ECON 439
Final: Extensive Form Games

Kevin Hasker

This exam will start at 18:40 and finish at 20:20.

Answer all questions in the space provided. Points will only be given for work shown.

1. *(16 points)* Please read and sign the following statement:

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2. *(16 points)* Consider a simultaneous move game, change it into a sequential game by having player one go first. Compare the payoffs in all pure strategy Nash equilibria of the simultaneous game to the subgame perfect equilibrium of the sequential game. Do player one's payoffs increase, decrease or are we unable to tell? Likewise for player two. Either prove your conclusion or give examples illustrating your statement.

3. *(6 points)* Give a precise definition of an *empty threat*. Do empty threat's have to help the person who is making it? Why or why not?

4. *(23 points total)* Consider an infinitely repeated game where payoffs t periods in the future are discounted by δ^{t-1}, for $\delta \in (0,1)$. The stage game is:

	L	C	R
U	$11; -2$	$3; 1$	$-1; -1$
M	$3; -7$	$6; -1$	$0; 0$
F	$10; 8$	$-1; 9$	$-3; -3$

(a) *(3 points)* Find the pure strategy best responses for both players in the stage game. You may mark them on the table above, but you will loose a point if you do not explain your notation below.

(b) *(2 points)* Find the unique Nash equilibrium of this stage game. Explain why it is a Nash equilibrium.

(c) *(4 points)* Define a minimax (or *minmax*) strategy and find the pure strategy minimax in this stage game for both players.

(d) *(3 points)* Find a trigger or Grimm strategy that can support (U, C) as the initial path of a subgame perfect equilibrium.

(e) *(5 points)* Prove that this is a subgame perfect equilibrium in all subgames, and find the minimal δ such that it is an equilibrium.

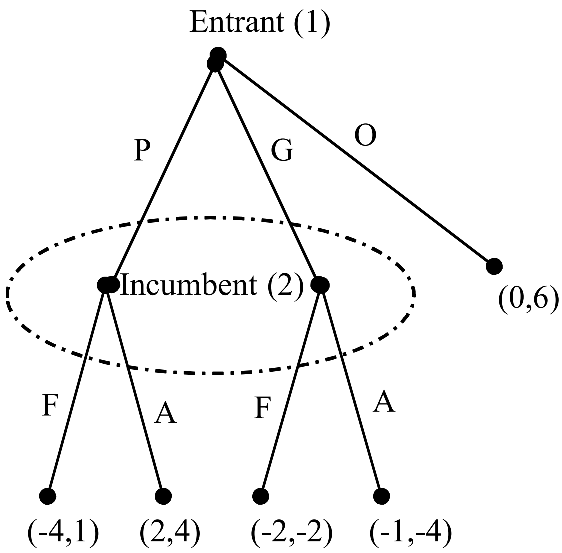
(f) *(3 points)* Prove that the following strategy is not a subgame perfect equilibrium for any δ.

$$s_t = \begin{cases} (U, C) & \text{if Player 1 played } X \text{ in the last period} \\ (M, R) & \text{else} \end{cases}$$

(g) *(3 points)* For a correlated strategy c, what can we conclude about $(u_1(c), u_2(c))$ in order for it to be the initial path of a subgame perfect equilibrium for some δ? Explain your answer.

5. *(3 points)* Krazy Ken and Straight Stu both grade on the curve. Stu, like all reasonable professors, adjusts the grades at the end of the semester so that the average grade is what Stu wants. Ken adjusts the grades after each assignment so that those who completed the assignment get the average grade Ken wants. Ken has noticed that a higher percentage of students are showing up for his exams than Stu's. Assuming students are rational and want to maximize their grade explain this phenomena.

6. *(16 points total)* Consider an entry game where the entrant (P1) now has the choice of being aggressive. The incumbent (P2) does not know if the entrant is aggressive (G) or passive (P). The extensive form game is:



(a) *(5 points)* Solve this game as if the incumbent knows whether the Entrant chooses P or G. Is this a strategy of the game as specified? Is it a weak sequential equilibrium? (You may indicate your best response on the game above, but you will loose two points if you do not explain your notation.)

(b) *(6 points)* Find the other weak sequential equilibrium of this game, be sure to specify all beliefs with intervals if possible.

(c) *(2 points)* Notice that G is a strictly dominated strategy, does this mean that it can't affect the set of weak sequential equilibria? Why or why not?

(d) *(3 points)* In this game there would be a benefit to the entrant of revealing her plans to the incumbent, or letting the incumbent know whether she chose P or G. Would this benefit occur in an arbitrary extensive form game? Why or why not? (We will continue to assume sequential rationality in this new game.)

7. *(20 points total)* Consider an industry which has the demand curve $P(t, Q) = 66 - t - 3Q$, where $t \in \{0, 1, 2, ...\}$ is the time period and Q is the total output. There are two firms in this industry, their strategy is simply the last period in which they produce—$t_i \in \{0, 1, 2, ...\}$. In period t, if $t \leq t_i$ they produce k_i ($k_1 = 9$, $k_2 = 3$) if $t > t_i$ they produce no output. They have constant marginal cost, and their total costs are $c_1(q_1) = 3q_1$, $c_2(q_2) = 25q_2$. Their profit in period t is $\pi_{it} = (P(t, Q_t) - c_i)\, q_{it}$ where $q_{it} = 0$ if $t > t_i$ and k_i if $t \leq t_i$. Their objective is to maximize the sum of their profits in all periods.

   Assume a firm will choose to produce if indifferent. And please notice that for simplicity price can be negative.

   (a) *(6 points)* Let $\underline{t}$ be the last period in which if both firms produce they will make at least zero profit, $\bar{t}_i$ be the last period in which firm i produces alone then they can make at least zero profit. Find $(\underline{t}, \bar{t}_1, \bar{t}_2)$ note that $\bar{t}_2 \neq \bar{t}_1 > \underline{t}$.

(b) *(4 points)* Prove that $t_i \in \{\underline{t}, \underline{t}+1, \underline{t}+2, ..., \bar{t}_i\}$ for both i.

(c) *(3 points)* Assuming that $\min\{t_1, t_2\} \geq \min\{\bar{t}_1, \bar{t}_2\} - 1$, find the unique equilibrium values of (t_1, t_2).

(d) *(3 points)* Given the proof in the last step, assuming that $\min\{t_1, t_2\} \geq \min\{\bar{t}_1, \bar{t}_2\} - 2$, find the unique equilibrium values of (t_1, t_2).

(e) *(4 points)* Find the unique equilibrium of the entire game, proving your conclusion.

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3. *(6 points)* Give a precise definition of an *empty threat*. Do empty threat's have to help the person who is making it? Why or why not?

4. *(23 points total)* Consider an infinitely repeated game where payoffs t periods in the future are discounted by δ^{t-1}, for $\delta \in (0, 1)$. The stage game is:

	L	C	R
U	$8; -1$	$-1; 0$	$4; 2$
M	$4; -3$	$0; 0$	$7; -2$
D	$7; 5$	$-3; -4$	$-2; 6$

(a) *(3 points)* Find the pure strategy best responses for both players in the stage game. You may mark them on the table above, but you will loose a point if you do not explain your notation below.

(b) *(2 points)* Find the unique Nash equilibrium of this stage game. Explain why it is a Nash equilibrium.

(c) *(4 points)* Define a minimax (or *minmax*) strategy and find the pure strategy minimax in this stage game for both players.

(d) *(3 points)* Find a trigger or Grimm strategy that can support (U, R) as the initial path of a subgame perfect equilibrium.

(e) *(5 points)* Prove that this is a subgame perfect equilibrium in all subgames, and find the minimal δ such that it is an equilibrium.

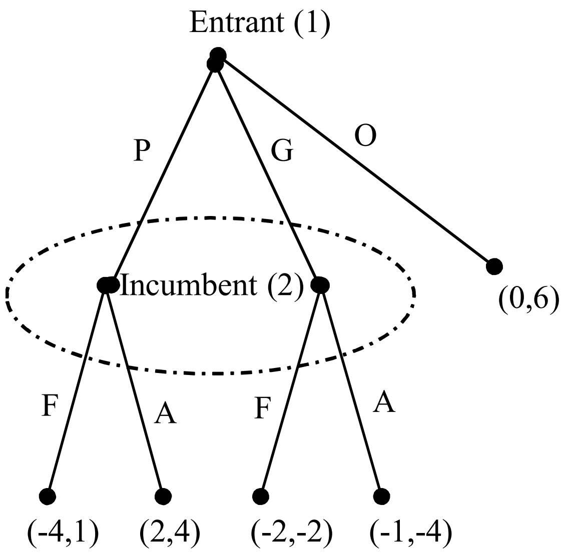
(f) *(3 points)* Prove that the following strategy is not a subgame perfect equilibrium for any δ.

$$s_t = \begin{cases} (U, R) & \text{if Player 1 played } X \text{ in the last period} \\ (M, C) & \text{else} \end{cases}$$

(g) *(3 points)* For a correlated strategy c, what can we conclude about $(u_1(c), u_2(c))$ in order for it to be the initial path of a subgame perfect equilibrium for some δ? Explain your answer.

5. *(3 points)* Krazy Ken and Straight Stu both grade on the curve. Stu, like all reasonable professors, adjusts the grades at the end of the semester so that the average grade is what Stu wants. Ken adjusts the grades after each assignment so that those who completed the assignment get the average grade Ken wants. Ken has noticed that a higher percentage of students are showing up for his exams than Stu's. Assuming students are rational and want to maximize their grade explain this phenomena.

6. *(16 points total)* Consider an entry game where the entrant (P1) now has the choice of being aggressive. The incumbent (P2) does not know if the entrant is aggressive (G) or passive (P). The extensive form game is:



(a) *(5 points)* Solve this game as if the incumbent knows whether the Entrant chooses P or G. Is this a strategy of the game as specified? Is it a weak sequential equilibrium? (You may indicate your best response on the game above, but you will loose two points if you do not explain your notation.)

(b) *(6 points)* Find the other weak sequential equilibrium of this game, be sure to specify all beliefs with intervals if possible.

(c) *(2 points)* Notice that G is a strictly dominated strategy, does this mean that it can't affect the set of weak sequential equilibria? Why or why not?

(d) *(3 points)* In this game there would be a benefit to the entrant of revealing her plans to the incumbent, or letting the incumbent know whether she chose P or G. Would this benefit occur in an arbitrary extensive form game? Why or why not? (We will continue to assume sequential rationality in this new game.)

7. *(20 points total)* Consider an industry which has the demand curve $P(t,Q) = 30 - t - \frac{1}{2}Q$, where $t \in \{0,1,2,...\}$ is the time period and Q is the total output. There are two firms in this industry, their strategy is simply the last period in which they produce—$t_i \in \{0,1,2,...\}$. In period t, if $t \leq t_i$ they produce k_i ($k_1 = 20$, $k_2 = 16$) if $t > t_i$ they produce no output. They have constant marginal cost, and their total costs are $c_1(q_1) = 4q_1$, $c_2(q_2) = 10q_2$. Their profit in period t is $\pi_{it} = (P(t,Q_t) - c_i)\, q_{it}$ where $q_{it} = 0$ if $t > t_i$ and k_i if $t \leq t_i$. Their objective is to maximize the sum of their profits in all periods.

   Assume a firm will choose to produce if indifferent. And please notice that for simplicity price can be negative.

   (a) *(6 points)* Let $\underline{t}$ be the last period in which if both firms produce they will make at least zero profit, $\bar{t}_i$ be the last period in which firm i produces alone then they can make at least zero profit. Find $(\underline{t}, \bar{t}_1, \bar{t}_2)$ note that $\bar{t}_2 \neq \bar{t}_1 > \underline{t}$.

(b) *(4 points)* Prove that $t_i \in \{\underline{t}, \underline{t}+1, \underline{t}+2, ..., \bar{t}_i\}$ for both i.

(c) *(3 points)* Assuming that $\min\{t_1, t_2\} \geq \min\{\bar{t}_1, \bar{t}_2\} - 1$, find the unique equilibrium values of (t_1, t_2).

(d) *(3 points)* Given the proof in the last step, assuming that $\min\{t_1, t_2\} \geq \min\{\bar{t}_1, \bar{t}_2\} - 2$, find the unique equilibrium values of (t_1, t_2).

(e) *(4 points)* Find the unique equilibrium of the entire game, proving your conclusion.

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3. *(6 points)* Give a precise definition of an *empty threat*. Do empty threat's have to help the person who is making it? Why or why not?

4. *(23 points total)* Consider an infinitely repeated game where payoffs t periods in the future are discounted by δ^{t-1}, for $\delta \in (0, 1)$. The stage game is:

	L	C	R
U	$5; -2$	$8; -3$	$0; 0$
M	$7; 5$	$-3; 6$	$-3; -5$
D	$8; 0$	$5; 3$	$-1; 1$

(a) *(3 points)* Find the pure strategy best responses for both players in the stage game. You may mark them on the table above, but you will loose a point if you do not explain your notation below.

(b) *(2 points)* Find the unique Nash equilibrium of this stage game. Explain why it is a Nash equilibrium.

(c) *(4 points)* Define a minimax (or *minmax*) strategy and find the pure strategy minimax in this stage game for both players.

(d) *(3 points)* Find a trigger or Grimm strategy that can support (D, C) as the initial path of a subgame perfect equilibrium.

(e) *(5 points)* Prove that this is a subgame perfect equilibrium in all subgames, and find the minimal δ such that it is an equilibrium.

(f) *(3 points)* Prove that the following strategy is not a subgame perfect equilibrium for any δ.

$$s_t = \begin{cases} (D, C) & \text{if Player 1 played } X \text{ in the last period} \\ (U, R) & \text{else} \end{cases}$$

(g) *(3 points)* For a correlated strategy c, what can we conclude about $(u_1(c), u_2(c))$ in order for it to be the initial path of a subgame perfect equilibrium for some δ? Explain your answer.

5. *(3 points)* Krazy Ken and Straight Stu both grade on the curve. Stu, like all reasonable professors, adjusts the grades at the end of the semester so that the average grade is what Stu wants. Ken adjusts the grades after each assignment so that those who completed the assignment get the average grade Ken wants. Ken has noticed that a higher percentage of students are showing up for his exams than Stu's. Assuming students are rational and want to maximize their grade explain this phenomena.

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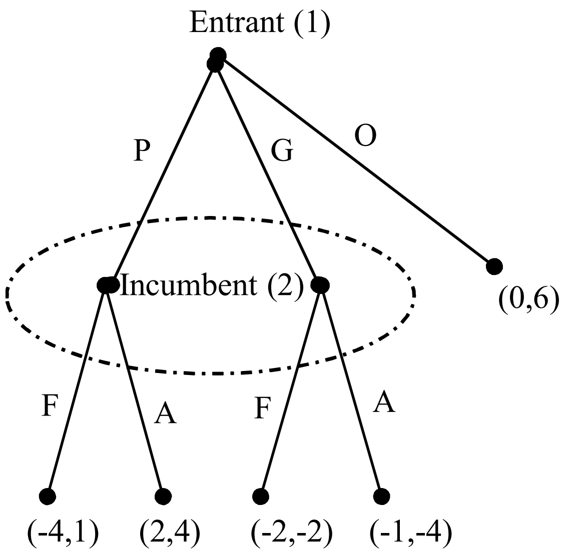

(a) *(5 points)* Solve this game as if the incumbent knows whether the Entrant chooses P or G. Is this a strategy of the game as specified? Is it a weak sequential equilibrium? (You may indicate your best response on the game above, but you will loose two points if you do not explain your notation.)

(b) *(6 points)* Find the other weak sequential equilibrium of this game, be sure to specify all beliefs with intervals if possible.

(c) *(2 points)* Notice that G is a strictly dominated strategy, does this mean that it can't affect the set of weak sequential equilibria? Why or why not?

(d) *(3 points)* In this game there would be a benefit to the entrant of revealing her plans to the incumbent, or letting the incumbent know whether she chose P or G. Would this benefit occur in an arbitrary extensive form game? Why or why not? (We will continue to assume sequential rationality in this new game.)

7. *(20 points total)* Consider an industry which has the demand curve $P(t,Q) = 60 - t - 2Q$, where $t \in \{0,1,2,...\}$ is the time period and Q is the total output. There are two firms in this industry, their strategy is simply the last period in which they produce—$t_i \in \{0,1,2,...\}$. In period t, if $t \leq t_i$ they produce k_i ($k_1 = 4$, $k_2 = 12$) if $t > t_i$ they produce no output. They have constant marginal cost, and their total costs are $c_1(q_1) = 24q_1$, $c_2(q_2) = 4q_2$. Their profit in period t is $\pi_{it} = (P(t,Q_t) - c_i)\, q_{it}$ where $q_{it} = 0$ if $t > t_i$ and k_i if $t \leq t_i$. Their objective is to maximize the sum of their profits in all periods.

   Assume a firm will choose to produce if indifferent. And please notice that for simplicity price can be negative.

   (a) *(6 points)* Let $\underline{t}$ be the last period in which if both firms produce they will make at least zero profit, $\bar{t}_i$ be the last period in which firm i produces alone then they can make at least zero profit. Find $(\underline{t}, \bar{t}_1, \bar{t}_2)$ note that $\bar{t}_2 \neq \bar{t}_1 > \underline{t}$.

(b) *(4 points)* Prove that $t_i \in \{\underline{t}, \underline{t}+1, \underline{t}+2, ..., \bar{t}_i\}$ for both i.

(c) *(3 points)* Assuming that $\min\{t_1, t_2\} \geq \min\{\bar{t}_1, \bar{t}_2\} - 1$, find the unique equilibrium values of (t_1, t_2).

(d) *(3 points)* Given the proof in the last step, assuming that $\min\{t_1, t_2\} \geq \min\{\bar{t}_1, \bar{t}_2\} - 2$, find the unique equilibrium values of (t_1, t_2).

(e) *(4 points)* Find the unique equilibrium of the entire game, proving your conclusion.

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4. *(23 points total)* Consider an infinitely repeated game where payoffs t periods in the future are discounted by δ^{t-1}, for $\delta \in (0, 1)$. The stage game is:

	L	C	R
U	$13; 10$	$-3; -5$	$-2; 11$
M	$14; -1$	$-1; 0$	$5; 2$
D	$5; -8$	$0; 0$	$8; -2$

(a) *(3 points)* Find the pure strategy best responses for both players in the stage game. You may mark them on the table above, but you will loose a point if you do not explain your notation below.

(b) *(2 points)* Find the unique Nash equilibrium of this stage game. Explain why it is a Nash equilibrium.

(c) *(4 points)* Define a minimax (or *minmax*) strategy and find the pure strategy minimax in this stage game for both players.

(d) *(3 points)* Find a trigger or Grimm strategy that can support (M, R) as the initial path of a subgame perfect equilibrium.

(e) *(5 points)* Prove that this is a subgame perfect equilibrium in all subgames, and find the minimal δ such that it is an equilibrium.

(f) *(3 points)* Prove that the following strategy is not a subgame perfect equilibrium for any δ.

$$s_t = \begin{cases} (M, R) & \text{if Player 1 played } X \text{ in the last period} \\ (D, C) & \text{else} \end{cases}$$

(g) *(3 points)* For a correlated strategy c, what can we conclude about $(u_1(c), u_2(c))$ in order for it to be the initial path of a subgame perfect equilibrium for some δ? Explain your answer.

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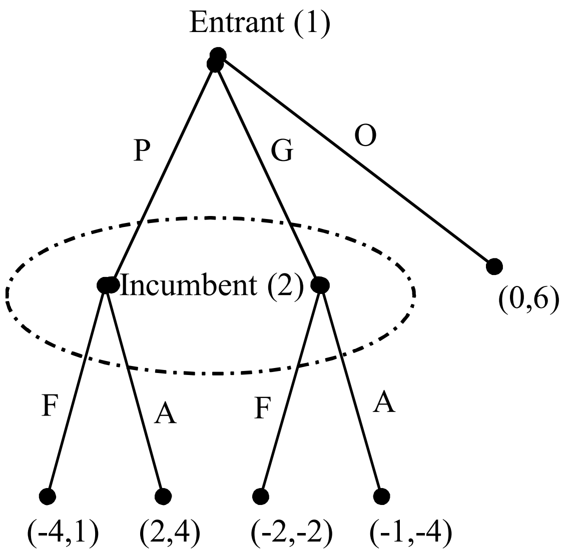

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(b) *(6 points)* Find the other weak sequential equilibrium of this game, be sure to specify all beliefs with intervals if possible.

(c) *(2 points)* Notice that G is a strictly dominated strategy, does this mean that it can't affect the set of weak sequential equilibria? Why or why not?

(d) *(3 points)* In this game there would be a benefit to the entrant of revealing her plans to the incumbent, or letting the incumbent know whether she chose P or G. Would this benefit occur in an arbitrary extensive form game? Why or why not? (We will continue to assume sequential rationality in this new game.)

7. *(20 points total)* Consider an industry which has the demand curve $P(t, Q) = 32 - t - \frac{1}{2}Q$, where $t \in \{0, 1, 2, ...\}$ is the time period and Q is the total output. There are two firms in this industry, their strategy is simply the last period in which they produce—$t_i \in \{0, 1, 2, ...\}$. In period t, if $t \leq t_i$ they produce k_i ($k_1 = 14$, $k_2 = 20$) if $t > t_i$ they produce no output. They have constant marginal cost, and their total costs are $c_1(q_1) = 9q_1$, $c_2(q_2) = 2q_2$. Their profit in period t is $\pi_{it} = (P(t, Q_t) - c_i)\, q_{it}$ where $q_{it} = 0$ if $t > t_i$ and k_i if $t \leq t_i$. Their objective is to maximize the sum of their profits in all periods.

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   (a) *(6 points)* Let $\underline{t}$ be the last period in which if both firms produce they will make at least zero profit, $\bar{t}_i$ be the last period in which firm i produces alone then they can make at least zero profit. Find $(\underline{t}, \bar{t}_1, \bar{t}_2)$ note that $\bar{t}_2 \neq \bar{t}_1 > \underline{t}$.

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(e) *(4 points)* Find the unique equilibrium of the entire game, proving your conclusion.