

# ECON 433

## Quiz 03

Kevin Hasker

1. (2 points) **Honor Statement:** Please read and sign the following statement:

I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not use a calculator or other electronic aid for calculation during this test.

Name and Surname: \_\_\_\_\_  
 Student ID: \_\_\_\_\_  
 Signature: \_\_\_\_\_  
 \_\_\_\_\_

**Remark 1** *LOL, you might have noticed that this problem is almost **exactly** like the problem we analyzed on Monday, even though both were constructed at random. The number of consumers is the same, the median location is the same, and even the number of consumers in location 1 is the same. Pure chance! I swear!*

*Why did this mistake happen? I guess... I can give you a few clues. First of all it is ideal to have one or two cells with **very large** populations because then those are obvious place(s) to locate your firm. Second of all for this trick to really work it has to be a very low population cell where the firms actually locate. Third it can't be the middle because that is also an obvious guess. Finally, a minor concern is that large numbers are bad... so... coincidences happen.*

2. (18 points total) Consider the following model of firm location. There are two firms that choose a location at the same time: for  $i \in \{a, b\}$ ,  $l_i \in \{1, 2, 3, 4, 5\}$ . Each firm's objective is to maximize its number of customers. Each consumer is endowed with a location ( $v_k \in \{1, 2, 3, 4, 5\}$   $k \in (1, 2, 3, \dots, 22)$ ) and go to the firm that is closest to them, choosing each firm with equal likelihood if both firms are equally close. The number of consumers at each location is:

1	2	3	4	5
10	2	2	4	4

notice the total number of consumers is 22.

- (a) (10 points) Fill out the following table with the payoffs of firm  $a$  from being at location  $l$  when firm  $b$  is at location  $m \in \{1, 2, 3, 4, 5\}$ . In

the last row right the location that is the best response for firm  $a$ .

Location of firm $b \rightarrow$	1	2	3	4	5
Location of firm $a \downarrow$					
1	11	10	11	12	13
2	12	11	12	13	14
3	11	10	11	14	16
4	10	9	8	11	18
5	9	8	6	4	11
Best Response of $a$	2	2	2	3	4

**Remark 2** If they have marked up this table so that it is hard to read the numbers they have written down feel encouraged to not give them credit. Notice they are thirty numbers to fill out, and only ten points to allocate. If you insist you could make it a third of a point per number, but that's not really worth it. Just be generous and round to the nearest whole point—perhaps giving greater weight to the best responses.

- (b) (2 points) What is the equilibrium location for both firms?

**Solution 3** The equilibrium is location 2, because for both firms  $BR(2) = 2$

- (c) (6 points) Find the equilibrium by iterated removal of strictly dominated strategies. **Do not mark the table, instead explain your reasoning below.**

**Solution 4** First we notice that  $\pi(4, x) > \pi(5, x)$  for  $x \in \{1, 2, 3, 4, 5\}$  thus location 4 dominates location 5.

We can also see that  $\pi(2, x) > \pi(1, x)$  for  $x \in \{1, 2, 3, 4, 5\}$  thus location 2 dominates location 1.

Since we know that both firms are rational this means that both will only choose a location  $y \in \{2, 3, 4\}$ .

Given this we can now see that  $\pi(3, y) > \pi(4, y)$  for all  $y$ , 4 is a better response against 5, but we know the other firm will not choose location 5. Thus 3 dominates 4.

Thus we know firms will only choose  $z \in \{2, 3\}$ , but again  $\pi(2, z) > \pi(3, z)$  and location 2 dominates location 3.

Thus in equilibrium both firms must choose the unique strategy to survive iterated deletion of dominated strategies, or location 2.