

ECON 439

Extensive Form Games

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 9:30 and end at 11:10

1. (16 points) **Honor Statement:** Please read and sign the following statement:

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2. (26 points total) About Equilibrium.

- (a) (6 points) Define a Nash equilibrium and explain why (perhaps with an example) a subgame perfect equilibrium is generally a better equilibrium concept than a Nash equilibrium.

- (b) (2 points) Give an example of a game where empirically people do not play the subgame perfect equilibrium. (In all examples I can think of the verification is from experiments.)

- (c) (6 points) Define a *subgame perfect equilibrium* and explain why (perhaps with an example) weak sequential equilibrium is generally a better equilibrium concept than a subgame perfect equilibrium. **NOTE:** Be sure to define a subgame.

(d) (8 points) Define a *weak sequential equilibrium*.

(e) (4 points) What is the general problem with Nash equilibrium that means we need refinements of it like subgame perfect equilibrium and weak sequential equilibrium?

3. (36 points total) Consider a group of three people $\{1, 2, 3\}$ who has to choose among five alternatives: $\{A, B, C, D, E\}$. Their preferences are as follows:

1	2	3
E	B	A
B	D	C
A	A	E
C	C	D
D	E	B

For some reason they have agreed on the agenda—order of considering options—of (C, D, E, B, A) , but have not agreed whether to use the standard committee model or the status quo model to select the outcome. First, let's do some preliminary analysis.

(a) (2 points) What does it mean if a person's vote is *pivotal*?

(b) (4 points) Prove that any $X \in \{A, B, C, D, E\}$ can be the subgame perfect equilibrium with either the standard committee model or the status quo model using the agenda above. **Note:** you may not assume people always vote as if they were pivotal.

(c) (10 points) Fill out the following table with which of the two options a majority of the people prefer—after the option write who prefers

it. For example if people 1 and 3 prefer B to D then you would write $B\{1, 3\}$ in the second row and third column of this table.

vs.	B	C	D	E
A				
B	NA			
C	NA	NA		
D	NA	NA	NA	

- (d) (5 points) Define the *top cycle*, and then find it for this example.

From this point on you may assume that everyone votes as if they were always pivotal.

- (e) (7 points total) Consider the standard committee model of selecting an option. In round one they vote to either accept or reject the first option in the agenda, with a majority vote determining which occurs. If they reject they do the same to the second option in the second round, and so on until if they reject the next to last option they are accepting the final option by default.
- (2 points) In the fourth round, will they accept or reject the fourth option? Why?
 - (1 point) In the third round, will they accept or reject the third option? Why?
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- iv. (1 point) In the first round, will they accept or reject the first option?

- v. (2 points) What will be the option that is selected. Why?

- f) (6 points total) Consider the status quo model. In the first round they will vote between the first two options in the agenda, the option that gets the majority of the votes will become the *status quo*. Thereafter in the n 'th round they will vote between the status quo and the $n + 1$ 'st option.
 - i. (2 points) Considering the option that the status quo could be any of the options before it, what would be the outcome in a vote between that status quo and the final option in the agenda? For one of them explain why.

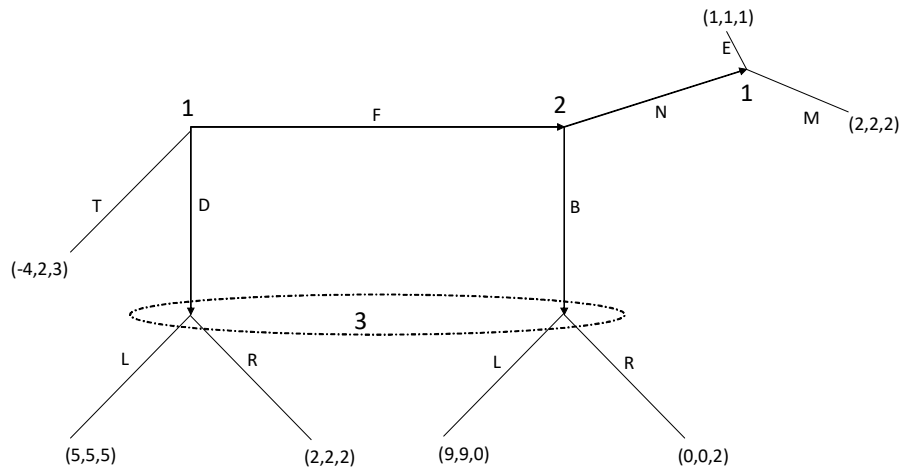
 - ii. (2 points) Given this analysis, now repeat the same exercise for the third round.

 - iii. (1 point) Do the same analysis for the second round.

 - iv. (1 point) Finally, given all your previous analysis what will be the winner in the first round. Does it matter which wins?

- (g) (2 points) Now the people have decided to vote on whether to use the standard committee model or the status quo model. Which one would win and who would vote for which option?

4. (22 points total) Consider the general extensive form game below, In this game player 1 chooses first in the upper left corner, after that point the choices flow in the direction of the arrows.



- (a) (4 points) Write down the strategies of this game.
- (b) (3 points) If this was a game of perfect information (everyone knew what had happened at every decision point in the game) how would the strategies change?
- (c) (6 points) Treating this as a game of perfect information, find the equilibrium strategies and write them out below.

- (d) (*5 points*) Rewrite these strategies as strategies of the game as given. Are they a weak sequential equilibrium? Are they a subgame perfect equilibrium? Why or why not?
- (e) (*4 points*) Find the weak sequential equilibrium strategies, be sure to clearly specify the *range* of off-path beliefs that support this as an equilibrium.

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2. (26 points total) About Equilibrium.

- (a) (6 points) Define a Nash equilibrium and explain why (perhaps with an example) a subgame perfect equilibrium is generally a better equilibrium concept than a Nash equilibrium.

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(d) (8 points) Define a *weak sequential equilibrium*.

(e) (4 points) What is the general problem with Nash equilibrium that means we need refinements of it like subgame perfect equilibrium and weak sequential equilibrium?

3. (36 points total) Consider a group of three people $\{1, 2, 3\}$ who has to choose among five alternatives: $\{A, B, C, D, E\}$. Their preferences are as follows:

1	<i>E</i>	3
<i>B</i>	<i>A</i>	<i>D</i>
<i>A</i>	<i>C</i>	<i>E</i>
<i>D</i>	<i>D</i>	<i>B</i>
<i>E</i>	<i>E</i>	<i>C</i>
<i>C</i>	<i>B</i>	<i>A</i>

For some reason they have agreed on the agenda—order of considering options—of (E, C, B, A, D) , but have not agreed whether to use the standard committee model or the status quo model to select the outcome. First, let's do some preliminary analysis.

(a) (2 points) What does it mean if a person's vote is *pivotal*?

(b) (4 points) Prove that any $X \in \{A, B, C, D, E\}$ can be the subgame perfect equilibrium with either the standard committee model or the status quo model using the agenda above. **Note:** you may not assume people always vote as if they were pivotal.

(c) (10 points) Fill out the following table with which of the two options a majority of the people prefer—after the option write who prefers

it. For example if people 1 and 3 prefer B to D then you would write $B\{1, 3\}$ in the second row and third column of this table.

	vs.	B	C	D	E
A					
B		NA			
C		NA	NA		
D		NA	NA	NA	

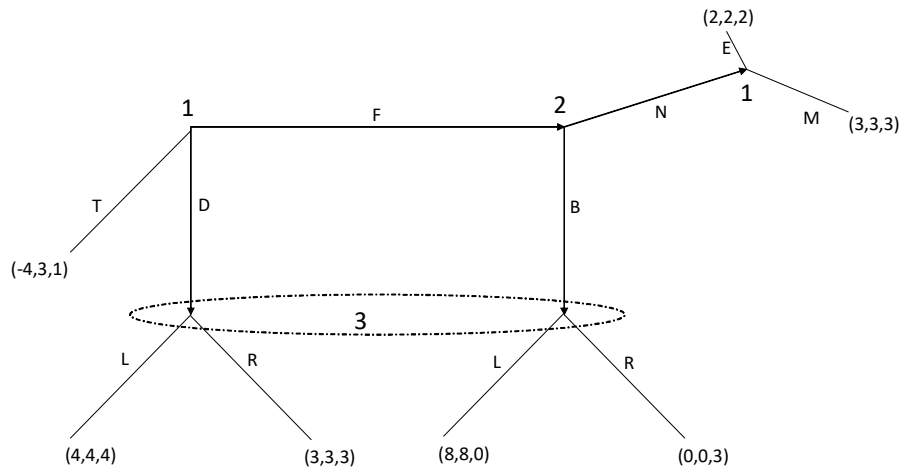
- (d) (5 points) Define the *top cycle*, and then find it for this example.

From this point on you may assume that everyone votes as if they were always pivotal.

- (e) (7 points total) Consider the standard committee model of selecting an option. In round one they vote to either accept or reject the first option in the agenda, with a majority vote determining which occurs. If they reject they do the same to the second option in the second round, and so on until if they reject the next to last option they are accepting the final option by default.
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- (a) (4 points) Write down the strategies of this game.
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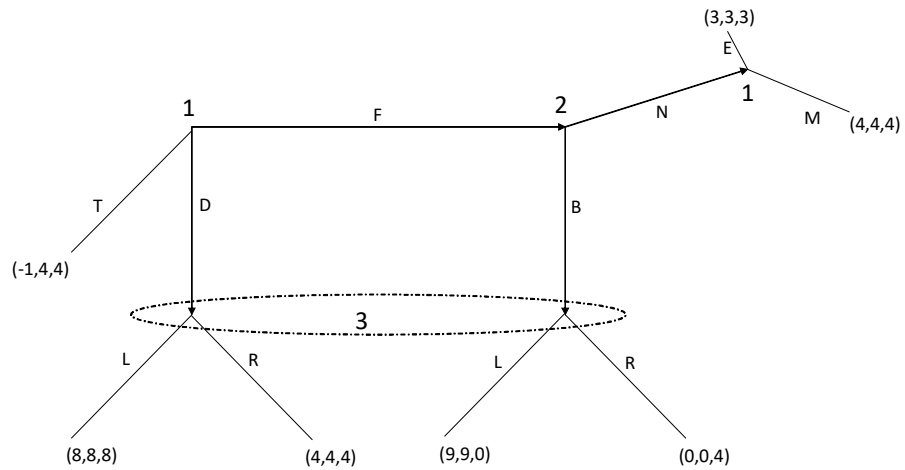
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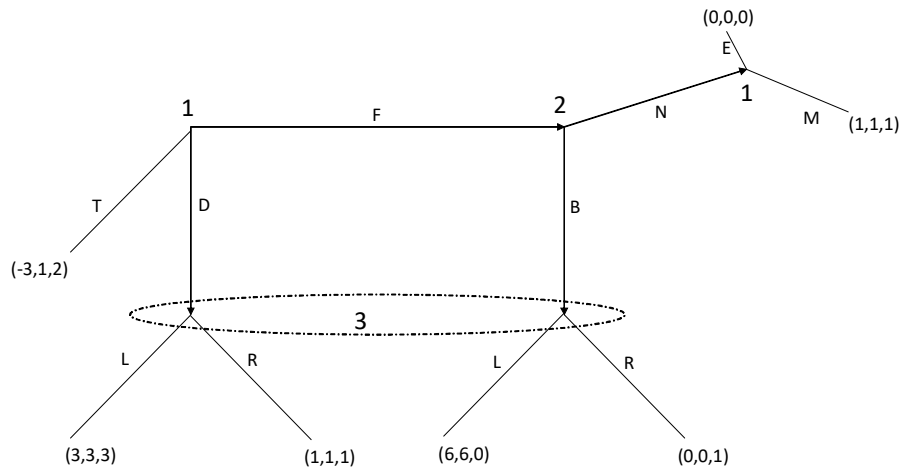
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