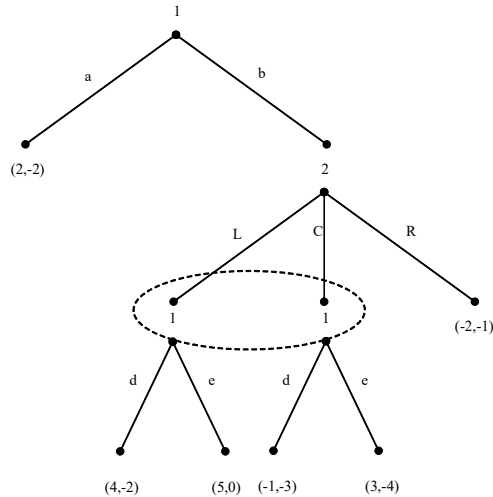# ECON 439
Final: Extensive Form Games
Kevin Hasker

This exam will start at or after 17:45 and must be turned into Moodle by 20:15

Points will only be given for work shown.

1. *(14 points)* Please write the following statement on the first page of your answers and below it write your full name, student ID, and sign it.

   I promise that I have neither given nor received aid from another person while taking this final exam. The following answers are in my own words and based on my own work using class notes, books, and online resources but no human assistance.

2. *(28 points total)* Consider the following extensive form game:

   (a) *(4 points)* Find the strategies of both players.

   (b) *(8 points)* Convert this into a strategic or normal form game, find the best responses and pure strategy Nash equilibria.

   (c) *(4 points)* Treating this as a sequential game where player 1 knows whether player 2 played L or C find the subgame perfect equilibrium.

   (d) *(4 points)* Convert this into a strategy of the game as given, is it a weak sequential equilibrium? If it is state the range of beliefs at the information set $\{L, C\}$ that support it as an equilibrium.

   (e) *(4 points)* Find a subgame perfect equilibrium that is not a weak sequential equilibrium and explain why it is subgame perfect. Hint: It must be a Nash equilibrium.

(f) *(4 points)* Find a Nash equilibrium that is neither subgame perfect nor a weak sequential equilibrium. Identify who makes an empty threat in this equilibrium, does it help the player who makes it?

3. *(26 points)* Consider the following economy, throughout this question you may assume people always vote as if they were pivotal.

1	2	3
D	C	E
C	E	B
E	B	A
A	D	D
B	A	C

(a) *(3 points)* An option is Pareto efficient if no other is better than it for all people. Identify the Pareto efficient options and for those that are not Pareto efficient state something that Pareto dominates it.

(b) *(5 points)* Fill out the empty squares in the following table indicating which option will win a majority of votes of each pair. In row X and column Y first indicate which gets the majority (X or Y) and then who will vote for the winning option.

vs.	B	C	D	E
A				
B	XX			
C	XX	XX		
D	XX	XX	XX	

(c) *(3 points)* Find the top cycle of this economy.

In the following three parts, consider the agenda (A, B, D, E, C).

(d) *(5 points total)* In the committee model: one first votes to accept the first option in the agenda or reject it and move on to the second option. This process is repeated until the last option is accepted if no other option has been. Find the winner with the agenda (A, B, D, E, C) in this model.

   i. *(1 point)* First will C be accepted or will we reject C and accept E?

   ii. *(1 point)* Then will we accept D or reject it and move on?

   iii. *(1 point)* Will we accept B or reject it and move on?

   iv. *(1 point)* Will we accept A or reject it and move on?

   v. *(1 point)* Which option will be selected?

(e) *(6 points)* In the incumbency model: first one votes over the first two options in the agenda. The one that gets a majority of the votes becomes the incumbent, and moves on to compete against the third option. This process repeats until all options are voted on. Find the winner with the agenda (A, B, D, E, C) in this model.

    i. *(1 point)* In the 4th round it will be C versus some X for $X \in \{A, B, D, E\}$. State which option will in a contest of C versus X for all X.

    ii. *(2 points)* In the 3rd round the contest will be E versus X for $X \in \{A, B, D\}$. Given the outcomes you found above, which option will win each contest?

    iii. *(1 point)* In the 2nd round the contest will be D versus X for $X \in \{A, B\}$. Given the outcomes you found above, which option will win each contest?

    iv. *(1 point)* In the first round the contest will be A versus B. Given the outcomes you found above, which option will win?

    v. *(1 point)* Which option will be selected?

(f) *(4 points)* Consider the incumbency model where agents vote naively. I.e. they simply vote for the option in front of them that they like the most. Find which option will be selected with this type of voting and identify at least one voter who would like to change their vote and vote strategically.

4. *(32 points total)* Consider a Spence signalling model, where agents use education to signal their ability. Their are two types of workers, h and l with $\Pr(h) = \frac{2}{3}$. Education does not affect productivity thus regardless of their level of education a high productivity worker has a value of $\pi_h = 36$ to a firm and a low productivity worker has a value of $\pi_l = 6$ to a firm. Education costs less for the high types, the marginal cost of a year of education is $c_h = 3$ for a high type, for a low type it is $c_l = 10$. Firms are Bertrand competitors thus if a worker goes to school for e years they will get $w(e) = \beta(e)\pi_h + (1 - \beta(e))\pi_l$ where $\beta(e) = \Pr(h|e)$ is the common beliefs of the firms that a workers is a high type given e years of education. For $x \in \{h, l\}$ the utility of a worker is $u_x(w, e) = w - c_x e$. Without loss of generality we can characterize an equilibrium as a (w_h, e_h, w_l, e_l) where a high type worker chooses to go to school for e_h years to earn a wage of w_h and a low type worker chooses to go for e_l and earn w_l.

(a) *(8 points)* Write down the four constraints that summarize a worker's optimal behavior for a given (w_h, e_h, w_l, e_l).

(b) *(4 points)* Which of these four constraints will never be used when analyzing equilibria? Prove your statement.

(c) *(8 points total)* For each of the following state whether it can be a pure strategy weak sequential equilibrium or not. If it can be prove this, if it can not explain why.

    i. *(2 points)* $(w_h, e_h, w_l, e_l) = (36, 11, 6, 0)$

    ii. *(2 points)* $(w_h, e_h, w_l, e_l) = (30, 6, 6, 0)$

    iii. *(2 points)* $(w_h, e_h, w_l, e_l) = (36, 4, 8, 0)$

    iv. *(2 points)* $(w_h, e_h, w_l, e_l) = (26, 1, 26, 1)$

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(d) *(6 points)* Characterize the equilibria where $e_h \neq e_l$. What is the name of this class of equilibria? Who is signalling in these equilibria and what does the signal do?

(e) *(6 points)* Characterize the equilibria where $e_h = e_l$. What is the name of this class of equilibria? Who is signalling in these equilibria and what does the signal do? What is the one equilibrium in this class where no one is signalling?