{2}Q - 6 = 0$$

iii. Find the quantity she will produce, the price she sells grapes for, and her profits. (For clarity you should assume she can only charge a per-unit price.)

$$\begin{aligned} 10 &= Q \\ P &= 16 - \frac{1}{2}(10) = 11 \end{aligned}$$

$$\Pi_g^m = PQ - 6Q = (P-6)\,Q = (11-6)\,10 = 50$$

(b) Now Henry the Highland Goat has gotten interested in growing grapes. Unfortunately he has to grow them in the high hills, thus he has higher costs and a large fixed cost of setting up production. His cost function is $c_h(q_h) = 8q_h + 1$. At this point Giggles can not commit to any fixed level of production, so if Henry produces Henry and Giggles act as Cournot Competitors.

i. Set up the objective function of both firms.

$$\max_{q_g} \left(16 - \frac{1}{2}(q_h + q_g)\right) q_g - 6q_g$$

$$\max_{q_h} \left(16 - \frac{1}{2}(q_h + q_g)\right) q_h - 8q_h - 1$$

ii. Find their first order conditions.

$$\begin{aligned} \left(16 - \frac{1}{2}(q_h + q_g)\right) - \frac{1}{2}q_g - 6 &= 0 \\ \left(16 - \frac{1}{2}(q_h + q_g)\right) - \frac{1}{2}q_h - 8 &= 0 \end{aligned}$$

iii. Find their best responses or reaction functions.

$$\begin{aligned} 10 - \frac{1}{2}q_h &= q_g \\ 8 - \frac{1}{2}q_g &= q_h \end{aligned}$$

iv. Find the equilibrium quantities they will produce, the market price, and the profits of both firms.

$$\begin{aligned}
8-\frac{1}{2}\left(10-\frac{1}{2}q_h\right) &= q_h \\
3+\frac{1}{4}q_h &= q_h \\
3 &= \frac{3}{4}q_h \\
q_h &= 4 \\
10-\frac{1}{2}(4) &= 8=q_g
\end{aligned}$$

$$P=16-\frac{1}{2}(4+8)=10$$

$$\begin{aligned}
\Pi_g^c &= (P-6)\,q_g=(10-6)\,8=32 \\
\Pi_h^c &= (P-8)\,q_h-1=(11-8)\,4-1=11
\end{aligned}$$

(c) Now Giggles realizes she can commit to a fixed level of production, so she can effectively choose her level of output before Henry chooses his. In other words she can act as a Stackelberg leader.

i. Explain the general principle that says this commitment will always have some value to Giggles. Your answer should apply to all oligopoly games, not just this particular game.
This is First Mover's Advantage, the first mover in a sequential variant of a game can always get a higher payoff than in any (pure strategy) Nash equilibrium of the simultaneous variant of the same game.

ii. Explain why Henry's best response or reaction function will be the same in this game as it was in the Cournot game.
Because now he is being asked what he will produce **given** *Giggles has produced X, while before he was being asked what to produce when he* **expected** *Giggles would produce X. The answer to the two questions will always be the same.*

iii. Find Giggle's new objective function.

$$\max_{q_g}\left(16-\frac{1}{2}(q_h+q_g)\right)q_g-6q_g$$

given

$$8-\frac{1}{2}q_g=q_h$$

$$\max_{q_g}\left(16-\frac{1}{2}\left(\left(8-\frac{1}{2}q_g\right)+q_g\right)\right)q_g-6q_g$$

$$\max_{q_g}\frac{1}{4}q_g\,(24-q_g)$$

iv. Find her first order condition.

$$\frac{1}{4}(24 - q_g) - \frac{1}{4}q_g = 0$$

v. Find the quantity Giggle's will produce, the quantity Henry will produce, and the market price.

$$\begin{aligned} \frac{1}{4}(24 - q_g) - \frac{1}{4}q_g &= 0 \\ 24 &= 2q_g \\ 12 &= q_g \\ 8 - \frac{1}{2}(12) &= 2 = q_h \end{aligned}$$

$$P = 16 - \frac{1}{2}(12 + 2) = 9$$

A. Find the profits of both firms in this game, use this to explain the value of commitment to Giggles.

$$\begin{aligned} \Pi_g^s &= (P - 6) q_g = (9 - 6) 12 = 36 \\ \Pi_h^s &= (P - 8) q_h - 1 = (9 - 8) 2 - 1 = 1 \end{aligned}$$

Since $\Pi_g^s > \Pi_g^c$ *Giggles has a strictly positive value of being able to commit. Giggles would be willing to pay up to* $\Pi_g^s - \Pi_g^c = 36 - 32 = 4$ *to be able to commit, because this is the amount her profits will increase.*

(d) Now assume that Henry's fixed costs are 6 instead of 1. Explain why you do not need to recalculate Henry's best responses when you make this change. Given this find the equilibrium of both games you just analyzed (the Cournot and Stackelberg), and use the profits in equilibrium to express Giggle's **new** value to being able to commit.

If Henry produces, then the amount he will want to produce will be determined by his first order condition, which will not be affected by his fixed costs. So assuming Henry produces he will produce the same amount. But then if $q_g = 12$ $q_h = 2$ *and the profits of Henry are now reduced by 5, so he will not produce. Thus Giggles should produce assuming that Henry does not, or she should produce* 10. *To be precise (which is not required for the answer) you should check that Henry will not want to produce if Giggles produces* 10, $q_h = 8 - \frac{1}{2}(10) = 3$ $\Pi_h = \left(16 - \frac{1}{2}(10 + 3)\right) 3 - 8 * 3 - 6 = -\frac{3}{2} < 0$, *so Henry will still not want to produce. Thus now Giggles is a Monopolist. When they produced simultaneously,* $\Pi_h^c = 11$, *so in the simultaneous game Henry will still want to produce if his fixed costs increase by 5. So now* $\Pi_g^s = \Pi_g^m = 50$, *and her value of being able to commit has increased from* 4 *to* $\Pi_g^m - \Pi_g^c = 50 - 32 = 18$.

6. Consider a standard Hoetelling model. There are a finite number of locations $l \in (1, 2, 3, 4, 5)$, and a fixed number of customers at each location given by this table:

1	2	3	4	5
6	12	8	4	4

There are two firms, firm A has location l_A and firm B has location l_B. (We allow $l_A = l_B$). For fixed locations (l_A, l_B) customers always buy from the firm that is the closest, and split their business if both firms are equally close. Firm's choose their location to maximize the number of customers they have.

(a) For each location of firm B, $l_B \in (1, 2, 3, 4, 5)$ find the best response of firm A. You can use the table below to help you, and at least for one location you should figure out the profit of each location for firm A.

$l_B =$	1	2	3	4	5
$BR_A(l_B) =$	2	2	2	3	4
$\Pi_A(1, l_B) =$	17	6	12	18	22
$\Pi_A(2, l_B) =$	28	17	18	22	26
$\Pi_A(3, l_B) =$	22	16	17	26	28
$\Pi_A(4, l_B) =$	16	12	8	17	30
$\Pi_A(5, l_B) =$	12	8	6	4	17

(b) Why can you assume that the best responses of firm B are the same as the best responses of firm A?

Because they have the same payoff, so obviously they must have the same best responses. To be precise their payoffs are symmetric.

(c) Find the Nash equilibrium of this game.

It is always $l_A = l_B = l_M$ where l_M is the location of the median customer. This location is 2 in this case.

(d) Why can't the Nash equilibrium be welfare optimal if your welfare function decreases in the distance customers have to travel?

Since in equilibrium $l_A = l_B$ I can always get a better outcome by enforcing that $l_A < l_B$, in this alternative model everyone has to travel a shorter distance and the total profits of the firms are the same. Thus this can not be welfare optimal.

7. Consider two firms that are competing in a market by choosing their quantity, the price is then set to clear the market. The costs of both firms are $c_1(q_1) = 12q_1$ and $c_2(q_2) = 12q_2$ and the market demand curve is $P = 24 - \frac{1}{2}Q$, where $Q = q_1 + q_2$.

(a) Assume that these firms choose quantity simultaneously, or act as Cournot Competitors.

i. Set up the objective function of one of the firms.

$$\left(24 - \frac{1}{2}(q_1 + q_2)\right) q_1 - 12q_1$$

ii. Find that firm's reaction function.

$$\begin{aligned} 24 - q_1 - \frac{1}{2}q_2 - 12 &= 0 \\ 12 - \frac{1}{2}q_2 &= q_1 \end{aligned}$$

iii. Why didn't I ask you to set up the objective function of both firms? How can this help you solve the rest of this problem? *Because they have the same profit function it was unnecessary, it can be found by symmetry, or exchanging q_1 and q_2 in the reaction function of firm 1. This can be helpful because when symmetry occurs usually there will be an equilibrium where both firms produce the same quantity.*

iv. Find the equilibrium quantity produced by both firms and the equilibrium price.

$$\begin{aligned} (24 - 12) - \frac{1}{2}q &= q \\ q &= \frac{1}{3(\frac{1}{2})}(24 - 12) = 8 \\ Q &= \frac{2}{3(\frac{1}{2})}(24 - 12) = 16 \\ P &= 24 - \frac{1}{2}\frac{2}{3(\frac{1}{2})}(24 - 12) = \frac{1}{3}24 + \frac{2}{3}12 = 16 \end{aligned}$$

v. Find the profit of both firms in equilibrium.

$$\begin{aligned} \pi &= (P - c)\, q \\ \pi^c &= \left(\frac{1}{3}24 + \frac{2}{3}12 - 12\right)\frac{1}{3(\frac{1}{2})}(24 - 12) = \frac{1}{9(\frac{1}{2})}(24 - 12)^2 = 32 \end{aligned}$$

(b) Now assume that firm 1 is the incumbent, and assume that means they get to choose their output first, then firm 2 gets to choose their output after they have seen the output of firm 1.

i. Find the reaction function of firm 2. *Hint: No calculation is required to do this.*

$$12 - \frac{1}{2}q_1 = q_2$$

this is because the second firm will have the same reaction function in a Cournot and Stackelberg game.

ii. Set up firm 1's objective function.

$$\left(24 - \frac{1}{2}\left(q_1 + q_2\left(q_1\right)\right)\right) q_2 - 12q_1$$
$$\left(24 - \frac{1}{2}\left(q_1 + \left(\left(24-12\right) - \frac{1}{2}q_1\right)\right)\right) q_1 - 12q_1$$
$$-\frac{1}{2}q_1\left(-12 + \frac{1}{2}q_1\right) = 6q_1 - \frac{1}{4}q_1^2$$

iii. Find the equilibrium quantity produced by both firms and the equilibrium price.

$$\begin{aligned}
-\frac{1}{2}\left(12 - 24 + \frac{1}{2}q_1\right) - \frac{1}{2}q_1\frac{1}{2} &= 0 \\
q_1 &= (24-12) = 12 \\
\frac{1}{2}(24-12) &= (24-12) - \frac{1}{2}(24-12) = q_2 = 6 \\
Q &= \frac{1}{2(\frac{1}{2})}(24-12) + \frac{1}{2}(24-12) = \frac{3}{2}(24-12) = 18 \\
P &= 24 - \frac{1}{2}\frac{3}{2}(24-12) = \frac{1}{4}24 + \frac{3}{4}12 = 6 + 9 = 15
\end{aligned}$$

iv. Find the profit of both firms in equilibrium.

$$\begin{aligned}
P - 12 &= \frac{1}{4}24 + \frac{3}{4}12 - 12 = 15 \\
\pi_1^s &= (P-12)\,q_1 = \frac{1}{4}24 - \frac{1}{4}12)\,(24-12) = \frac{1}{4}(24-12)^2 = 36 \\
\pi_2^s &= \left(\frac{1}{4}24 - \frac{1}{4}12\right)\frac{1}{4(\frac{1}{2})}(24-12) = \frac{1}{8}(24-12)^2 = 18
\end{aligned}$$

(c) Now assume that there is a fixed start up cost of $F = 60$, briefly explain how the equilibrium in both models would change.

First of all in both models it is clear that there will never be two firms, because the profits above are never higher than 60. *However it may be that one firm enters and produces as a monopolist, this would happen if profits are high enough. Notice that the monopoly output is found by setting* $q_2 = 0$ *in the reaction function.*

$$\begin{aligned}
\frac{1}{2\left(\frac{1}{2}\right)}(24-12)-\frac{1}{2}(0) &= q_1 = 12 \\
P &= 24-\frac{1}{2}\left(\frac{1}{2(\frac{1}{2})}(24-12)\right)=\frac{1}{2}24+\frac{1}{2}12=18 \\
\pi &= \frac{1}{2}(24-12)^2 = 72 > 60
\end{aligned}$$

So in equilibrium one firm will enter and the other will stay out. In the Stackelberg model it must be firm 1, in the Cournot equilibrium it could be either firm.

This can also be found by solving the problem directly.

8. Consider the standard Bertrand model, where firms split demand if they charge the same price and get all of demand if they charge a strictly lower price than their competitors. Assume that all firms have the same constant marginal cost.

   (a) What will be the equilibrium price?
   Price will be equal to marginal cost.

   (b) Explain why this will be the equilibrium.
   Because if they charge a higher price than one of the other firms will undercut them by a little amount to capture all of the demand.

   (c) What is the underlying assumption of the model that makes this the equilibrium? Is this assumption reasonable? Why or why not?
   It is because the cross-price elasticity of demand when all firms are charging the same price is infinite. In other words if all of your competitors raise their price by a trivial amount then you will get all of the demand. This is not reasonable, even in essentially perfectly competitive markets like the market for simits very few people would go to a different simitci just because one simitci was charging 5 kurus more. This is obvious because almost no one in Turkey gives change in exact kurus. They almost always round it up to the nearest 5 kurus or 10 kurus. If people were this price sensitive store owners would not follow this practice.

9. Assume that there are two firms who act by choosing quantities, price is then set so that the market clears. The demand curve in their market is $P = 48 - Q$ and the costs for firm one are $c_1(q_1) = 8q_1$ and for firm 2 are $c_2(q_2) = 16q_2$.

   (a) If they act as Cournot Competitors:

      i. Find the objective functions for both firms.

$$\max_{q_1} \left((48) - (1) \left(q_1 + q_2 \right) \right) q_1 - (8) q_1$$
$$\max_{q_2} \left((48) - (1) \left(q_1 + q_2 \right) \right) q_2 - (16) q_2$$

ii. Find the first order conditions for both firms.

$$\begin{aligned} (48) - (1) \left(q_1 + q_2 \right) - (1) q_1 - (8) &= 0 \\ (48) - (1) \left(q_1 + q_2 \right) - (1) q_2 - (16) &= 0 \end{aligned}$$

iii. Find the reaction functions for both firms.

$$\begin{aligned} 48 - \left(q_1 + q_2 \right) - q_1 - 8 &= 0 \\ 48 - \left(q_1 + q_2 \right) - q_2 - 16 &= 0 \end{aligned}$$

$$\begin{aligned} q_1 &= R_1 \left(q_2 \right) = 20 - \frac{1}{2} q_2 \\ q_2 &= R_2 \left(q_1 \right) = 16 - \frac{1}{2} q_1 \end{aligned}$$

iv. Find the quantity each firm produces in equilibrium, the market quantity, the price, and both firm's profits.

$$\begin{aligned} q_1 &= R_1 \left(R_2 \left(q_1 \right) \right) \\ q_1 &= 20 - \frac{1}{2} \left(16 - \frac{1}{2} q_1 \right) \\ q_1 &= \frac{1}{4} q_1 + 12 \\ q_1^c &= 16 \\ q_2 &= R_2 \left(q_1 \right) \\ &= 16 - \frac{1}{2} (16) \\ q_2^c &= 8 \end{aligned}$$

$$\begin{aligned} P^s &= (48) - Q \\ &= (48) - 28 \\ &= 20 \end{aligned}$$

: :

$$\begin{aligned} \Pi_1^s &= (20 - 8)\, 24 \\ \Pi_1^s &= 288 \\ \Pi_2^s &= 16 \end{aligned}$$

(b) If they act as Stackelberg competitors, firm 1 choosing quantity first.

i. Find firm 2's reaction function.

$$\max_{q_2} \left((48) - (q_1 + q_2)\right) q_2 - (16)q_2$$
$$(48) - (q_1 + q_2) - q_2 - (16) = 0$$
$$q_2 = R_2(q_1) = 16 - \frac{1}{2}q_1$$

not many points are given to this question since it is already answered above, and the answer can just be copied here

ii. Find firm 1's objective function.

$$\max_{q_1} \left((48) - (q_1 + R_2(q_1))\right) q_1 - (8)q_1$$
$$\max_{q_1} \left((48) - \left(q_1 + 16 - \frac{1}{2}q_1\right)\right) q_1 - (8)q_1$$

:

iii. Find firm 1's first order condition.

$$\left(48 - \left(q_1 + 16 - \frac{1}{2}q_1\right)\right) - \left(1 - \frac{1}{2}\right) q_1 - 8 = 0$$
$$\frac{1}{2}(48) + \frac{1}{2}(16) - (1)q_1 - (8) = 0$$

iv. Find the quantity firms 1 and 2 produce in equilibrium, the market price, and the profit of both firms.

$$\begin{aligned}
q_1^s &= 24 \\
q_2 &= 16 - \frac{1}{2}q_1 \\
q_2^s &= 4 \\
Q &= q_1 + q_2 = \frac{1}{2(1)}\left((48) - 2(8) + (16)\right) + \frac{1}{4(1)}\left((48) + 2(8) - 3(16)\right) \\
&= 28
\end{aligned}$$

$$\begin{aligned}
P^s &= (48) - Q \\
&= (48) - 28 \\
&= 20
\end{aligned}$$

$$\begin{aligned}
\Pi_1^s &= (20 - 8)\, 24 \\
\Pi_1^s &= 288 \\
\Pi_2^s &= 16
\end{aligned}$$

v. Without any mathematical analysis guess at whether the consumers would prefer the high cost or the low cost firm choosing quantity first in this model. Explain your reasoning.
It is better to have the low cost firm produce first. The intuitive reason for this is that in the Stackelberg equilibrium the firm that produces first produces more, since the low cost firm is already producing more giving them this additional incentive is probably best. This is born out by this example, but showing this is not part of a good answer.

$$P^s\left(1 \text{ first}\right) = 20 < 22 = P^s\left(2 \text{ first}\right)$$

if you give the wrong answer but argue it clearly then you can get full credit for this question.

(c) Is the Stackelberg model a Pareto Improvement over the Cournot Model? Who is better off with the equilibrium in the Stackelberg model? Who is worse off? Explain your reasoning. (Please consider all three parties, the two firms and the consumers.)

- *Consumers—get a lower price and so they are happier with the Stackelberg equilibrium.*

$$P^s = 20 < 24 = P^c$$

- *Firm 1—gets a higher profit by first mover's advantage:*

$$\Pi_1^c = 256 < 288 = \Pi_1^s$$

- *Firm 2—gets a lower profit.*

$$\Pi_2^c = 256 > 16 = \Pi_2^s$$

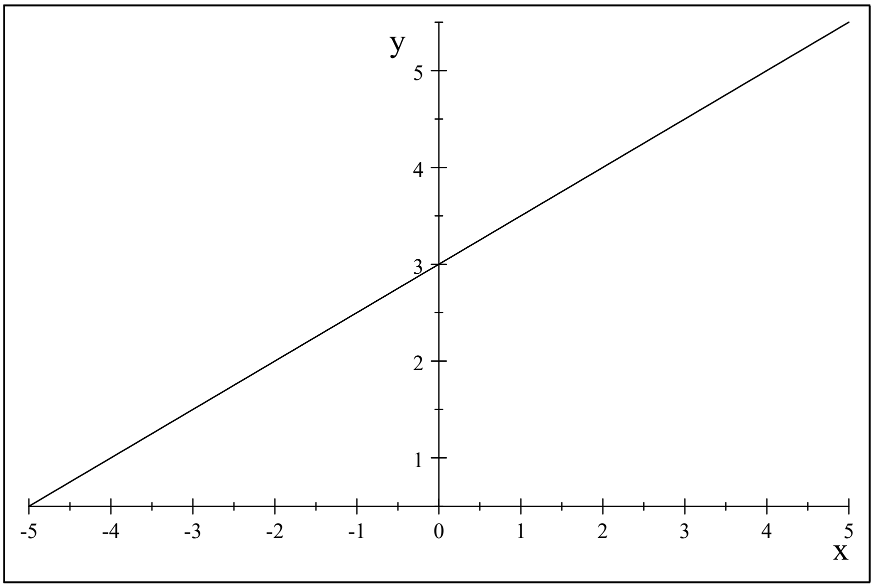
Thus this is not a Pareto improvement because firm 2 does worse, even though consumers and firm 1 does better.

10. Consider a model of Differentiated Bertrand, where both firms choose the price that maximizes their profits and meet the quantity demanded at that price. The demand of firm one is $q_1 = 44 - 11p_1 + 11p_2$ and of firm two is $q_2 = 44 - 11p_2 + 11p_1$ and the cost function is $c\left(q\right) = 2q$.

(a) Set up firm 2's profit maximization problem and find his best response. Use symmetry to find the best response of firm 1. Graph their best responses in the graph below.

$$\begin{aligned} \pi &= \left(p_1 - c\right) q_1\left(p_1, p_2\right) \\ \pi &= \left(p_1 - 2\right)\left(44 - 11p_1 + 11p_2\right) \end{aligned}$$

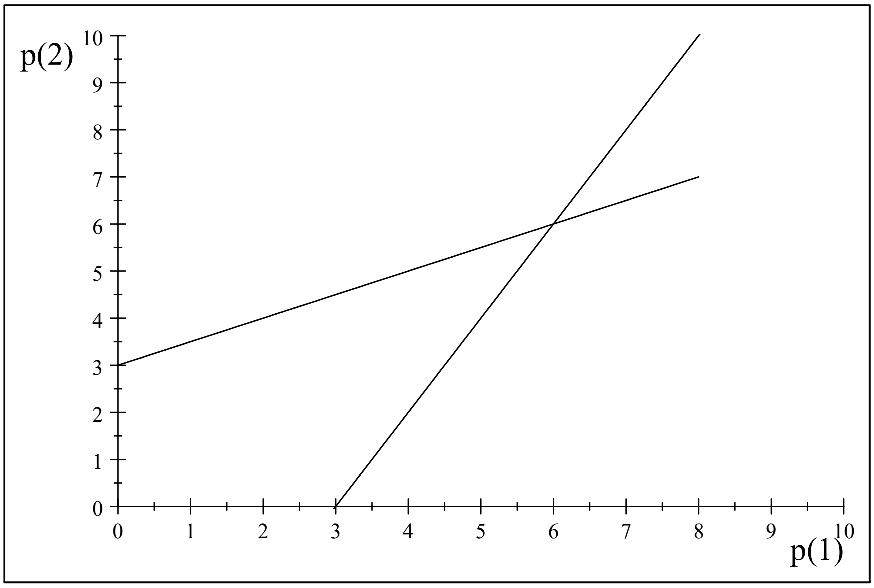
$$\begin{aligned}(44-11p_1+11p_2)-11\,(p_1-2) &= 0\\ 11p_2-22p_1+66 &= 0\\ p_1 &= \frac{1}{2}p_2+3\end{aligned}$$



by symmetry:

$$p_2=\frac{1}{2}p_1+3$$

the best responses look like the graph below. Notice they are upward sloping.



(b) Find the equilibrium price both firms charge, the quantity they sell, and the profits of both firms.

$$\begin{aligned} p &= \frac{1}{2}p + 3 \\ p &= 6 \\ q_1 &= q_2 = 44 - 11\,(6) + 11\,(6) = 44 \\ \pi &= (6-2)\,44 = 176 \end{aligned}$$

Now consider a variation on the model where firm 1 sets its price first, and then firm 2 sets its price after observing firm 1's price.

(c) Argue that firm 2's best response will be the same as in the first part of this problem.

The only difference between the two models is that in the first model firm 2 chooses its price assuming firm 1 has a constant price, in part 2 firm 2 chooses it's price knowing the fixed price of firm 1, thus the BR is the same.

$$p_2 = \frac{1}{2}p_1 + 3$$

(d) Using this set up firm 1's problem and find the price it will charge and the quantity it will sell. Also do this for firm 2 and find the profits of both firms.

$$\begin{aligned}
q_1 &= 44 - 11p_1 + 11p_2(p_1) \\
&= 44 - 11p_1 + 11\left(\frac{1}{2}p_1 + 3\right) \\
&= 77 - \frac{11}{2}p_1 \\
&\max_{p_1}(p_1 - 2)\left(77 - \frac{11}{2}p_1\right)
\end{aligned}$$

$$\begin{aligned}
\left(77 - \frac{11}{2}p_1\right) - \frac{11}{2}(p_1 - 2) &= 0 \\
88 - 11p_1 &= 0 \\
p_1 &= 8 \\
q_1 &= 77 - \frac{11}{2}(8) = 33 \\
\Pi_1 &= (8 - 2)\,33 = 198 \\
p_2 &= \frac{1}{2}(8) + 3 = 7 \\
q_2 &= 44 - 11(7) + 11(8) = 55 \\
\pi_2 &= (7 - 2)\,55 = 275
\end{aligned}$$

(e) Would both firms be willing to sign a legally binding document making sure that firm 1 chooses its price first and will not change its price after firm 2 sets its price? How much would firm 2 be willing to pay to be the second firm to set their price? Is there any way that these firms could implicitly come to such an agreement?

Yes, firm 2 would be willing to pay the difference in the profits between the first and second mover, or $275 - 198 = 77$. *It might be hard to get an implicit agreement to do this, but both firms would prefer to do it.*

11. .Assume there are two Cournot Competitors in a market with a demand curve of $Q = 144 - 3P$. Firm one has the cost function of $C_1(q_1) = 2q_1$, firm 2 has a cost function of $C_2(q_2) = q_2$.

   (a) Set up both firms' objective functions.

$$\begin{aligned}
P &= 48 - \frac{1}{3}Q \\
&\max_{q_1}\left(48 - \frac{1}{3}(q_1 + q_2)\right)q_1 - 2q_1 \\
&\max_{q_2}\left(48 - \frac{1}{3}(q_1 + q_2)\right)q_2 - q_2
\end{aligned}$$

(b) Find the first order conditions and their reaction functions.

$$\begin{aligned}
\left(48 - \frac{1}{3}(q_1 + q_2)\right) - \frac{1}{3}q_1 - 2 &= 0 \\
q_1 &= 69 - \frac{1}{2}q_2 \\
\left(48 - \frac{1}{3}(q_1 + q_2)\right) - \frac{1}{3}q_2 - 1 &= 0 \\
q_2 &= \frac{141}{2} - \frac{1}{2}q_1
\end{aligned}$$

(c) Find the quantity produced and the price in the Cournot equilibrium.

$$\begin{aligned}
q_1 &= 69 - \frac{1}{2}q_2 \\
q_2 &= \frac{141}{2} - \frac{1}{2}\left(69 - \frac{1}{2}q_2\right) \\
q_2 &= 48 \\
q_1 &= 69 - \frac{1}{2}(48) = 45 \\
P &= 48 - \frac{1}{3}Q = 48 - \frac{1}{3}(45 + 48) = 17
\end{aligned}$$

(d) What is the Pareto Efficient price? Which firm will not produce if the two firms produce efficiently?

$$\begin{aligned}
P &= MC = \min_{1,2}\{MC_1, MC_2\} = 1 \\
Q &= 144 - 3P = 141
\end{aligned}$$

so firm one will not produce.

(e) Discuss the two types of inefficiencies in a Cournot Equilibrium, using this question to illustrate both of them.

Under production—the firms are producing too little.

Inefficient production—firm one should not produce but it does.

12. Assume that a monopolist faces the demand curve $Q = 32 - 2P$, and that their cost function is $c(q) = 30 + 4q$. (Note that all fixed costs are start up costs.)

(a) Find the monopoly quantity and price.

i. What is their marginal revenue (Notice this should be in terms of Q, not P).

$$
\begin{aligned}
P &= 16 - \frac{1}{2}Q \\
R(Q) &= PQ = \left(16 - \frac{1}{2}Q\right)Q \\
MR &= 16 - Q
\end{aligned}
$$

ii. What Quantity will they produce?

$$
\begin{aligned}
MR &= 16 - Q = 4 = MC \\
Q &= 12
\end{aligned}
$$

iii. What price will they sell at?

$$
P = 16 - \frac{1}{2}Q = 16 - \frac{1}{2}(12) = 10
$$

iv. What Profits will they make?

$$
\begin{aligned}
R(Q) &= \left(16 - \frac{1}{2}(12)\right)12 = 120 \\
C(Q) &= 30 + 4q = 30 + 4(12) = 78 \\
\Pi &= 42
\end{aligned}
$$

(b) Assume there is a new firm with a new technology that enters the market, they have a cost function $c(q) = 4q$. Assuming that the incumbent knows about this entry (and therefore will choose their quantity first) find the quantities produced by the entrant and the incumbent and the market price.

i. Find what quantity the entrant will produce as a function of the quantity the incumbent produces.

A. Find the entrant's marginal revenue.

$$
\begin{aligned}
Pq_e &= \left(16 - \frac{1}{2}(q_i + q_e)\right)q_e \\
MR &= 16 - \frac{1}{2}q_i - q_e
\end{aligned}
$$

B. Find the entrant's reaction function.

$$
\begin{aligned}
MR &= 16 - \frac{1}{2}q_i - q_e = 4 = MC \\
q_e &= 12 - \frac{1}{2}q_i
\end{aligned}
$$

ii. Find what quantity the incumbent will produce.

A. Set up the incumbent's objective function.

$$
\begin{aligned}
&\max_{q_i} \left(16 - \frac{1}{2}\left(q_i + q_e\left(q_i\right)\right)\right) q_i - 4q_i - 30 \\
&\max_{q_i} \left(16 - \frac{1}{2}\left(q_i + \left(12 - \frac{1}{2}q_i\right)\right)\right) q_i - 4q_i - 30 \\
&\max_{q_i} 6q_i - \frac{1}{4}q_i^2 - 30
\end{aligned}
$$

B. Solve the first order condition for the quantity they will produce.

$$
\begin{aligned}
6 - \frac{2}{4}q_i &= 0 \\
q_i &= 12
\end{aligned}
$$

C. What will be the price in this market?

$$
\begin{aligned}
q_e &= 12 - \frac{1}{2}q_i = 12 - \frac{1}{2}(12) = 6 \\
Q &= 18 \\
P &= 16 - \frac{1}{2}Q = 16 - \frac{1}{2}(18) = 7
\end{aligned}
$$

D. What will be the profits of the incumbent and the entrant?

$$
\begin{aligned}
\Pi_i &= 7 * 12 - 4 * 12 - 30 = 6 \\
\Pi_e &= 7 * 6 - 4 * 6 = 18
\end{aligned}
$$

(c) Assume that the incumbent's fixed start up costs are higher, specifically that they are 70 instead of 30. What will happen when the entrant tries to enter? I do not want to know what the price and quantity will be, just explain what will occur. Answering this question does not require an answer to part *b*, a good guess well defended by an argument will be sufficient.

The incumbent will have to shut down because their fixed start up costs are to high. You can see this because their profit as a monopolist is only 6 when $F_{st} = 30$ *so if the entrant produces at all they will have negative profits. To be precise form part b you can see that their profits will be* -34.

13. Tekel used to be the monopoly producer of Raki in Turkey. Assume that the demand curve is $Q = 60 - P$ and that Tekel's costs are $c_t(q_t) = 8q_t$.

(a) When Tekel is a Monopolist:

i. Find Tekel's objective function and their first order condition.

$$\begin{aligned} P &= 60 - Q \\ &\max_Q P(Q)Q - C(Q) \\ &\max_Q (60 - Q)Q - (8)Q \end{aligned}$$

$$60 - Q - Q - (8) = 0$$

ii. Find the optimal quantity for Tekel to produce, and the price it will sell Raki at.

$$\begin{aligned} Q_m &= 26 \\ P_m &= 60 - Q = 34 \end{aligned}$$

(b) Now assume that the Turkish government has decided to allow one more firm into the market for Raki, Efe, and that Efe's cost is $c_e(q_e) = 4q_e$.

Assume that these firms are Cournot Competitors.

i. Set up the objective function of Tekel and Efe.

$$\max_{q_t} (60 - (q_t + q_e))\, q_t - 8q_t, \max_{q_e} (60 - (q_t + q_e))\, q_e - 4q_e$$

ii. Find their first order conditions and their reaction functions.

$$(60 - (q_t + q_e)) - q_t - (8) = 0, ((60) - (q_t + q_e)) - q_e - (4) = 0$$

$$q_t = 26 - \frac{1}{2}q_e, q_e = 28 - \frac{1}{2}q_t$$

iii. Find the Cournot-Nash Equilibrium quantities.

$$\begin{aligned} q_t &= 26 - \frac{1}{2}q_e \\ q_t &= 26 - \frac{1}{2}\left(28 - \frac{1}{2}q_t\right) \\ q_t &= \frac{48}{3} = 16 \\ q_e &= 28 - \frac{1}{2}q_t = 24 - \frac{1}{2}(16) = 16 \end{aligned}$$

(c) Now assume that Tekel, as the incumbent firm, chooses their production level first and that Efe chooses their production level second. Or that these firms are in a Stackelberg game, with Tekel as the first mover.

i. Set up the objective function of Efe.

$$\max_{q_e} \left(\left(60\right) - \left(q_t + q_e\right)\right) q_e - \left(4\right) q_e$$

ii. Find Efe's first order conditions and its reaction function.

$$\left(60 - \left(q_t + q_e\right)\right) - q_e - 4 = 0$$
$$q_e = 28 - \frac{1}{2} q_t$$

iii. Set up Tekel's objective function incorporating the reaction function of Efe.

$$\max_{q_t} \left(60 - \left(q_t + q_e\left(q_t\right)\right)\right) q_t - 8q_t$$
$$\max_{q_t} \left(60 - \left(q_t + \left(28 - \frac{1}{2} q_t\right)\right)\right) q_t - 8q_t$$
$$\max_{q_t} \left(32 - \frac{1}{2} q_t\right) q_t - 8q_t$$

iv. Find Tekel's first order condition and how much Tekel will produce.

$$\begin{aligned} \left(32 - \frac{1}{2} q_t\right) - \frac{1}{2} q_t - (8) &= 0 \\ q_t &= 24 \end{aligned}$$

v. Find out how much Efe will produce.

$$q_e = 28 - \frac{1}{2} q_t = 28 - \frac{1}{2}\left(24\right) = 16$$

vi. If Efe was the incumbent firm do you think that more or less output would be produced? Why is assuming that Tekel has higher marginal costs than Efe reasonable?
Either answer is correct if it is well argued. Actually more will be produced, in this problem the low cost firm is more aggressive, and since the first firm naturally produces more (consider the case where they have symmetric costs) more total output will be produced, resulting in lower prices and higher consumer surplus. Unfortunately it is more reasonable to assume that the new firm has newer technology, and thus lower costs.

(d) Assume that Tekel's profits are used to fund government expenditures and that the government cannot tax Efe. Why might having both Tekel and Efe produce not be Pareto Superior to giving Tekel a monopoly?

Essentially a correct answer will realize that using Tekel's profits to fund government projects can be an efficient way of raising revenue, better than taxing people in other markets.

Thus while the owners of Tekirdag will be doing better if competition is allowed, it would be worse for people in general thus allow Tekirdag to compete it is not Pareto superior to allow competition in this market.

14. Assume there are three firms that face a demand curve for Baklava of:

$$Q = 280 - P$$

each firm has a cost function of $c(q) = 2q^2$.

(a) (Competition) Assuming that the firms act as if they are perfect competitors:

i. Solve for their short run supply curve. BE SURE TO FIND THE SHUTDOWN POINT! (The answer is a little surprising but you need to show how to do it.)

$$\begin{aligned} p &= MC \geq AVC \\ MC &= q \\ AVC &= 2q \\ MC &\geq AVC \end{aligned}$$

for all q thus the short run supply is

$$\begin{aligned} p &= 4q \\ q &= \frac{p}{4} \end{aligned}$$

ii. Solve for the aggregate supply curve.

$$Q = 3q = \frac{3}{4}P$$

iii. Solve for the equilibrium price and quantity.

$$\begin{aligned} \frac{3}{4}P &= Q = 280 - P \\ \frac{3}{4}P &= 280 - P \\ P &= 160 \\ Q &= 120 \end{aligned}$$

(b) (Cartel) Assuming that the firms collude to produce the monopoly output:

i. Using *only* their cost functions solve for a relationship between the firm's outputs. Is this relationship the same as when they acted like perfect competitors?
In a cartel the firms will minimize total cost, or set

$$\begin{aligned} MC_1 &= MC_2 \\ 4q_1 &= 4q_2 \\ q_1 &= q_2 \end{aligned}$$

and yes, this is the same as in perfect competition.

ii. Solve for the monopoly output.

$$\begin{aligned} Q &= q_1 + q_2 + q_3 \\ Q &= 3q \\ q &= \frac{Q}{3} \end{aligned}$$

$$\begin{aligned} P &= (280) - Q \\ TR &= ((280) - Q) Q \\ TC &= 3(2)\left(\frac{Q}{3}\right)^2 = \frac{2}{3}Q^2 \end{aligned}$$

$$\Pi(Q) = ((280) - Q) Q - \frac{2}{3}Q^2$$

$$\begin{aligned} (280) - 2Q - \frac{4}{3}Q &= 0 \\ Q &= 84 \end{aligned}$$

iii. Solve for the price they will sell at.

$$P = 196$$

(c) (Cournot) Assuming each firm chooses the quantity that maximizes their profit given the other firm's outputs:

i. How is the concept of *symmetry* useful in solving the following problem? Give a general definition of this concept.
Since the firms have the same costs and the same demand curve the problem is symmetric and we know that $q_1 = q_2 = q_3$ in the solution. In general a problem is symmetric if, up to relabeling the identity of the firms, the firms face the same objective function.

ii. Solve for one firm's reaction function.

$$\Pi(q_1) = (280 - (q_1 + q_2 + q_3))\, q_1 - 2q_1^2$$

$$\begin{aligned} 280 - 2q_1 - (q_2 + q_3) - 4q_1 &= 0 \\ q_1 &= \frac{140}{3} - \frac{1}{6}(q_2 + q_3) \end{aligned}$$

iii. Find the Cournot-Nash equilibrium outputs.
$q_1 = q_2 = q_3 = q$

$$\begin{aligned} q &= \frac{140}{3} - \frac{1}{6}(q + q) \\ q &= 35 \\ Q &= 105 \\ P &= 175 \end{aligned}$$

15. In the market for Cranberries there are two producers, market demand is given by:

$$Q = 126 - 6P$$

(a) Assume that firm one has a total cost of $c(q_1) = 0$ and that firm two has a total cost of $c(q_2) = 6q_2$, what is the Cournot-Nash equilibrium? What is the price that goods are sold at?
Firm 1's problem is given by:

$$\max_{q_1} \frac{126 - (q_1 + q_2)}{6} q_1$$

which takes its maximum at $q_1^* = \frac{126 - q_2}{2}$ (1)
Firm 2's problem is given by:

$$\max_{q_1} \frac{126 - (q_1 + q_2)}{6} q_2 - 6q_2$$

which takes its maximum at $q_2^* = \frac{90 - q_1}{2}$ (2)
Substituting (2) *into* (1) *gives:*
$q_1^* = \frac{126 - \frac{90 - q_1^*}{2}}{2} \Rightarrow q_1^* = 54 \Rightarrow q_2^* = \frac{90 - 54}{2} = 18$
Then
$Q^* = q_1^* + q_2^* = 54 + 18 = 72 = 126 - 6P^* \Rightarrow P^* = 9$

(b) What is elasticity of demand at the Cournot-Nash equilibrium?

$$\begin{aligned} e_d &= \frac{dQ}{dP}\frac{P}{Q} \\ &= (-6)\frac{9}{72} \\ &= -\frac{3}{4} \end{aligned}$$

(c) Assume that you are a consultant for the prosecution in a federal anti-trust case. The firm under question points out that demand is inelastic, and argues that therefore they have no market power. Given this analysis, how would you respond?

This is only true if the firm is a monopolist, they can still have market power and have inelastic demand.

2 From ECON439 (Game Theory)

1. Consider a Cournot Oligopoly where the inverse demand curve is given by $P = 17 - Q$ and the costs of a type a firm is $c_a\left(q^a, q^b\right) = q^a$ and the costs of a type b firm is $c_b\left(q^a, q^b\right) = 3q^b$.

   (a) In this part of the question assume that there is one firm of both types.

      i. Set up the objective function of both firms.

$$\max_{q_a} (17 - q_a - q_b) q_a - q_a$$
$$\max_{q_b} (17 - q_a - q_b) q_b - 3q_b$$

      ii. Find the best responses of both firms.

$$\begin{aligned}(17 - q_a - q_b) - q_a - 1 &= 0\\ q_a &= 8 - \frac{1}{2}q_b\end{aligned}$$

$$\begin{aligned}(17 - q_a - q_b) - q_b - 3 &= 0\\ q_b &= 7 - \frac{1}{2}q_a\end{aligned}$$

      iii. Find the Nash equilibrium quantities.

$$\begin{aligned}q_a &= 8 - \frac{1}{2}\left(7 - \frac{1}{2}q_a\right)\\ q_a &= \frac{1}{4}q_a + \frac{9}{2}\\ q_a &= 6\\ q_b &= 7 - \frac{1}{2}(6) = 4\end{aligned}$$

      iv. Find the profit of both firms in the Nash equilibrium.

$$\begin{aligned}\pi_a &= (17 - 6 - 4)\,6 - 6 = 36\\ \pi_b &= (17 - 6 - 4)\,4 - 3 * 4 = 16\end{aligned}$$

(b) Now assume that there are two firms of type b, firm 1 and firm 2, firm 1 produces q_1^b and firm 2 produces q_2^b.

i. Set up the objective function of both types of firms.

$$(17 - q_a - q_{1b} - q_{2b})\, q_{ib} - 3q_{ib}$$

ii. Find the best responses of both types of firms.

$$(17 - q_a - q_{ib} - q_{jb}) - q_{ib} - 3 = 0$$

$$q_{ib} = 7 - \frac{1}{2}q_a - \frac{1}{2}q_{jb}$$

iii. Why can you assume that $q_1^b = q_2^b$ in equilibrium?

Solution 6 *By symmetry this follows.*

iv. Using the insight in the last part of the question, find the Nash equilibrium quantities.

Solution 7 *For firm a*

$$q_a = 8 - \left(\frac{1}{2}\right) 2q_b$$

because there are now two firms who both will produce q_b. For firm's of type b we have:

$$\begin{aligned} q_b &= 7 - \frac{1}{2}q_a - \frac{1}{2}q_b \\ q_b &= \frac{14}{3} - \frac{1}{3}q_a \end{aligned}$$

Thus we have

$$\begin{aligned} q_a &= 8 - \left(\frac{1}{2}\right) 2 \left(\frac{14}{3} - \frac{1}{3}q_a\right) \\ q_a &= 5 \end{aligned}$$

$$q_b = \frac{14}{3} - \frac{1}{3}(5) = 3$$

and $Q = 11 = 5 + 2 * 3$

v. Find the profit of both types of firms in the Nash equilibrium.

$$\begin{aligned} \pi_a &= (17 - Q)\, q_a - q_a \\ &= (17 - 11)\, 5 - 5 = 25 \end{aligned}$$

$$\begin{aligned} \pi_b &= (17 - Q)\, q_b - 3q_b \\ &= (17 - 11)\, 3 - 3 * 3 \\ &= 9 \end{aligned}$$

(c) Now assume that the costs of firms of type b is $c_b\left(q^a, q^b\right) = 3q^b + F$, where F is a fixed cost. Further consider a *free entry equilibrium* where as many firms of type b can enter as want. (Note that only one firm of type a will be in the market, and that the costs of that type of firm do not change.) This means that for firms of type b, $\pi_i^b = 0$ in equilibrium.

i. If $F = 9$, what will be the equilibrium number of firms of type b? Why?

Solution 8 *Looking at the solutions above, this means that there will be two firms of type b*

ii. If $F = 20$, what will be the equilibrium number of firms of type b? What will be total quantity produced by firms in the market?

Solution 9 *Hmmm, this is a tough question, or it can be. When there is only one firm of type b their profits are 16, so that firm will not enter. In principle this means you have to solve the monopoly problem, but actually you don't because that is the Cournot best response when other firms produce no output. So using that:*

$$q_a = 8 - \frac{1}{2}q_b = q_a = 8 - \frac{1}{2}(0) = 8$$

Of course if you don't remember you might want to set up the problem again:

$$\max_{q_a} (17 - q_a - (0))\, q_a - q_a$$

$$\begin{aligned} (17 - q_a - (0)) - q_a - 1 &= 0 \\ q_a &= 8 \end{aligned}$$