ECON 439
Extensive Form Games

Be sure to show your work for all answers, even if the work is simple.

This exam will begin at 9:30 and end at 11:10 on Friday, 16 June

1. *(10 points)* **Honor Statement:** Please read and sign the following statement:

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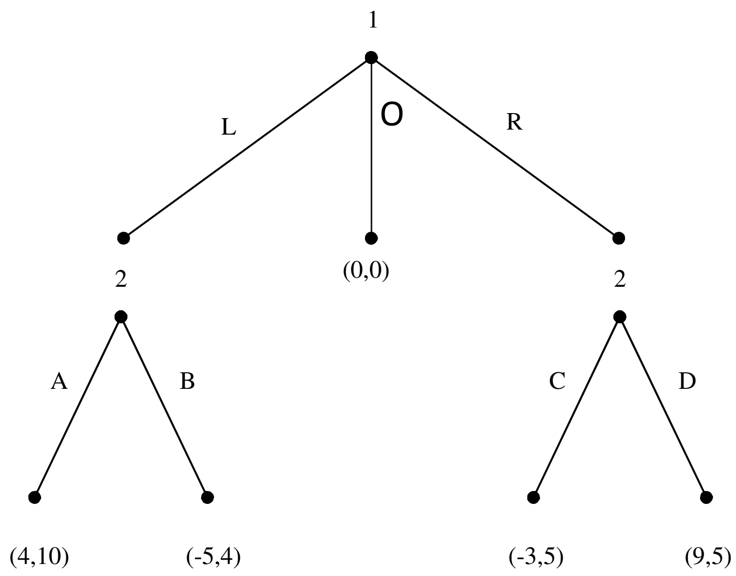
2. *(10 points total)* Remember that in a sequential game h^T is a terminal history—i.e. a potential complete history of the game, and that the set is denoted H^T. In this question we assume that players have *wholly unique payoffs* or that for all i $u_i\left(h^T\right) \neq u_i\left(\tilde{h}^T\right)$ when $h^T \neq \tilde{h}^T$, $\left\{h^T, \tilde{h}^T\right\} \subseteq H^T$.

   (a) *(6 points)* Consider the simplest sequential game. First player 1 makes a choice between A and B and then player 2 makes a choice between C and D if player 1 chooses A. Thus $H^T = \{(A,C), (A,D), B\}$. Prove that for all u_1 and u_2 with wholly unique payoffs there is a unique subgame perfect equilibrium.

   (b) *(4 points)* Generalize this, show that in a finite sequential game with wholly unique payoffs their is a unique subgame perfect equilibrium—or Zermelo's Theorem.

3. *(26 points total)* Consider the following sequential game:



(a) *(8 points)* Write down all the strategies for player 2.

(b) *(6 points)* Write down the subgame perfect *equilibria* strategies below. You may mark your best responses on the game above for partial credit, but only if you explain your notation.

(c) *(4 points)* Using the table below convert this game into a strategic

or normal form game. Their are too many rows and columns.

$S_1 \downarrow$ $S_2 \longrightarrow$

_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _
_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _
_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _
_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _
_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _
_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _	_ _;_ _

(d) *(4 points)* Mark the best responses on the table above and write down the equilibrium strategies below.

(e) *(4 points)* Categorize these equilibria as either subgame perfect, empty threat, or "other" (where the payoff is the same as in a SPE but the strategy is wrong.) Who makes the empty threat? Is it helping him or her?

4. *(6 points)* What is a *signal* in game theory?

5. *(16 points total)* Consider the following normal form game as the stage game of an infinitely repeated game, let the common discount factor be δ.

	L	C	R
U	$-2; 8$	$5; 7$	$10; 10$
M	$-3; 5$	$2; 2$	$5; -2$
D	$0; 0$	$0; -5$	$14; -4$

(a) *(3 points)* Find the pure strategy best responses for both players. Explain your notation, whether you use the table above or write them down below.

(b) *(1 point)* Find the pure strategy Nash equilibrium and write the strategy below.

(c) *(6 points)* Write down a **Grimm** or **Trigger** strategy such that if δ is high enough people expect to play (M, C) forever.

(d) *(6 points)* Prove that it is a subgame perfect equilibrium for high enough δ, and find the critical δ^* such that it is a SPE.

6. *(12 points total)* Consider a certification model of signalling. A seller has a good that is worth $w \in \{w_1, w_2, w_3, ..., w_n\}$ to themselves, where $w_1 > w_2 > w_3 > ... > w_n > 0$, and assume that for all $i \in \{1, 2, 3, ...n\}$ $\Pr(w = w_i) > 0$. It has a value to the customer of vw where $v > 1$. At a cost of $c > 0$ the good can be *certified*, which makes the worth of the good common knowledge. The profits of the seller can be expressed as $\pi(p, w) = p - w$ if they make a sale, otherwise it is zero. The utility of the buyers can be written as $u(p, w) = vw - p$ if they purchase a good of worth w, otherwise it is zero.

   Like in the Spence signalling model, we assume their is Bertrand competition among the buyers who all have the same expectations. This means that if the good is certified $p = vw$, if it is not then $p = vE[w|w \in NC]$ where NC is the set of worths that will not be certified. (Only consider pure strategy equilibria, i.e. either all or none of the sellers for whom $w = w_i$ will get certified).

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7. *(14 points total)* Define the following terms:

   (a) *(3 points)* an *assessment*

   (b) *(3 points)* *sequential rationality.*

   (c) *(3 points)* *consistency of beliefs.*

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8. *(6 points)* It is common to believe that optimizing (rational) agents will always play an equilibrium. Explain what is wrong with this belief by explaining what else we need to get agents to play a equilibrium. (Your answer should be correct for all versions of equilibrium we have analyzed this semester.)

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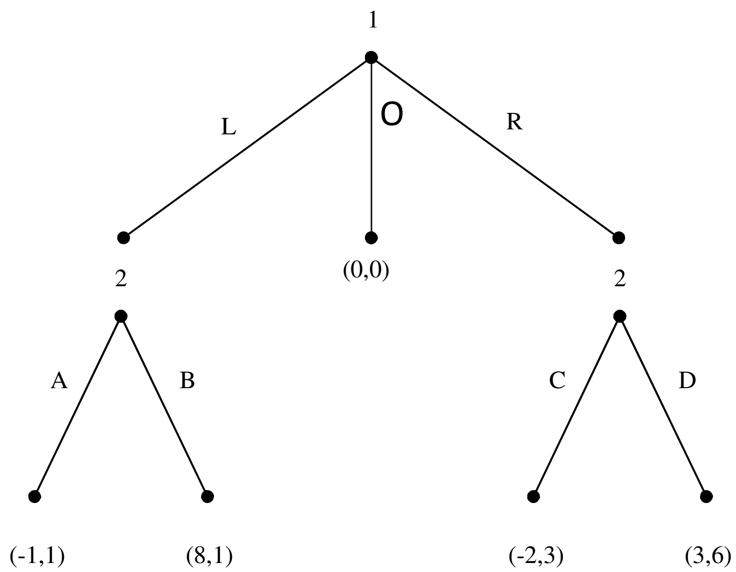

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5. *(16 points total)* Consider the following normal form game as the stage game of an infinitely repeated game, let the common discount factor be δ.

	L	C	R
U	$10; 10$	$9; 5$	$-4; 6$
M	$9; -4$	$4; 4$	$-5; 9$
D	$12; -2$	$0; -1$	$0; 0$

(a) *(3 points)* Find the pure strategy best responses for both players. Explain your notation, whether you use the table above or write them down below.

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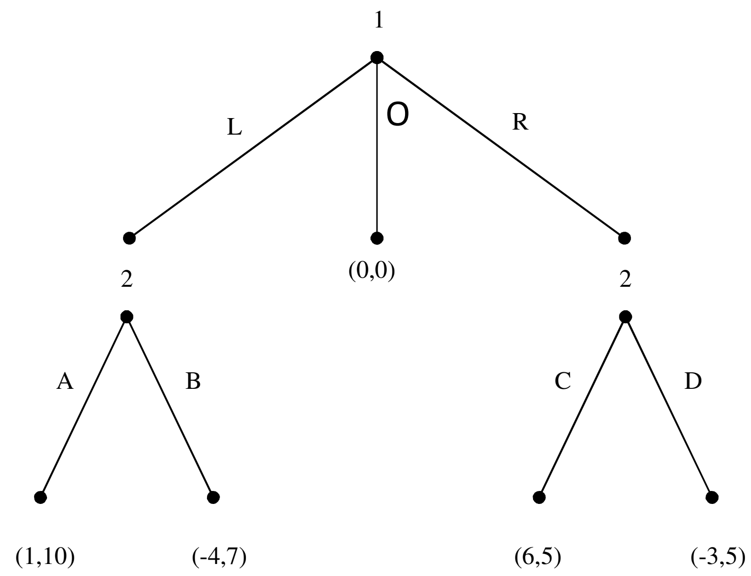

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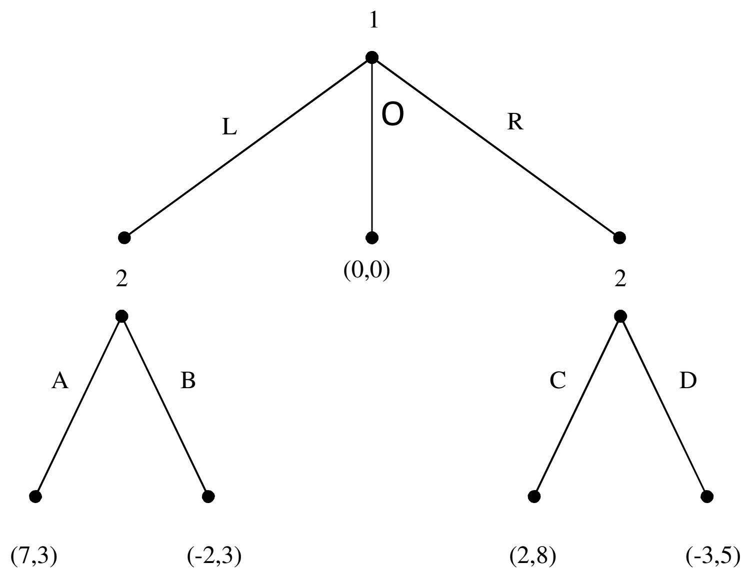

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