ECON 439
Midterm: Normal Form Games

Kevin Hasker

This exam will start at 17:40 and finish at 19:20.

Answer all questions in the space provided. Points will only be given for work shown.

1. *(12 points)* Please read and sign the following statement:

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   Name and Surname: _
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   _

2. *(20 points total)* Consider a second price auction where player i's values is $v_i \in [0, v^+]$. Each player submits a bid, $b_i \in [0, v^+]$, let $\bar{b}_{-i} = \max_{j \in I \setminus i} b_j$, then their utility function can be written as:

$$u_i\left(b_i, \bar{b}_{-i}\right) = \begin{cases} v_i - \bar{b}_{-i} & \text{If } b_i > \bar{b}_{-i} \\ \frac{1}{J}\left(v_i - \bar{b}_{-i}\right) & \text{If } b_i = \bar{b}_{-i} \text{ where } J = \#\left(\{j | j \in I, b_j = b_i\}\right) \\ 0 & \text{If } b_i < \bar{b}_{-i} \end{cases}$$

   (a) *(12 points total)* If $[v_i]_{i \in I}$ are common knowledge for all players:

      i. *(6 points)* Find the best response of player i. You may assume that bids are in very small units (κ, kurus) if you want but this is not necessary.

      ii. *(2 points)* Find an equilibrium in weakly dominant strategies.

iii. *(4 points)* Characterize the full set of equilibria where one player wins for sure.

(b) *(8 points total)* If i knows v_i and that $[v_j]_{j\in I\backslash i}$ are i.i.d. with a distribution function $F(\cdot)$. Assume that $F(a) > 0$ for any $a \in (0, v^+)$.

i. *(2 points)* Show that the equilibrium in weakly dominant strategies is still an equilibrium.

ii. *(2 points)* Show that there can be an equilibrium where for arbitrary $j \in I$, $b_j = v^+$, and for all other $i \in I\backslash j$, $b_i = 0$.

iii. *(4 points)* A *reservation price*, denoted r, is a minimal price at which the unit will be sold. If only one person bids more than r they buy it at the price r, if none bid more than r then no one buys the good.
Prove that if $r > 0$ then there is no equilibrium where for arbitrary $j \in I$, $b_j = v^+$, and for all other $i \in I\backslash j$, $b_i \leq r$.

3. *(30 points total)* Consider the following normal form game:

	α	β	δ	γ
A	$2; -6$	$4; 1$	$0; -3$	$6; -2$
B	$4; 4$	$0; 1$	$0; 6$	$-3; 4$
C	$4; -1$	$7; 3$	$3; 1$	$3; 4$
D	$7; 1$	$4; 7$	$2; 4$	$1; -2$

(a) *(6 points)* Explain, in detail, how you find and what are the best responses to B and β.

(b) *(6 points)* Find all the other pure strategy best responses for both players. You may mark them on the table above but you will loose four points if you do not explain your notation below.

(c) *(6 points)* In this game there is one dominated strategy for each player. What is it, what dominates it, and why?

(d) *(4 points)* In this game there are two strategies that can be ruled out by iterated removal of dominated strategies, which strategies are these and what dominates each one?

(e) *(8 points)* Write the remaining game in the table below and find the Nash equilibrium.

	_ _ _ _ _ _	_ _ _ _ _ _ _
_ _ _ _		
_ _ _ _		

4. *(38 points total)* Consider a differentiated Bertrand duopoly. Firm $i \in \{1,2\}$ chooses a price, $p_i \in [0,\infty)$ and firm i's demand is:

$$q_i = p_j - 2p_i + 13$$

where $j = \{1,2\} \setminus i$, or the price of the other firm. Firm one has the cost $c(q) = 9q_1$. **HINTS: In this question what I do *not* ask contains information. Also, there is a simple way that it seems you can reformulate this problem. If you do this you will get zero points.**

(a) *(11 points total)* Symmetric: Assume that firm 2's costs are $c(q) = 9q_2$.

i. *(2 points)* Set up the objective function of a representative firm.

ii. *(3 points)* Find it's first order condition.

iii. *(3 points)* Find the best response of this representative firm.

iv. *(3 points)* Find the Nash equilibrium prices, you may assume the equilibrium is symmetric.

(b) *(12 points total)* Asymmetric: Assume that firm 2's costs are $c(q) = q_2$.

i. *(2 points)* Set up the objective function of firm two.

ii. *(3 points)* Find its first order condition.

iii. *(3 points)* Find its best response.

iv. *(4 points)* Find the Nash equilibrium prices.

(c) *(15 points total)* Bayesian: Assume that firm one only knows that firm 2's costs are:

$$c(q) = \begin{cases} 9q_2 & \text{with probability } \mu \\ q_2 & \text{with probability } 1-\mu \end{cases},$$

and firm 2 knows their own costs.

i. *(2 points)* Carefully set up the objective function for firm one. Explain why you do not need to find it's best response again.

ii. *(2 points)* Find the expected best response of firm two, it should be a function of μ.

iii. *(3 points)* Find the equilibrium price of firm one, it should be a function of μ.

iv. *(2 points)* Find the equilibrium prices for firm two, they should be functions of μ.

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2. *(20 points total)* Consider a second price auction where player i's values is $v_i \in [0, v^+]$. Each player submits a bid, $b_i \in [0, v^+]$, let $\bar{b}_{-i} = \max_{j \in I \setminus i} b_j$, then their utility function can be written as:

$$u_i\left(b_i, \bar{b}_{-i}\right) = \begin{cases} v_i - \bar{b}_{-i} & \text{If } b_i > \bar{b}_{-i} \\ \frac{1}{J}\left(v_i - \bar{b}_{-i}\right) & \text{If } b_i = \bar{b}_{-i} \text{ where } J = \#\left(\{j | j \in I, b_j = b_i\}\right) \\ 0 & \text{If } b_i < \bar{b}_{-i} \end{cases}$$

   (a) *(12 points total)* If $[v_i]_{i \in I}$ are common knowledge for all players:

      i. *(6 points)* Find the best response of player i. You may assume that bids are in very small units (κ, kurus) if you want but this is not necessary.

      ii. *(2 points)* Find an equilibrium in weakly dominant strategies.

iii. *(4 points)* Characterize the full set of equilibria where one player wins for sure.

(b) *(8 points total)* If i knows v_i and that $[v_j]_{j \in I \backslash i}$ are i.i.d. with a distribution function $F(\cdot)$. Assume that $F(a) > 0$ for any $a \in (0, v^+)$.

i. *(2 points)* Show that the equilibrium in weakly dominant strategies is still an equilibrium.

ii. *(2 points)* Show that there can be an equilibrium where for arbitrary $j \in I$, $b_j = v^+$, and for all other $i \in I \backslash j$, $b_i = 0$.

iii. *(4 points)* A *reservation price*, denoted r, is a minimal price at which the unit will be sold. If only one person bids more than r they buy it at the price r, if none bid more than r then no one buys the good.
Prove that if $r > 0$ then there is no equilibrium where for arbitrary $j \in I$, $b_j = v^+$, and for all other $i \in I \backslash j$, $b_i \leq r$.

3. *(30 points total)* Consider the following normal form game:

	α	β	δ	γ
A	$3; 1$	$8; 6$	$2; 4$	$9; 4$
B	$3; 3$	$2; 3$	$-2; 6$	$6; 1$
C	$0; -5$	$12; -2$	$-2; -3$	$3; 2$
D	$5; 1$	$5; -3$	$1; 3$	$7; 5$

(a) *(6 points)* Explain, in detail, how you find and what are the best responses to B and β.

(b) *(6 points)* Find all the other pure strategy best responses for both players. You may mark them on the table above but you will loose four points if you do not explain your notation below.

(c) *(6 points)* In this game there is one dominated strategy for each player. What is it, what dominates it, and why?

(d) *(4 points)* In this game there are two strategies that can be ruled out by iterated removal of dominated strategies, which strategies are these and what dominates each one?

(e) *(8 points)* Write the remaining game in the table below and find the Nash equilibrium.

	_ _ _ _ _ _	_ _ _ _ _ _
_ _ _ _		
_ _ _ _		

4. *(38 points total)* Consider a differentiated Bertrand duopoly. Firm $i \in \{1,2\}$ chooses a price, $p_i \in [0,\infty)$ and firm i's demand is:

$$q_i = p_j - 2p_i + 20$$

where $j = \{1,2\} \backslash i$, or the price of the other firm. Firm one has the cost $c(q) = 2q_1$. **HINTS: In this question what I do *not* ask contains information. Also, there is a simple way that it seems you can reformulate this problem. If you do this you will get zero points.**

(a) *(11 points total)* Symmetric: Assume that firm 2's costs are $c(q) = 2q_2$.

i. *(2 points)* Set up the objective function of a representative firm.

ii. *(3 points)* Find it's first order condition.

iii. *(3 points)* Find the best response of this representative firm.

iv. *(3 points)* Find the Nash equilibrium prices, you may assume the equilibrium is symmetric.

(b) *(12 points total)* Asymmetric: Assume that firm 2's costs are $c(q) = 10q_2$.

i. *(2 points)* Set up the objective function of firm two.

ii. *(3 points)* Find its first order condition.

iii. *(3 points)* Find its best response.

iv. *(4 points)* Find the Nash equilibrium prices.

(c) *(15 points total)* Bayesian: Assume that firm one only knows that firm 2's costs are:

$$c(q) = \begin{cases} 2q_2 & \text{with probability } \mu \\ 10q_2 & \text{with probability } 1-\mu \end{cases},$$

and firm 2 knows their own costs.

i. *(2 points)* Carefully set up the objective function for firm one. Explain why you do not need to find it's best response again.

ii. *(2 points)* Find the expected best response of firm two, it should be a function of μ.

iii. *(3 points)* Find the equilibrium price of firm one, it should be a function of μ.

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$$u_i\left(b_i, \bar{b}_{-i}\right) = \begin{cases} v_i - \bar{b}_{-i} & \text{If } b_i > \bar{b}_{-i} \\ \frac{1}{J}\left(v_i - \bar{b}_{-i}\right) & \text{If } b_i = \bar{b}_{-i} \text{ where } J = \#\left(\{j | j \in I, b_j = b_i\}\right) \\ 0 & \text{If } b_i < \bar{b}_{-i} \end{cases}$$

   (a) *(12 points total)* If $[v_i]_{i \in I}$ are common knowledge for all players:

      i. *(6 points)* Find the best response of player i. You may assume that bids are in very small units (κ, kurus) if you want but this is not necessary.

      ii. *(2 points)* Find an equilibrium in weakly dominant strategies.

iii. *(4 points)* Characterize the full set of equilibria where one player wins for sure.

(b) *(8 points total)* If i knows v_i and that $[v_j]_{j \in I \backslash i}$ are i.i.d. with a distribution function $F(\cdot)$. Assume that $F(a) > 0$ for any $a \in (0, v^+)$.

i. *(2 points)* Show that the equilibrium in weakly dominant strategies is still an equilibrium.

ii. *(2 points)* Show that there can be an equilibrium where for arbitrary $j \in I$, $b_j = v^+$, and for all other $i \in I \backslash j$, $b_i = 0$.

iii. *(4 points)* A *reservation price*, denoted r, is a minimal price at which the unit will be sold. If only one person bids more than r they buy it at the price r, if none bid more than r then no one buys the good.
Prove that if $r > 0$ then there is no equilibrium where for arbitrary $j \in I$, $b_j = v^+$, and for all other $i \in I \backslash j$, $b_i \leq r$.

3. *(30 points total)* Consider the following normal form game:

	α	β	δ	γ
A	$-2; -1$	$3; 3$	$-3; 3$	$1; 5$
B	$3; 4$	$1; -3$	$4; 2$	$1; 1$
C	$1; 7$	$7; -1$	$0; -3$	$3; 3$
D	$5; 2$	$3; 0$	$2; 6$	$4; 2$

(a) *(6 points)* Explain, in detail, how you find and what are the best responses to B and β.

(b) *(6 points)* Find all the other pure strategy best responses for both players. You may mark them on the table above but you will loose four points if you do not explain your notation below.

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	_ _ _ _ _ _	_ _ _ _ _ _
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_ _ _ _		

4. *(38 points total)* Consider a differentiated Bertrand duopoly. Firm $i \in \{1, 2\}$ chooses a price, $p_i \in [0, \infty)$ and firm i's demand is:

$$q_i = p_j - 2p_i + 18$$

where $j = \{1, 2\} \backslash i$, or the price of the other firm. Firm one has the cost $c(q) = 12q_1$. **HINTS: In this question what I do *not* ask contains information. Also, there is a simple way that it seems you can reformulate this problem. If you do this you will get zero points.**

(a) *(11 points total)* Symmetric: Assume that firm 2's costs are $c(q) = 12q_2$.

i. *(2 points)* Set up the objective function of a representative firm.

ii. *(3 points)* Find it's first order condition.

iii. *(3 points)* Find the best response of this representative firm.

iv. *(3 points)* Find the Nash equilibrium prices, you may assume the equilibrium is symmetric.

(b) *(12 points total)* Asymmetric: Assume that firm 2's costs are $c(q) = 4q_2$.

i. *(2 points)* Set up the objective function of firm two.

ii. *(3 points)* Find its first order condition.

iii. *(3 points)* Find its best response.

iv. *(4 points)* Find the Nash equilibrium prices.

(c) *(15 points total)* Bayesian: Assume that firm one only knows that firm 2's costs are:

$$c(q) = \begin{cases} 12q_2 & \text{with probability } \mu \\ 4q_2 & \text{with probability } 1-\mu \end{cases},$$

and firm 2 knows their own costs.

i. *(2 points)* Carefully set up the objective function for firm one. Explain why you do not need to find it's best response again.

ii. *(2 points)* Find the expected best response of firm two, it should be a function of μ.

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Prove that if $r > 0$ then there is no equilibrium where for arbitrary $j \in I$, $b_j = v^+$, and for all other $i \in I \setminus j$, $b_i \leq r$.

3. *(30 points total)* Consider the following normal form game:

	α	β	δ	γ
A	$3; 6$	$0; 0$	$0; 9$	$3; 5$
B	$-1; -7$	$6; -4$	$-4; 6$	$4; 1$
C	$0; 12$	$1; 6$	$1; 3$	$5; 7$
D	$-2; 1$	$1; 1$	$-5; -4$	$3; 2$

(a) *(6 points)* Explain, in detail, how you find and what are the best responses to B and β.

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_ _ _ _		
_ _ _ _		

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$$q_i = p_j - 2p_i + 16$$

where $j = \{1,2\} \backslash i$, or the price of the other firm. Firm one has the cost $c(q) = 2q_1$. **HINTS: In this question what I do *not* ask contains information. Also, there is a simple way that it seems you can reformulate this problem. If you do this you will get zero points.**

(a) *(11 points total)* Symmetric: Assume that firm 2's costs are $c(q) = 2q_2$.

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(b) *(12 points total)* Asymmetric: Assume that firm 2's costs are $c(q) = 10q_2$.

   i. *(2 points)* Set up the objective function of firm two.

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