# ECON 439
# Midterm: Normal Form Games

Kevin Hasker

This exam will start at about 10:30 and will end around 12:10

Points will only be given for work shown.

1. *(14 points)* **Honor Statement:** Please read and sign the following statement:

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2. *(8 points)* Consider an $n$ player game where each player has a finite number of strategies. Will there be a Nash equilibrium of this game? Explain you answer and discuss why this information is important.

3. *(22 points total)* Consider a game where there are two states of the world, $\omega_1$ and $\omega_2$. Player 1 does not know the state of the world and believes $\Pr(\omega_1) = \frac{1}{2}$. Player 2 knows the state of the world. Player 1 has the strategies $\{A, B, C\}$ and player 2 has the strategies $\{\alpha, \beta, \gamma\}$ if the state is $\omega_1$ and $\{\delta, \tau, \chi\}$ if the state is $\omega_2$. The Normal form games are:

$\omega_1$

| | $\alpha$ | $\beta$ | $\gamma$ |
|---|---|---|---|
| $A$ | $0;3$ | $0;4$ | $7;7$ |
| $B$ | $4;1$ | $4;2$ | $4;3$ |
| $C$ | $0;1$ | $5;5$ | $0;4$ |

$\omega_2$

| | $\delta$ | $\tau$ | $\chi$ |
|---|---|---|---|
| $A$ | $7;5$ | $0;4$ | $0;1$ |
| $B$ | $4;2$ | $4;5$ | $4;3$ |
| $C$ | $0;2$ | $0;3$ | $5;5$ |

(a) *(6 points)* Treating the game as if player 1 knows whether the state is $\omega_1$ or $\omega_2$, find the pure strategy best responses of both players in both games. You may use the table to mark your best response but you will loose two points if you do not explain your notation below.

(b) *(4 points)* In the game as described, fill out the following table for $x \in \{A, B, C\}$.

| | $BR_2(A)$ | $BR_2(B)$ | $BR_2(C)$ |
|---|---|---|---|
| $U_1(A, BR_2(x))$ | | | |
| $U_1(B, BR_2(x))$ | | | |
| $U_1(C, BR_2(x))$ | | | |
| | ____________ | ____________ | ____________ |

(c) *(5 points)* Using the table you just constructed, find the pure strategy Bayesian Nash equilibrium of this game.

(d) *(7 points)* Compare the Bayesian Nash equilibrium you found to the best responses of the games played under full information (in part *a* of this question.) What is peculiar about this equilibrium? What does it show us about why we can not remove pure strategies from consideration when they are never a weak best response?

4. *(20 points)* About dominance and weak dominance.

(a) *(4 points)* Define a dominated strategy.

(b) *(4 points)* Explain why we can iterate the concept of dominance, and conclude that as long as players are rational and this is common knowledge no one will use a strategy that does not survive this process.

(c) *(4 points)* Define a weakly dominated strategy.

(d) *(4 points)* Explain why if players are rational and this is common knowledge we *can not* safely iteratively remove weakly dominated strategies.

(e) *(4 points)* Consider a modified game where every strategy will be played with some small probability $\varepsilon > 0$. Will there be any weakly dominated strategies that are not dominated with this modification?

5. *(30 points)* Consider a war of attrition where they can fight for up to two periods. Each player ($i \in \{1, 2\}$) can choose $t_i \in \{0, 1, 2\}$. They have a symmetric payoff function which is:

$$u_i(t_i, t_j) = \begin{cases} 8 - 6t_j & \text{if} \quad t_i > t_j \\ 4 - 6t_j & \text{if} \quad t_i = t_j \\ -6t_i & \text{if} \quad t_i < t_j \end{cases}$$

for $i \in \{1, 2\}$, $j \neq i$.

(a) *(9 points)* Convert this to a standard normal/strategic form game, drawing the table below.

(b) *(6 points)* Find the pure strategy best responses of both players, you may use the table you just drew but you will loose two points if you do not explain your notation below.

(c) *(6 points)* Find the pure strategy Nash equilibria, explain why they are Nash equilibria.

(d) *(9 points)* Note that all the pure strategy equilibria are asymmetric, or $t_i^* \neq t_j^*$. Find the symmetric equilibrium, heavy partial credit will be given for work towards the correct answer.

6. *(6 points)* Consider a Hotelling location model. There are $L$ locations, with $c_l$ ($l \in \{1, 2, 3, ..., L\}$) customers at each location, $c_l > 0$. These customers will go to firm $a$ if $|l_a - l| < |l_b - l|$, to firm $b$ if $|l_a - l| > |l_b - l|$ and otherwise half will go to each firm. Firms choose their location ($l_x \in \{1, 2, 3, ..., L\}$ for $x \in \{a, b\}$) to maximize their number of customers. Prove *business stealing* or that $BR_a(l_b) \in \{l_b - 1, l_b, l_b + 1\}$.

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3. *(22 points total)* Consider a game where there are two states of the world, $\omega_1$ and $\omega_2$. Player 1 does not know the state of the world and believes $\Pr(\omega_1) = \frac{1}{2}$. Player 2 knows the state of the world. Player 1 has the strategies $\{A, B, C\}$ and player 2 has the strategies $\{\alpha, \beta, \gamma\}$ if the state is $\omega_1$ and $\{\delta, \tau, \chi\}$ if the state is $\omega_2$. The Normal form games are:

$\omega_1$

| | $\alpha$ | $\beta$ | $\gamma$ |
|---|---|---|---|
| $A$ | 0; 2 | 0; 3 | 7; 5 |
| $B$ | 4; 7 | 4; 3 | 4; 4 |
| $C$ | 5; 3 | 0; 1 | 0; 2 |

$\omega_2$

| | $\delta$ | $\tau$ | $\chi$ |
|---|---|---|---|
| $A$ | 0; 1 | 0; 4 | 7; 5 |
| $B$ | 4; 5 | 4; 4 | 4; 1 |
| $C$ | 0; 2 | 5; 5 | 0; 3 |

(a) *(6 points)* Treating the game as if player 1 knows whether the state is $\omega_1$ or $\omega_2$, find the pure strategy best responses of both players in both games. You may use the table to mark your best response but you will loose two points if you do not explain your notation below.

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| | $BR_2(A)$ | $BR_2(B)$ | $BR_2(C)$ |
|---|---|---|---|
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(c) *(5 points)* Using the table you just constructed, find the pure strategy Bayesian Nash equilibrium of this game.

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5. *(30 points)* Consider a war of attrition where they can fight for up to two periods. Each player ($i \in \{1, 2\}$) can choose $t_i \in \{0, 1, 2\}$. They have a symmetric payoff function which is:

$$u_i(t_i, t_j) = \begin{cases} 6 - 9t_j & \text{if} \quad t_i > t_j \\ 3 - 9t_j & \text{if} \quad t_i = t_j \\ -9t_i & \text{if} \quad t_i < t_j \end{cases}$$

for $i \in \{1, 2\}$, $j \neq i$.

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$\omega_1$

| | $\alpha$ | $\beta$ | $\gamma$ |
|---|---|---|---|
| $A$ | 6; 1 | 6; 5 | 6; 4 |
| $B$ | 0; 3 | 8; 5 | 0; 2 |
| $C$ | 10; 7 | 0; 3 | 0; 4 |

$\omega_2$

| | $\delta$ | $\tau$ | $\chi$ |
|---|---|---|---|
| $A$ | 6; 2 | 6; 1 | 6; 3 |
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| | $\alpha$ | $\beta$ | $\gamma$ |
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| $B$ | $8;3$ | $0;2$ | $0;1$ |
| $C$ | $6;3$ | $6;2$ | $6;5$ |

$\omega_2$

| | $\delta$ | $\tau$ | $\chi$ |
|---|---|---|---|
| $A$ | $0;2$ | $0;1$ | $10;3$ |
| $B$ | $8;7$ | $0;4$ | $0;3$ |
| $C$ | $6;4$ | $6;7$ | $6;3$ |

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