ECON 439
Final: Extensive Form Games

Kevin Hasker

This exam will start at 18:40 and finish at 20:20.

Answer all questions in the space provided. Points will only be given for work shown.

1. *(12 points)* Please read and sign the following statement:

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   _

2. *(35 points total)* Consider the Spence signalling model of education. There is one worker and multiple firms. The worker may be high quality with probability $\lambda = \frac{3}{4}$—in which case they are worth $\pi_h = 18$ to a firm—or they may be low quality—in which case they are worth $\pi_l = 6$ to the firm. The worker may choose to get education, $e \in [0, \infty)$. This education has no impact on their value to the firm, and has a marginal cost of $c_h = 2$ if the worker is high quality and $c_l = 3$ if the worker is low quality. The timing of the game is as follows:

   0. Nature chooses the worker's type.
   1. The worker chooses her education level.
   2. The firms compete for the worker.
   3. The worker chooses which—if any—firm she will work for.

   The worker's utility of choosing (w, e) is $u_x(w, e) = w - c_x e$ where $x \in \{l, h\}$ and the firm's profit is of hiring a worker of type x is $\pi = \pi_x - w$.

   We are interested in *self-selection equilibria* which is a (w_h, e_h, w_l, e_l) where type h chooses (w_h, e_h) and type l chooses (w_l, e_l)—these must also be weak sequential equilibria.

   (a) *(3 points)* Assume that some type of worker chooses an e, what are the possible values for $w(e)$ and why?

(b) *(8 points)* Write down the four constraints on the worker in a self-selection equilibrium.

(c) *(4 points)* Which of these four constraints is unnecessary? Prove that it is implied by the other constraints.

(d) *(9 points total)* If $e_h \neq e_l$:

i. *(2 points)* What will be the value of e_l?

ii. *(4 points)* What will be the range of values possible for e_h?

iii. *(3 points)* What type of equilibria are these? Who is signalling in these equilibria?

(e) *(7 points total)* If $e_h = e_l = e^*$:

i. *(4 points)* What will be the range of values possible for e^*?

ii. *(3 points)* What type of equilibria are these? Who is signalling in these equilibria?

(f) *(4 points)* Give a general definition of a signal and find the unique equilibrium in this game where there is no signalling.

3. *(28 points total)* Consider the following game as the stage game of a T period repeated game (where $T < \infty$ or $T = \infty$.)

		Player 2		
		α	β	ε
	A	$10; 2$	$-3; 2$	$4; 3$
Player 1	B	$7; 4$	$-3; -3$	$3; 3$
	C	$1; -3$	$0; 0$	$10; -1$

(a) *(8 points)* Find the best responses and all of the Nash equilibria of this stage game. You may mark your best responses above but explain your notation below.

(b) *(3 points)* Find the *pure strategy* minimax strategies in this game.

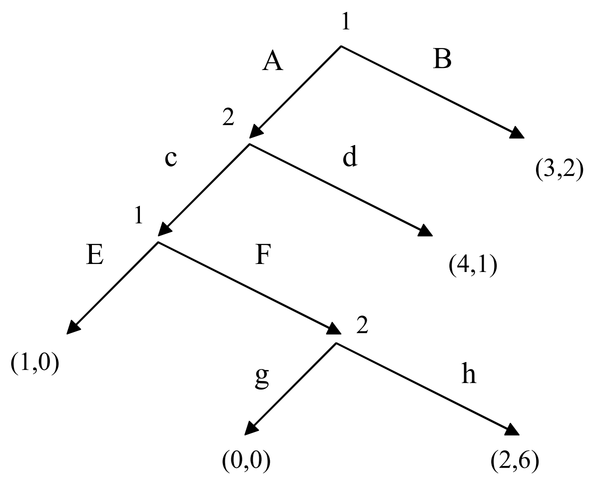
(c) *(6 points)* Assume $T < \infty$, or the finite repeated game. Characterize the set of equilibrium payoffs for all T. Prove your result. *Be careful to prove your result in every subgame.*

(d) *(2 points)* For $x \in \{A, B, C\}$ and $y \in \{\alpha, \beta, \varepsilon\}$ show that the value of playing (x, y) in every period is $u_i(x, y) / (1 - \delta)$, where $\delta \in (0, 1)$ is the discount factor.

(e) *(6 points)* Assume $T = \infty$, using a Grimm or Trigger strategy find the minimal δ such that there is a subgame perfect equilibrium where players expect to play (A, ε) in every period. *Be careful to prove your result in every subgame.*

(f) *(3 points)* Note that in the above strategy Player 1 is the only person that has an incentive problem. Rewrite the strategy so that it is only a function of Player 1's actions and prove that this is never an equilibrium.

4. *(25 points total)* Consider the following sequential game (extensive form game of perfect information).



(a) *(4 points)* Find all the strategies of both players.

(b) *(6 points)* Find the subgame perfect or backward induction equilibrium *strategies* and write them down below.

(c) *(3 points)* Using this game explain why writing down the tactics (the actions that are taken on the equilibrium path) or the outcome (the utilities of both players in equilibrium) are not sufficient.

(d) *(6 points)* Using the table below, create a Normal form game that is strategically equivalent to this extensive form game. (There are more rows and columns than necessary.)

		Player 2					
		_ _ _	_ _ _	_ _ _	_ _ _	_ _ _	_ _ _
	_ _ _						
	_ _ _						
Player 1	_ _ _						
	_ _ _						
	_ _ _						
	_ _ _						

(e) *(6 points)* Find the all of the best responses and pure strategy Nash equilibria of this game. For those Nash equilibria that are not subgame perfect equilibria, explain who makes an empty threat and discuss whether this empty threat benefits them or not.

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2. *(35 points total)* Consider the Spence signalling model of education. There is one worker and multiple firms. The worker may be high quality with probability $\lambda = \frac{2}{3}$—in which case they are worth $\pi_h = 20$ to a firm—or they may be low quality—in which case they are worth $\pi_l = 2$ to the firm. The worker may choose to get education, $e \in [0, \infty)$. This education has no impact on their value to the firm, and has a marginal cost of $c_h = 3$ if the worker is high quality and $c_l = 6$ if the worker is low quality. The timing of the game is as follows:

   0. Nature chooses the worker's type.
   1. The worker chooses her education level.
   2. The firms compete for the worker.
   3. The worker chooses which—if any—firm she will work for.

   The worker's utility of choosing (w, e) is $u_x(w, e) = w - c_x e$ where $x \in \{l, h\}$ and the firm's profit is of hiring a worker of type x is $\pi = \pi_x - w$.

   We are interested in *self-selection equilibria* which is a (w_h, e_h, w_l, e_l) where type h chooses (w_h, e_h) and type l chooses (w_l, e_l)—these must also be weak sequential equilibria.

   (a) *(3 points)* Assume that some type of worker chooses an e, what are the possible values for $w(e)$ and why?

(b) *(8 points)* Write down the four constraints on the worker in a self-selection equilibrium.

(c) *(4 points)* Which of these four constraints is unnecessary? Prove that it is implied by the other constraints.

(d) *(9 points total)* If $e_h \neq e_l$:

i. *(2 points)* What will be the value of e_l?

ii. *(4 points)* What will be the range of values possible for e_h?

iii. *(3 points)* What type of equilibria are these? Who is signalling in these equilibria?

(e) *(7 points total)* If $e_h = e_l = e^*$:

i. *(4 points)* What will be the range of values possible for e^*?

ii. *(3 points)* What type of equilibria are these? Who is signalling in these equilibria?

(f) *(4 points)* Give a general definition of a signal and find the unique equilibrium in this game where there is no signalling.

3. *(29 points total)* Consider the following game as the stage game of a T period repeated game (where $T < \infty$ or $T = \infty$.)

		Player 2		
		α	β	ε
	A	$2; -1$	$5; -7$	$0; 0$
Player 1	B	$4; 8$	$-4; 1$	$-1; 4$
	C	$5; -6$	$3; 1$	$-1; -6$

(a) *(8 points)* Find the best responses and all of the Nash equilibria of this stage game. You may mark your best responses above but explain your notation below.

(b) *(3 points)* Find the *pure strategy* minimax strategies in this game.

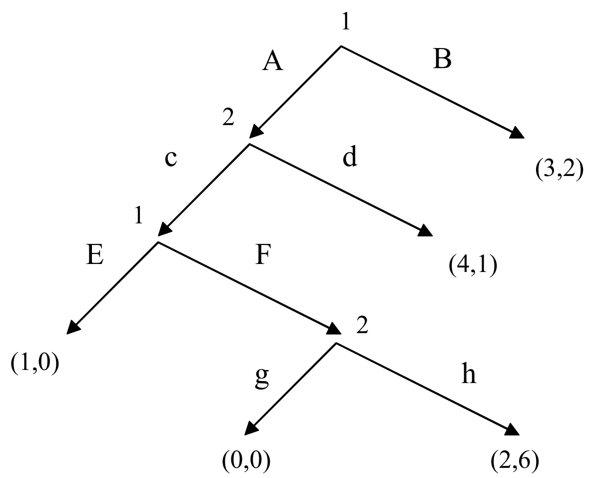
(c) *(6 points)* Assume $T < \infty$, or the finite repeated game. Characterize the set of equilibrium payoffs for all T. Prove your result. *Be careful to prove your result in every subgame.*

(d) *(2 points)* For $x \in \{A, B, C\}$ and $y \in \{\alpha, \beta, \varepsilon\}$ show that the value of playing (x, y) in every period is $u_i(x, y) / (1 - \delta)$, where $\delta \in (0, 1)$ is the discount factor.

(e) *(6 points)* Assume $T = \infty$, using a Grimm or Trigger strategy find the minimal δ such that there is a subgame perfect equilibrium where players expect to play (C, β) in every period. *Be careful to prove your result in every subgame.*

(f) *(3 points)* Note that in the above strategy Player 1 is the only person that has an incentive problem. Rewrite the strategy so that it is only a function of Player 1's actions and prove that this is never an equilibrium.

4. *(25 points total)* Consider the following sequential game (extensive form game of perfect information).



(a) *(4 points)* Find all the strategies of both players.

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		_ _ _	_ _ _	_ _ _	_ _ _	_ _ _	_ _ _
	_ _ _						
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Player 1	_ _ _						
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		Player 2		
		α	β	ε
Player 1	A	$10; -1$	$-4; -1$	$2; 4$
	B	$-2; -4$	$0; 0$	$10; -5$
	C	$6; 9$	$-4; 3$	$-3; 4$

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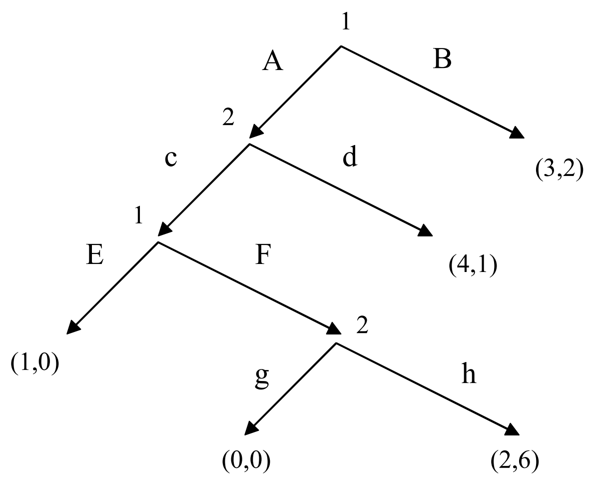

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