ECON 439
Midterm: Normal Form Games

Kevin Hasker

This exam will start at 17:40 and finish at 21:20.

Answer all questions in the space provided. Points will only be given for work shown.

1. *(12 points)* Please read and sign the following statement:

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   Name and Surname: _

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   _

2. *(21 points total)* Consider a normal form game with a finite number of players, each of which has a finite number of strategies S_i, $\#(S_i) < \infty$.

   (a) *(4 points)* Give a mathematical definition of a mixed strategy for player i.

   (b) *(2 points)* How does this definition describe a person's actual behavior in a game of pure conflict like Rock/Paper/Scissors?

   (c) *(2 points)* What do you think of this as a model of how a person behaves in a game of pure conflict like Rock/Paper/Scissors?

   (d) *(6 points)* Define a Nash equilibrium in (potentially) mixed strategies. (Your answer does not have to be the one I use.)

(e) *(4 points)* What is the best method to find a strictly mixed strategy equilibrium? You might want to consider a game like matching pennies or Rock/Paper/Scissors when answering this.

(f) *(3 points)* If you use the method above what check do you have to do after you find your potential equilibrium? [**Hint:** You might want to think about Dinner with the Enemy or the Speeding game.]

3. *(36 points total)* Consider a Cournot oligopoly. Firm choose quantity, $q_i \geq 0$, and the price is based on market clearing at the total quantity produced, $Q = \sum_{i=1}^{n} q_i$: $P = 11 - \frac{1}{6}Q$ and all firms have the constant marginal cost, $c_i(q) = c_i q_i$; let $Q_{-i} = \sum_{j \neq i} q_j$.

   (a) *(10 points total)* First consider the two firm case, where $c_1(q) = 3q_1$ and $c_2(q) = 4q_2$.

      i. *(2 points)* Set up both firm's objective functions.

      ii. *(4 points)* Find the first order conditions for both firms.

      iii. *(6 points)* Find the quantity each firm will produce in equilibrium, the total quantity, and the market price.

(b) *(10 points total)* Now assume that firm 2's costs are

$$c_2(q) = \begin{cases} 4q_2 & \text{with probability } \tau \\ 3q_2 & \text{with probability } 1-\tau \end{cases}$$

where $\tau \in [0,1]$.

i. *(4 points)* Find the best response of both types of firm 2 and firm 1. [**Hint:** There is probably a reason I am not asking you to set up the objective function again.]

ii. *(6 points)* Find the quantity each type of firm will produce in equilibrium. All answers should be a function of τ.

(c) *(7 points total)* Now assume there are n firms, all of which have $c_i(q) = 3q_i$. Assume that each firm produces a strictly positive level of output in equilibrium ($q_i^* > 0$):

i. *(1 point)* Find the first order condition of a representative firm.

ii. *(6 points)* Find the total output that will be produced, the market price, and the amount produced by each firm. All answers should be a function of n. [**Hint:** There is probably a reason I don't say you can use symmetry, I will allow it but you don't need it.]

(d) *(9 points total)* Now assume there are n firms, but all you know is that the average constant marginal cost is 3 and that that each firm produces a strictly positive level of output in equilibrium ($q_i^* > 0$):

i. *(3 points)* Find the objective function and first order condition of a representative firm. It should be a function of c_i.

ii. *(4 points)* Find the total output that will be produced, the market price, and the amount produced by each firm. All answers should be a function of n, and the quantity of firm i will depend on firm i's marginal cost (c_i).

iii. *(2 points)* Find an upper bound on c_i such that they will produce a strictly positive amount with n firms. What happens to this bound as $n \to \infty$? What does this tell us about market heterogeneity?

4. *(29 points total)* Consider the following Hotelling model of firm location where consumers are in discrete locations. Firms choose a location $l_i \in \{1, 2, 3, 4, 5\}$ in order to maximize the number of customers they have, and customers go to the nearest firm—splitting their business if both firms are equally close. The number of customers at each location are given by the following table.

Location	1	2	3	4	5
Number of Customers	12	2	2	8	10

(a) *(6 points total)* Fill out the following table with the number of customers firm 1 has if firm 2 is at the location across the top and firm 1 is at the location in the first column.

Location of firm 2→	1	2	3	4	5
Location of firm 1↓					
1					
2					
3					
4					
5					

(b) *(5 points total)* Find the best response for firm 1 to each location of firm 2.

(c) *(2 points)* Find the Nash equilibrium.

(d) *(8 points)* Show that it is the unique strategy that survives iterated deletion of dominated strategies, being clear in your argument at each step.

(e) *(4 points)* Prove that in a model with a finite number of locations that if firm two is at location l_2 then the best response for firm one is either $l_2 - 1$, $l_2 + 1$, or l_2. You may assume there are a strictly positive number of customers at each location.

(f) *(4 points)* Characterize the equilibrium for any finite number of locations, proving your assertion. You may assume the equilibrium is unique and that there are a strictly positive number of customers at each location.

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2. *(21 points total)* Consider a normal form game with a finite number of players, each of which has a finite number of strategies S_i, $\#(S_i) < \infty$.

   (a) *(4 points)* Give a mathematical definition of a mixed strategy for player i.

   (b) *(2 points)* How does this definition describe a person's actual behavior in a game of pure conflict like Rock/Paper/Scissors?

   (c) *(2 points)* What do you think of this as a model of how a person behaves in a game of pure conflict like Rock/Paper/Scissors?

   (d) *(6 points)* Define a Nash equilibrium in (potentially) mixed strategies. (Your answer does not have to be the one I use.)

(e) *(4 points)* What is the best method to find a strictly mixed strategy equilibrium? You might want to consider a game like matching pennies or Rock/Paper/Scissors when answering this.

(f) *(3 points)* If you use the method above what check do you have to do after you find your potential equilibrium? [**Hint:** You might want to think about Dinner with the Enemy or the Speeding game.]

3. *(36 points total)* Consider a Cournot oligopoly. Firm choose quantity, $q_i \geq 0$, and the price is based on market clearing at the total quantity produced, $Q = \sum_{i=1}^{n} q_i$: $P = 19 - \frac{1}{2}Q$ and all firms have the constant marginal cost, $c_i(q) = c_i q_i$; let $Q_{-i} = \sum_{j \neq i} q_j$.

   (a) *(10 points total)* First consider the two firm case, where $c_1(q) = 7q_1$ and $c_2(q) = 4q_2$.

      i. *(2 points)* Set up both firm's objective functions.

      ii. *(4 points)* Find the first order conditions for both firms.

      iii. *(6 points)* Find the quantity each firm will produce in equilibrium, the total quantity, and the market price.

(b) *(10 points total)* Now assume that firm 2's costs are

$$c_2(q) = \begin{cases} 4q_2 & \text{with probability } \tau \\ 7q_2 & \text{with probability } 1-\tau \end{cases}$$

where $\tau \in [0,1]$.

i. *(4 points)* Find the best response of both types of firm 2 and firm 1. [**Hint:** There is probably a reason I am not asking you to set up the objective function again.]

ii. *(6 points)* Find the quantity each type of firm will produce in equilibrium. All answers should be a function of τ.

(c) *(7 points total)* Now assume there are n firms, all of which have $c_i(q) = 7q_i$. Assume that each firm produces a strictly positive level of output in equilibrium ($q_i^* > 0$):

i. *(1 point)* Find the first order condition of a representative firm.

ii. *(6 points)* Find the total output that will be produced, the market price, and the amount produced by each firm. All answers should be a function of n. [**Hint:** There is probably a reason I don't say you can use symmetry, I will allow it but you don't need it.]

(d) *(9 points total)* Now assume there are n firms, but all you know is that the average constant marginal cost is 7 and that that each firm produces a strictly positive level of output in equilibrium ($q_i^* > 0$):

i. *(3 points)* Find the objective function and first order condition of a representative firm. It should be a function of c_i.

ii. *(4 points)* Find the total output that will be produced, the market price, and the amount produced by each firm. All answers should be a function of n, and the quantity of firm i will depend on firm i's marginal cost (c_i).

iii. *(2 points)* Find an upper bound on c_i such that they will produce a strictly positive amount with n firms. What happens to this bound as $n \to \infty$? What does this tell us about market heterogeneity?

4. *(29 points total)* Consider the following Hotelling model of firm location where consumers are in discrete locations. Firms choose a location $l_i \in \{1, 2, 3, 4, 5\}$ in order to maximize the number of customers they have, and customers go to the nearest firm—splitting their business if both firms are equally close. The number of customers at each location are given by the following table.

Location	1	2	3	4	5
Number of Customers	16	2	2	6	4

(a) *(6 points total)* Fill out the following table with the number of customers firm 1 has if firm 2 is at the location across the top and firm 1 is at the location in the first column.

Location of firm 2→	1	2	3	4	5
Location of firm 1↓					
1					
2					
3					
4					
5	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _

(b) *(5 points total)* Find the best response for firm 1 to each location of firm 2.

(c) *(2 points)* Find the Nash equilibrium.

(d) *(8 points)* Show that it is the unique strategy that survives iterated deletion of dominated strategies, being clear in your argument at each step.

(e) *(4 points)* Prove that in a model with a finite number of locations that if firm two is at location l_2 then the best response for firm one is either $l_2 - 1$, $l_2 + 1$, or l_2. You may assume there are a strictly positive number of customers at each location.

(f) *(4 points)* Characterize the equilibrium for any finite number of locations, proving your assertion. You may assume the equilibrium is unique and that there are a strictly positive number of customers at each location.

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   (a) *(4 points)* Give a mathematical definition of a mixed strategy for player i.

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(f) *(3 points)* If you use the method above what check do you have to do after you find your potential equilibrium? [**Hint:** You might want to think about Dinner with the Enemy or the Speeding game.]

3. *(36 points total)* Consider a Cournot oligopoly. Firm choose quantity, $q_i \geq 0$, and the price is based on market clearing at the total quantity produced, $Q = \sum_{i=1}^{n} q_i$: $P = 15 - \frac{1}{3}Q$ and all firms have the constant marginal cost, $c_i(q) = c_i q_i$; let $Q_{-i} = \sum_{j \neq i} q_j$.

   (a) *(10 points total)* First consider the two firm case, where $c_1(q) = 5q_1$ and $c_2(q) = q_2$.

      i. *(2 points)* Set up both firm's objective functions.

      ii. *(4 points)* Find the first order conditions for both firms.

      iii. *(6 points)* Find the quantity each firm will produce in equilibrium, the total quantity, and the market price.

(b) *(10 points total)* Now assume that firm 2's costs are

$$c_2(q) = \begin{cases} q_2 & \text{with probability } \tau \\ 5q_2 & \text{with probability } 1-\tau \end{cases}$$

where $\tau \in [0,1]$.

i. *(4 points)* Find the best response of both types of firm 2 and firm 1. [**Hint:** There is probably a reason I am not asking you to set up the objective function again.]

ii. *(6 points)* Find the quantity each type of firm will produce in equilibrium. All answers should be a function of τ.

(c) *(7 points total)* Now assume there are n firms, all of which have $c_i(q) = 5q_i$. Assume that each firm produces a strictly positive level of output in equilibrium ($q_i^* > 0$):

i. *(1 point)* Find the first order condition of a representative firm.

ii. *(6 points)* Find the total output that will be produced, the market price, and the amount produced by each firm. All answers should be a function of n. [**Hint:** There is probably a reason I don't say you can use symmetry, I will allow it but you don't need it.]

(d) *(9 points total)* Now assume there are n firms, but all you know is that the average constant marginal cost is 5 and that that each firm produces a strictly positive level of output in equilibrium ($q_i^* > 0$):

i. *(3 points)* Find the objective function and first order condition of a representative firm. It should be a function of c_i.

ii. *(4 points)* Find the total output that will be produced, the market price, and the amount produced by each firm. All answers should be a function of n, and the quantity of firm i will depend on firm i's marginal cost (c_i).

iii. *(2 points)* Find an upper bound on c_i such that they will produce a strictly positive amount with n firms. What happens to this bound as $n \to \infty$? What does this tell us about market heterogeneity?

4. *(29 points total)* Consider the following Hotelling model of firm location where consumers are in discrete locations. Firms choose a location $l_i \in \{1, 2, 3, 4, 5\}$ in order to maximize the number of customers they have, and customers go to the nearest firm—splitting their business if both firms are equally close. The number of customers at each location are given by the following table.

Location	1	2	3	4	5
Number of Customers	4	6	2	2	16

(a) *(6 points total)* Fill out the following table with the number of customers firm 1 has if firm 2 is at the location across the top and firm 1 is at the location in the first column.

Location of firm 2→	1	2	3	4	5
Location of firm 1↓					
1					
2					
3					
4					
5					

(b) *(5 points total)* Find the best response for firm 1 to each location of firm 2.

(c) *(2 points)* Find the Nash equilibrium.

(d) *(8 points)* Show that it is the unique strategy that survives iterated deletion of dominated strategies, being clear in your argument at each step.

(e) *(4 points)* Prove that in a model with a finite number of locations that if firm two is at location l_2 then the best response for firm one is either $l_2 - 1$, $l_2 + 1$, or l_2. You may assume there are a strictly positive number of customers at each location.

(f) *(4 points)* Characterize the equilibrium for any finite number of locations, proving your assertion. You may assume the equilibrium is unique and that there are a strictly positive number of customers at each location.

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   (a) *(10 points total)* First consider the two firm case, where $c_1(q) = 3q_1$ and $c_2(q) = 6q_2$.

      i. *(2 points)* Set up both firm's objective functions.

      ii. *(4 points)* Find the first order conditions for both firms.

      iii. *(6 points)* Find the quantity each firm will produce in equilibrium, the total quantity, and the market price.

(b) *(10 points total)* Now assume that firm 2's costs are

$$c_2(q) = \begin{cases} 6q_2 & \text{with probability } \tau \\ 3q_2 & \text{with probability } 1-\tau \end{cases}$$

where $\tau \in [0,1]$.

i. *(4 points)* Find the best response of both types of firm 2 and firm 1. [**Hint:** There is probably a reason I am not asking you to set up the objective function again.]

ii. *(6 points)* Find the quantity each type of firm will produce in equilibrium. All answers should be a function of τ.

(c) *(7 points total)* Now assume there are n firms, all of which have $c_i(q) = 3q_i$. Assume that each firm produces a strictly positive level of output in equilibrium ($q_i^* > 0$):

i. *(1 point)* Find the first order condition of a representative firm.

ii. *(6 points)* Find the total output that will be produced, the market price, and the amount produced by each firm. All answers should be a function of n. [**Hint:** There is probably a reason I don't say you can use symmetry, I will allow it but you don't need it.]

(d) *(9 points total)* Now assume there are n firms, but all you know is that the average constant marginal cost is 3 and that that each firm produces a strictly positive level of output in equilibrium ($q_i^* > 0$):

i. *(3 points)* Find the objective function and first order condition of a representative firm. It should be a function of c_i.

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4. *(29 points total)* Consider the following Hotelling model of firm location where consumers are in discrete locations. Firms choose a location $l_i \in \{1, 2, 3, 4, 5\}$ in order to maximize the number of customers they have, and customers go to the nearest firm—splitting their business if both firms are equally close. The number of customers at each location are given by the following table.

Location	1	2	3	4	5
Number of Customers	10	8	2	2	12

(a) *(6 points total)* Fill out the following table with the number of customers firm 1 has if firm 2 is at the location across the top and firm 1 is at the location in the first column.

Location of firm 2→	1	2	3	4	5
Location of firm 1↓					
1					
2					
3					
4					
5					

(b) *(5 points total)* Find the best response for firm 1 to each location of firm 2.

(c) *(2 points)* Find the Nash equilibrium.

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