ECON 439
Midterm: Normal Form Games

Kevin Hasker

This exam will start at or after 15:35 and must be turned into Moodle by 17:25

Points will only be given for work shown.

1. *(20 points)* Please write the following statement on the first page of your answers and below it write your full name, student ID, and sign it.

   I promise that I have neither given nor received aid from another person while taking this final exam. The following answers are in my own words and based on my own work using class notes, books, and online resources but no human assistance.

2. *(6 points)* In your own words prove that if in a Nash equilibrium the probability of $s_i \in S_i$ and $\tilde{s}_i \in S_i$ are both strictly positive and the other players strategy is $\sigma^*_{-i} \in \times_{j \neq i} \Delta(S_j)$ then $u_i\left(s_i, \sigma^*_{-i}\right) = u_i\left(\tilde{s}_i, \sigma^*_{-i}\right)$.

3. *(24 points total)*

	α	β	γ
A	$1;3$	$10;4$	$10;0$
B	$4;1$	$18;4$	$9;0$
C	$2;4$	$28;1$	$1;5$

   (a) *(4 points)* Carefully explain (as if to a high school student) what the best response of player 2 is to A. For all of the other pure strategies either write the best response below or mark them on the table above—but if you use the table you will loose a point if you do not explain your notation below.

   (b) *(4 points)* Write down the cycle in pure strategy best responses below, and in a table draw the game with these strategies deleted. The table should look like:

	s_2	$\hat{s}_2$
s_1	$u_1(s_1, s_2); u_2(s_1, s_2)$	$u_1(s_1, \hat{s}_2); u_2(s_1, \hat{s}_2)$
$\tilde{s}_1$	$u_1(\tilde{s}_1, s_2); u_2(\tilde{s}_1, s_2)$	$u_1(\tilde{s}_1, \hat{s}_2); u_2(\tilde{s}_1, \hat{s}_2)$

   (c) *(4 points)* Find a *candidate* for a Nash equilibrium where only strategies in the cycle over pure strategy best responses are given a positive probability of being used.

   (d) *(2 points)* Show that this candidate Nash equilibrium is not a Nash equilibrium of this game.

   (e) *(10 points)* Find the Nash equilibria of this game.

4. *(28 points in total)* Consider the following Hotelling model. Two firms choose $l_i \in \{1, 2, 3, 4, 5\}$ to maximize the number of customers they have.

Customers at location l either go to the closer firm or split their business if both firms are equally close to them. Let $D_i(l_i, l_j)$ be the number of customers firm i has if they are located at l_i and the other firm is located at l_j. The distribution of customers is:

Locations :	1	2	3	4	5
# *Customers* :	4	8	2	2	10

(a) *(5 points)* Copy the following table into your answers and fill it out with the number of customers firm 1 will have if firm 2 is in location l_2. (The column is the location of firm 2, the row the location of firm 1.)

If $l_2 =$	1	2	3	4	5
$D_1(1, l_2) =$					
$D_1(2, l_2) =$					
$D_1(3, l_2) =$					
$D_1(4, l_2) =$					
$D_1(5, l_2) =$					

(b) *(6 points)* From the table what are the best responses of firm 1 to firm 2 and the Nash equilibrium? Why do you not have to do a new table for firm 2?

(c) *(8 points)* Solve this game by iterated deletion of dominated strategies.

From here on consider a generic version of this model, $l_i \in \{1, 2, 3, ..., L\}$ and assume that the median location (l_m) is unique.

(d) *(6 points)* Assume that firm i and j can choose a location from the set $\{l_-, l_- + 1, l_- + 2, ..., l_+\}$. Show that if $l_+ > l_m$ then $l_+ - 1$ dominates l_+, and likewise if $l_- < l_m$ $l_- + 1$ dominates l_-.

(e) *(3 points)* Prove that this game can be solved by iterated deletion of dominated strategies.

5. *(22 points total)* Consider a model of adverse selection. A seller has a car who's worth is $w \in \{5, 20, 50\} = \{w_l, w_m, w_h\}$. The buyer does not know the worth of the car, the buyer only knows that $\Pr(w = w_h) = \Pr(w = w_m) = \frac{2}{5}$, and $\Pr(w = w_l) = \frac{1}{5}$. Their value of buying the car is $3w$. A seller's strategy is $p \geq 0$, and a buyers strategy is a p^+ where they will buy the car if $p \leq p^+$. We assume that p^+ is set at the maximum it can be given what cars are sold. Their utilities are:

$$\pi_{seller}(w, p, p^+) = \begin{cases} p - w & \text{if} \quad p \leq p^+ \\ 0 & \text{if} \quad p > p^+ \end{cases} ; u_{buyer}(w, p, p^+) = \begin{cases} vw - p & \text{if} \quad p \leq p^+ \\ 0 & \text{if} \quad p > p^+ \end{cases} .$$

(a) *(6 points total)* Find the expected value of the car if:

i. *(2 points)* All cars are sold.

ii. *(2 points)* High worth cars are not sold.

iii. *(2 points)* Only low worth cars are sold.

(b) *(9 points total)* Find out if there is an equilibrium when:

i. *(3 points)* All cars are sold.

ii. *(3 points)* High worth cars are not sold.

iii. *(3 points)* Only low worth cars are sold.

(c) *(3 points)* Consider a real world situation where currently only the low worth cars are being sold. Assuming that a trustworthy third party wanted to intervene in the market to get higher worth goods being sold, what problems would they face?

(d) *(4 points)* In this model we always assume $w_l > 0$. This is frankly unrealistic, as you should know there are some cars that will cost more to maintain and fix up than they could ever benefit you. Allowing $p < 0$ but changing nothing else about this model, what would happen if $w_l < 0$? You should consider the case when $|w_l|$ is small and $|w_l|$ is large.