ECON 439
Midterm: Normal Form Games

Kevin Hasker

This exam will start at 13:40 and finish at 15:20.

Answer all questions in the space provided. Points will only be given for work shown.

1. *(20 points)* Please read and sign the following statement:

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   _

2. *(16 points total)* Consider a Bertrand Duopoly where each firm chooses $p_i = k_i\delta$ where k_i is a natural number $k_i \in (0, 1, 2, 3, ...)$. The firms have a joint demand curve of $D(P)$ which is downward sloping. Firm i's demand is:

$$d_i(p_i, p_j) = \begin{cases} 0 & \text{if} \quad p_i > p_j \\ \frac{1}{2}D(p_i) & \text{if} \quad p_i = p_j \\ D(p_i) & \text{if} \quad p_i < p_j \end{cases}.$$

   Firm 2 has the marginal cost of 4, but Firm 1 has a marginal cost of 0 with probability $q \in (0,1)$ and 8 with probability $1 - q \in (0,1)$. To be clear:

$$\begin{aligned} c_2(q_2) &= 4q_2 \\ c_1(q_1) &= \begin{cases} 0 & \text{with probability} \quad q \\ 8q_2 & \text{with probability} \quad 1-q \end{cases} \end{aligned}$$

   Assume the δ is "very small" and that the monopoly price is high enough to always be irrelevant.

   (a) *(2 points)* Set up a firm's objective function assuming their marginal cost is c.

   (b) *(4 points)* Find this firm's best response to the other firm's price for all prices less than the monopoly price of this firm. Be very careful about the case when $p_j < c$.

(c) *(4 points)* If firm 2 knows that firm 1's marginal cost is zero, find the set of Nash Equilibria.

(d) *(1 point)* If firm 2 knows that firm 1's marginal cost is 8, find the set of Nash Equilibria.

(e) *(5 points)* Find the set of Nash equilibria of the original game, comment.

3. *(18 points total)* Consider a symmetric Cournot oligopoly with n firms. Firm choose quantity, $q_i \geq 0$. The price is set based on total quantity, $Q = \sum_{i=1}^{n} q_i$: $P = 45 - 4Q$ and all firms have the same constant marginal cost, $c_i(q) = q_i$; let $Q_{-i} = \sum_{j \neq i} q_j$.

   (a) *(2 points)* Find the firm's objective function using only q_i and Q_{-i} or Q.

   (b) *(4 points)* Find the firm's first order condition and best response.

(c) *(3 points)* Using symmetry, find a Nash equilibrium. (All answers from this point on will be a function of n.)

(d) *(4 points)* Find the unique Nash equilibrium **without** using symmetry. (**Hint:** First find a way to solve for the equilibrium market quantity Q^*, and then find q_i^*.)

(e) *(2 points)* Find the set of dominated strategies in this game.

(f) *(4 points)* Prove that if $n > 2$ then this equilibrium can not be found by using iterated deletion of dominated strategies by showing that there is no $\underline{q}$ and $\overline{q}$ ($\underline{q} < \overline{q}$) such that if $q_j \in [\underline{q}, \overline{q}]$ for all $j \neq i$ then the best response of i is in the interior of $(\underline{q}, \overline{q})$.

4. *(36 points)* Consider the following normal form game, where $\lambda > 1$.

(a) *(10 points)* Find all the pure strategy best responses of both players. You may mark them on the game but you will loose two points if you do not explain your notation below.

(b) *(4 points)* Find the unique dominated strategy in this game. State what dominates it and clearly explain why this strategy is dominated, making a payoff by payoff comparison.

(c) *(10 points)* Using iterated deletion of dominated strategies, find the two strategies that survive for both players. When you remove a strategy state what dominates the given strategy.

(d) *(4 points)* Write down the remaining game in the table provided below and find the unique Nash equilibrium of this game. The equilibrium will be a function of λ.

Player 2

Player 1	_ _ _;_ _ _	_ _ _;_ _ _
	_ _ _;_ _ _	_ _ _;_ _ _

(e) *(8 points total)* Now assume that this is a Bayesian game, with λ distributed uniformly over $[1, 13]$ or that $\Pr(\lambda \leq x) = F(x) = \frac{1}{12}x - \frac{1}{12}$.

i. *(2 points)* Argue now that Player 1 will use a cut off strategy, I.e. will play one action of $\lambda \geq \lambda^*$ and another if $\lambda \leq \lambda^*$. Specify this strategy.

ii. *(2 points)* Find a formula for λ^* as a function of the mixed strategy of player 2.

iii. *(4 points)* Find the equilibrium of this game.

5. *(10 points total)* We know that one of two events has occurred, X or Y, with $\Pr(X) = \pi \in (0, 1)$. We observe one or more independent signals. The signals are x or y, with $\Pr(x|X) = p \in \left(\frac{1}{2}, 1\right)$ and $\Pr(y|Y) = q \in \left(\frac{1}{2}, 1\right)$.

   (a) *(5 points)* Assume we have observed one signal and it was x. What is the probability that the true state is X?

   (b) *(3 points)* Assume we have observed two signals and both were x. What is the probability that the true state is X? Show it is higher than before.

   (c) *(2 points)* Explain why this model is fundamental tool for understanding the implications of data.

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2. *(16 points total)* Consider a Bertrand Duopoly where each firm chooses $p_i = k_i\delta$ where k_i is a natural number $k_i \in (0, 1, 2, 3, ...)$. The firms have a joint demand curve of $D(P)$ which is downward sloping. Firm i's demand is:

$$d_i(p_i, p_j) = \begin{cases} 0 & \text{if} \quad p_i > p_j \\ \frac{1}{2}D(p_i) & \text{if} \quad p_i = p_j \\ D(p_i) & \text{if} \quad p_i < p_j \end{cases}.$$

   Firm 2 has the marginal cost of 3, but Firm 1 has a marginal cost of 0 with probability $q \in (0, 1)$ and 6 with probability $1 - q \in (0, 1)$. To be clear:

$$\begin{aligned} c_2(q_2) &= 3q_2 \\ c_1(q_1) &= \begin{cases} 0 & \text{with probability} \quad q \\ 6q_2 & \text{with probability} \quad 1-q \end{cases} \end{aligned}$$

   Assume the δ is "very small" and that the monopoly price is high enough to always be irrelevant.

   (a) *(2 points)* Set up a firm's objective function assuming their marginal cost is c.

   (b) *(4 points)* Find this firm's best response to the other firm's price for all prices less than the monopoly price of this firm. Be very careful about the case when $p_j < c$.

(c) *(4 points)* If firm 2 knows that firm 1's marginal cost is zero, find the set of Nash Equilibria.

(d) *(1 point)* If firm 2 knows that firm 1's marginal cost is 6, find the set of Nash Equilibria.

(e) *(5 points)* Find the set of Nash equilibria of the original game, comment.

3. *(18 points total)* Consider a symmetric Cournot oligopoly with n firms. Firm choose quantity, $q_i \geq 0$. The price is set based on total quantity, $Q = \sum_{i=1}^{n} q_i$: $P = 40 - 2Q$ and all firms have the same constant marginal cost, $c_i(q) = 8q_i$; let $Q_{-i} = \sum_{j \neq i} q_j$.

(a) *(2 points)* Find the firm's objective function using only q_i and Q_{-i} or Q.

(b) *(4 points)* Find the firm's first order condition and best response.

(c) *(3 points)* Using symmetry, find a Nash equilibrium. (All answers from this point on will be a function of n.)

(d) *(4 points)* Find the unique Nash equilibrium **without** using symmetry. (**Hint:** First find a way to solve for the equilibrium market quantity Q^*, and then find q_i^*.)

(e) *(2 points)* Find the set of dominated strategies in this game.

(f) *(4 points)* Prove that if $n > 2$ then this equilibrium can not be found by using iterated deletion of dominated strategies by showing that there is no $\underline{q}$ and $\overline{q}$ ($\underline{q} < \overline{q}$) such that if $q_j \in [\underline{q}, \overline{q}]$ for all $j \neq i$ then the best response of i is in the interior of $(\underline{q}, \overline{q})$.

4. *(36 points)* Consider the following normal form game, where $\lambda > 6$.

Player 2

		α	β	δ	γ	ε
	A	$8; -1$	$-4; -1$	$1; -4$	$4; 4$	$\lambda; 1$
	B	$2; -4$	$1; -1$	$2; -3$	$6; 1$	$6; 4$
Player 1	C	$-1; -9$	$-2; -11$	$1; 1$	$3; -2$	$3; -4$
	D	$-3; -6$	$-1; 3$	$4; 1$	$-2; 1$	$4; 2$
	E	$-1; 6$	$-4; 8$	$3; 1$	$1; -2$	$5; 6$

(a) *(10 points)* Find all the pure strategy best responses of both players. You may mark them on the game but you will loose two points if you do not explain your notation below.

(b) *(4 points)* Find the unique dominated strategy in this game. State what dominates it and clearly explain why this strategy is dominated, making a payoff by payoff comparison.

(c) *(10 points)* Using iterated deletion of dominated strategies, find the two strategies that survive for both players. When you remove a strategy state what dominates the given strategy.

(d) *(4 points)* Write down the remaining game in the table provided below and find the unique Nash equilibrium of this game. The equilibrium will be a function of λ.

Player 2

	_ _ _;_ _ _	_ _ _;_ _ _
Player 1	_ _ _;_ _ _	_ _ _;_ _ _

(e) *(8 points total)* Now assume that this is a Bayesian game, with λ distributed uniformly over $[6, 14]$ or that $\Pr(\lambda \leq x) = F(x) = \frac{1}{8}x - \frac{3}{4}$.

i. *(2 points)* Argue now that Player 1 will use a cut off strategy, I.e. will play one action of $\lambda \geq \lambda^*$ and another if $\lambda \leq \lambda^*$. Specify this strategy.

ii. *(2 points)* Find a formula for λ^* as a function of the mixed strategy of player 2.

iii. *(4 points)* Find the equilibrium of this game.

5. *(10 points total)* We know that one of two events has occurred, X or Y, with $\Pr(X) = \pi \in (0, 1)$. We observe one or more independent signals. The signals are x or y, with $\Pr(x|X) = p \in \left(\frac{1}{2}, 1\right)$ and $\Pr(y|Y) = q \in \left(\frac{1}{2}, 1\right)$.

   (a) *(5 points)* Assume we have observed one signal and it was x. What is the probability that the true state is X?

   (b) *(3 points)* Assume we have observed two signals and both were x. What is the probability that the true state is X? Show it is higher than before.

   (c) *(2 points)* Explain why this model is fundamental tool for understanding the implications of data.

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2. *(16 points total)* Consider a Bertrand Duopoly where each firm chooses $p_i = k_i\delta$ where k_i is a natural number $k_i \in (0, 1, 2, 3, ...)$. The firms have a joint demand curve of $D(P)$ which is downward sloping. Firm i's demand is:

$$d_i(p_i, p_j) = \begin{cases} 0 & \text{if} \quad p_i > p_j \\ \frac{1}{2}D(p_i) & \text{if} \quad p_i = p_j \\ D(p_i) & \text{if} \quad p_i < p_j \end{cases}.$$

   Firm 2 has the marginal cost of 2, but Firm 1 has a marginal cost of 0 with probability $q \in (0, 1)$ and 4 with probability $1 - q \in (0, 1)$. To be clear:

$$\begin{aligned} c_2(q_2) &= 2q_2 \\ c_1(q_1) &= \begin{cases} 0 & \text{with probability} \quad q \\ 4q_2 & \text{with probability} \quad 1-q \end{cases} \end{aligned}$$

   Assume the δ is "very small" and that the monopoly price is high enough to always be irrelevant.

   (a) *(2 points)* Set up a firm's objective function assuming their marginal cost is c.

   (b) *(4 points)* Find this firm's best response to the other firm's price for all prices less than the monopoly price of this firm. Be very careful about the case when $p_j < c$.

(c) *(4 points)* If firm 2 knows that firm 1's marginal cost is zero, find the set of Nash Equilibria.

(d) *(1 point)* If firm 2 knows that firm 1's marginal cost is 4, find the set of Nash Equilibria.

(e) *(5 points)* Find the set of Nash equilibria of the original game, comment.

3. *(18 points total)* Consider a symmetric Cournot oligopoly with n firms. Firm choose quantity, $q_i \geq 0$. The price is set based on total quantity, $Q = \sum_{i=1}^{n} q_i$: $P = 40 - 2Q$ and all firms have the same constant marginal cost, $c_i(q) = 6q_i$; let $Q_{-i} = \sum_{j \neq i} q_j$.

   (a) *(2 points)* Find the firm's objective function using only q_i and Q_{-i} or Q.

   (b) *(4 points)* Find the firm's first order condition and best response.

(c) *(3 points)* Using symmetry, find a Nash equilibrium. (All answers from this point on will be a function of n.)

(d) *(4 points)* Find the unique Nash equilibrium **without** using symmetry. (**Hint:** First find a way to solve for the equilibrium market quantity Q^*, and then find q_i^*.)

(e) *(2 points)* Find the set of dominated strategies in this game.

(f) *(4 points)* Prove that if $n > 2$ then this equilibrium can not be found by using iterated deletion of dominated strategies by showing that there is no $\underline{q}$ and $\overline{q}$ ($\underline{q} < \overline{q}$) such that if $q_j \in [\underline{q}, \overline{q}]$ for all $j \neq i$ then the best response of i is in the interior of $(\underline{q}, \overline{q})$.

4. *(36 points)* Consider the following normal form game, where $\lambda > 4$.

Player 2

		α	β	δ	γ	ε
	A	$4; 5$	$6; 2$	$-4; -2$	$5; -1$	$4; 6$
	B	$3; 4$	$5; -1$	$-9; 5$	$4; -3$	$3; 2$
Player 1	C	$-2; 6$	$10; -1$	$-5; 2$	$2; 3$	$\lambda; 5$
	D	$1; -6$	$5; 4$	$1; 1$	$2; 10$	$-1; 4$
	E	$2; 5$	$3; -4$	$6; 5$	$-1; 11$	$-6; 6$

(a) *(10 points)* Find all the pure strategy best responses of both players. You may mark them on the game but you will loose two points if you do not explain your notation below.

(b) *(4 points)* Find the unique dominated strategy in this game. State what dominates it and clearly explain why this strategy is dominated, making a payoff by payoff comparison.

(c) *(10 points)* Using iterated deletion of dominated strategies, find the two strategies that survive for both players. When you remove a strategy state what dominates the given strategy.

(d) *(4 points)* Write down the remaining game in the table provided below and find the unique Nash equilibrium of this game. The equilibrium will be a function of λ.

Player 2

	_ _ _;_ _ _	_ _ _;_ _ _
Player 1	_ _ _;_ _ _	_ _ _;_ _ _

(e) *(8 points total)* Now assume that this is a Bayesian game, with λ distributed uniformly over $[4, 8]$ or that $\Pr(\lambda \leq x) = F(x) = \frac{1}{4}x - 1$.

i. *(2 points)* Argue now that Player 1 will use a cut off strategy, I.e. will play one action of $\lambda \geq \lambda^*$ and another if $\lambda \leq \lambda^*$. Specify this strategy.

ii. *(2 points)* Find a formula for λ^* as a function of the mixed strategy of player 2.

iii. *(4 points)* Find the equilibrium of this game.

5. *(10 points total)* We know that one of two events has occurred, X or Y, with $\Pr(X) = \pi \in (0,1)$. We observe one or more independent signals. The signals are x or y, with $\Pr(x|X) = p \in \left(\frac{1}{2}, 1\right)$ and $\Pr(y|Y) = q \in \left(\frac{1}{2}, 1\right)$.

   (a) *(5 points)* Assume we have observed one signal and it was x. What is the probability that the true state is X?

   (b) *(3 points)* Assume we have observed two signals and both were x. What is the probability that the true state is X? Show it is higher than before.

   (c) *(2 points)* Explain why this model is fundamental tool for understanding the implications of data.

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   $$d_i(p_i, p_j) = \begin{cases} 0 & \text{if} \quad p_i > p_j \\ \frac{1}{2}D(p_i) & \text{if} \quad p_i = p_j \\ D(p_i) & \text{if} \quad p_i < p_j \end{cases}.$$

   Firm 2 has the marginal cost of 5, but Firm 1 has a marginal cost of 0 with probability $q \in (0, 1)$ and 10 with probability $1 - q \in (0, 1)$. To be clear:

   $$\begin{aligned} c_2(q_2) &= 5q_2 \\ c_1(q_1) &= \begin{cases} 0 & \text{with probability} \quad q \\ 10q_2 & \text{with probability} \quad 1-q \end{cases} \end{aligned}$$

   Assume the δ is "very small" and that the monopoly price is high enough to always be irrelevant.

   (a) *(2 points)* Set up a firm's objective function assuming their marginal cost is c.

   (b) *(4 points)* Find this firm's best response to the other firm's price for all prices less than the monopoly price of this firm. Be very careful about the case when $p_j < c$.

(c) *(4 points)* If firm 2 knows that firm 1's marginal cost is zero, find the set of Nash Equilibria.

(d) *(1 point)* If firm 2 knows that firm 1's marginal cost is 10, find the set of Nash Equilibria.

(e) *(5 points)* Find the set of Nash equilibria of the original game, comment.

3. *(18 points total)* Consider a symmetric Cournot oligopoly with n firms. Firm choose quantity, $q_i \geq 0$. The price is set based on total quantity, $Q = \sum_{i=1}^{n} q_i$: $P = 45 - 3Q$ and all firms have the same constant marginal cost, $c_i(q) = 3q_i$; let $Q_{-i} = \sum_{j \neq i} q_j$.

   (a) *(2 points)* Find the firm's objective function using only q_i and Q_{-i} or Q.

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(c) *(3 points)* Using symmetry, find a Nash equilibrium. (All answers from this point on will be a function of n.)

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4. *(36 points)* Consider the following normal form game, where $\lambda > 5$.

Player 2

		α	β	δ	γ	ε
	A	$4; 8$	$3; 5$	$3; 5$	$1; 3$	$\lambda; 6$
	B	$5; 6$	$1; 3$	$6; 5$	$-4; 4$	$5; 8$
Player 1	C	$3; 4$	$-1; 0$	$4; -4$	$-10; 6$	$3; 3$
	D	$2; -1$	$-1; 5$	$3; 6$	$2; 2$	$-1; 5$
	E	$1; 6$	$-2; -5$	$5; 7$	$8; 6$	$-7; 1$

(a) *(10 points)* Find all the pure strategy best responses of both players. You may mark them on the game but you will loose two points if you do not explain your notation below.

(b) *(4 points)* Find the unique dominated strategy in this game. State what dominates it and clearly explain why this strategy is dominated, making a payoff by payoff comparison.

(c) *(10 points)* Using iterated deletion of dominated strategies, find the two strategies that survive for both players. When you remove a strategy state what dominates the given strategy.

(d) *(4 points)* Write down the remaining game in the table provided below and find the unique Nash equilibrium of this game. The equilibrium will be a function of λ.

Player 2

	_ _ _;_ _ _	_ _ _;_ _ _
Player 1	_ _ _;_ _ _	_ _ _;_ _ _

(e) *(8 points total)* Now assume that this is a Bayesian game, with λ distributed uniformly over $[5, 13]$ or that $\Pr(\lambda \leq x) = F(x) = \frac{1}{8}x - \frac{5}{8}$.

i. *(2 points)* Argue now that Player 1 will use a cut off strategy, I.e. will play one action of $\lambda \geq \lambda^*$ and another if $\lambda \leq \lambda^*$. Specify this strategy.

ii. *(2 points)* Find a formula for λ^* as a function of the mixed strategy of player 2.

iii. *(4 points)* Find the equilibrium of this game.

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