

# ECON 439

## Extensive Form Games

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This exam will begin at 9:30 and end at 11:10

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2. (26 points total) About Equilibrium.

- (a) (6 points) Define a Nash equilibrium and explain why (perhaps with an example) a subgame perfect equilibrium is generally a better equilibrium concept than a Nash equilibrium.

- (b) (2 points) Give an example of a game where empirically people do not play the subgame perfect equilibrium. (In all examples I can think of the verification is from experiments.)

- (c) (6 points) Define a *subgame perfect equilibrium* and explain why (perhaps with an example) weak sequential equilibrium is generally a better equilibrium concept than a subgame perfect equilibrium. **NOTE:** Be sure to define a subgame.

(d) (8 points) Define a *weak sequential equilibrium*.

(e) (4 points) What is the general problem with Nash equilibrium that means we need refinements of it like subgame perfect equilibrium and weak sequential equilibrium?

3. (36 points total) Consider a group of three people  $\{1, 2, 3\}$  who has to choose among five alternatives:  $\{A, B, C, D, E\}$ . Their preferences are as follows:

1	2	3
<i>E</i>	<i>B</i>	<i>A</i>
<i>B</i>	<i>D</i>	<i>C</i>
<i>A</i>	<i>A</i>	<i>E</i>
<i>C</i>	<i>C</i>	<i>D</i>
<i>D</i>	<i>E</i>	<i>B</i>

For some reason they have agreed on the agenda—order of considering options—of  $(C, D, E, B, A)$ , but have not agreed whether to use the standard committee model or the status quo model to select the outcome. First, let's do some preliminary analysis.

(a) (2 points) What does it mean if a person's vote is *pivotal*?

(b) (4 points) Prove that any  $X \in \{A, B, C, D, E\}$  can be the subgame perfect equilibrium with either the standard committee model or the status quo model using the agenda above. **Note:** you may not assume people always vote as if they were pivotal.

(c) (10 points) Fill out the following table with which of the two options a majority of the people prefer—after the option write who prefers

it. For example if people 1 and 3 prefer  $B$  to  $D$  then you would write  $B \{1, 3\}$  in the second row and third column of this table.

vs.	$B$	$C$	$D$	$E$
$A$				
$B$	NA			
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(d) (5 points) Define the *top cycle*, and then find it for this example.

From this point on you may assume that everyone votes as if they were always pivotal.

(e) (7 points total) Consider the standard committee model of selecting an option. In round one they vote to either accept or reject the first option in the agenda, with a majority vote determining which occurs. If they reject they do the same to the second option in the second round, and so on until if they reject the next to last option they are accepting the final option by default.

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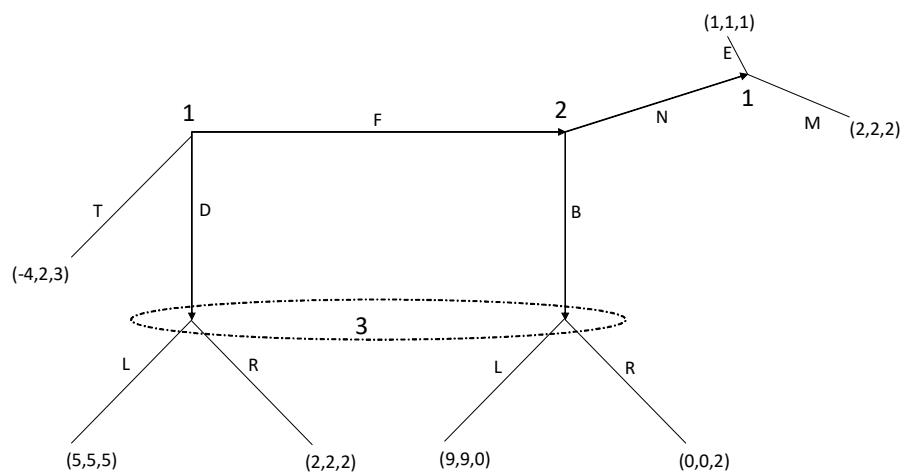
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4. (22 points total) Consider the general extensive form game below, In this game player 1 chooses first in the upper left corner, after that point the choices flow in the direction of the arrows.



(a) (4 points) Write down the strategies of this game.

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(d) (*5 points*) Rewrite these strategies as strategies of the game as given. Are they a weak sequential equilibrium? Are they a subgame perfect equilibrium? Why or why not?

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3. (36 points total) Consider a group of three people  $\{1, 2, 3\}$  who has to choose among five alternatives:  $\{A, B, C, D, E\}$ . Their preferences are as follows:

1	E	3
B	A	D
A	C	E
D	D	B
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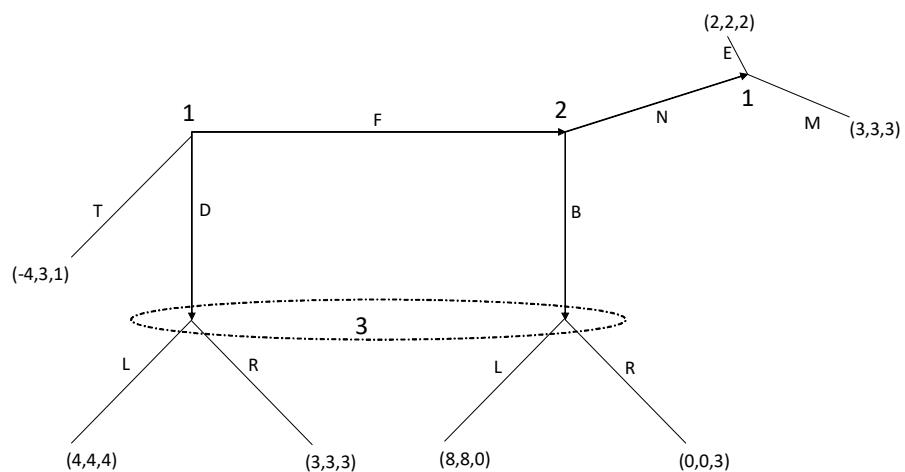
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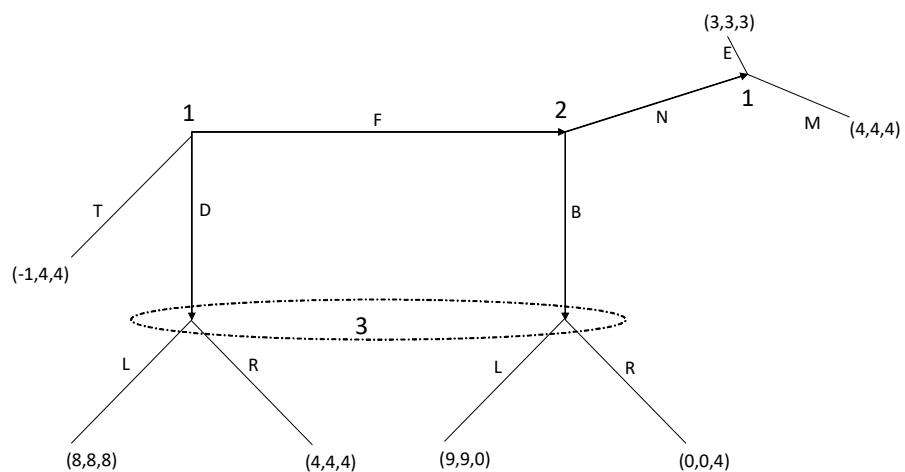
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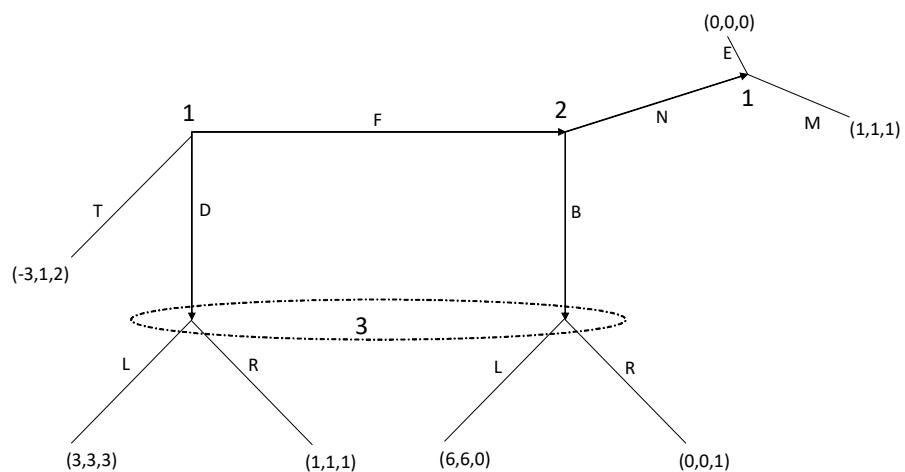
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