ECON 433

Quiz 2

Dr. Kevin Hasker

1. *(3 Points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(17 points total)* In a given market price is set to clear the market: $P = 62 - 3Q$, and firms choose their quantity to maximize their profit. Type 1 firms have the cost function $c_1(q) = 14q_1$ and type 2 have the cost function $c_2(q) = 2q_2$.

   Assuming there is one firm of each type.

   (a) *(4 points)* Set up the objective function of firm 1, find the first order condition, and the best response.

   $$\pi_1(q_1, q_2) = (62 - 3(q_1 + q_2))q_1 - 14q_1$$

   $$\begin{aligned}(62 - 3(q_1 + q_2)) - 3q_1 - 14 &= 0 \\ q_1 &= 8 - \frac{1}{2}q_2\end{aligned}$$

   (b) *(4 points)* Assuming there is one firm of type 2, set up its objective function, find the first order condition, and the best response.

   $$\pi_2(q_1, q_2) = (62 - 3(q_1 + q_2))q_2 - 2q_2$$

   $$\begin{aligned}(62 - 3(q_1 + q_2)) - 3q_2 - 2 &= 0 \\ q_2 &= 10 - \frac{1}{2}q_1\end{aligned}$$

   (c) *(3 points)* When there is one firm of each type find the equilibrium quantity of each firm and the market quantity.

   $$\begin{aligned}q_1 &= 8 - \frac{1}{2}\left(10 - \frac{1}{2}q_1\right) \\ q_1 &= \frac{1}{4}q_1 + 3 \\ q_1 &= 4\end{aligned}$$

$$\begin{aligned}
q_2 &= 10 - \frac{1}{2}(4) = 8 \\
q_2 &= 10 - \frac{1}{2}\left(8 - \frac{1}{2}q_2\right) \\
&= \frac{1}{4}q_2 + 6 \\
q_2 &= 8
\end{aligned}$$

$$Q = q_1 + q_2 = 8 + 4 = 12$$

If there are m firms of type 2 and no firms of type 1.

(d) *(3 points)* what is the equilibrium price (as a function of m)? You may assume symmetry.

Solution 1 *The first order condition of type 2 is:*

$$(62 - 3Q) - 3q - 2 = 0$$

where Q *is market quantity and* q *is this firms output. In a symmetric equilibrium* $Q = mq$ *or* $q = \frac{Q}{m}$

$$\begin{aligned}
(62 - 3Q) - 3\frac{Q}{m} - 2 &= 0 \\
-\frac{3}{m}(Q - 20m + Qm) &= 0 \\
Q &= 20\frac{m}{m+1}
\end{aligned}$$

and thus

$$P = 62 - 3Q = 62 - 3\left(20\frac{m}{m+1}\right) = \frac{62}{m+1} + 2\frac{m}{m+1}$$

(e) *(2 points)* How large does m need to be before firms of type 1 will decide not to enter?

Solution 2 *In order for a firm of type 1 to enter we need their profit when* $q_1 = 0$ *to be weakly negative:*

$$\pi = (P - c_1)\, q_1$$

or

$$P \leq c_1 = 14$$

$$\begin{aligned}
\frac{62}{m+1} + 2\frac{m}{m+1} &\leq 14 \\
\left(\frac{62}{m+1} + 2\frac{m}{m+1}\right)(m+1) &\leq 14\,(m+1) \\
2m + 62 &\leq 14m + 14 \\
62 - 14 &\leq 14m - 2m \\
48 &\leq 12m \\
4 &\leq m
\end{aligned}$$

Remark 3 *If they set up this:* $P \leq 14$ *properly I don't really care if they derive* $m = 4$*. Indeed they might think that* $m = 5$ *is necessary. They would be wrong but I don't care.*

Remark 4 *Notice that this is also the critical* m *such that all type 1 firms will stop producing and we will achieve production efficiency. The number of type 1 firms in the market at that time will not affect analysis.*