ECON 439
Midterm: Normal Form Games

Kevin Hasker

This exam will start at about 12:20 and will end around 14:00

Points will only be given for work shown.

1. *(10 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not offer assistance to others. Finally I will not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(8 points)* Lil' Kevin does not like mixed strategies, so he wants to study Game Theory without them. Explain to him why this will not work, be sure to mention both the mathematical and intuitive problem with this approach.

   Solution 1 *Obviously their can be many answers for this question, but here is my thoughts—which will be based on your answers when appropriate.*

   Mathematical:

   (a) *Existance: In order for their to be a Nash equilbrium the best responses of all parties need to be Upper Hemi-Continuous—or their can be no suddenly dissapearing points. With a discrete function it will be technically UHC (in the space it is defined on) but we can not be sure their will be a Nash equilibrium. As a colleague once said "their is nothing more difficult than trying to find an empty set."*

   (b) *Say that you are analyzing an interaction, and you find a cycle in the pure strategy best responses. What are you going to do? Are you going to say their is no equilibrium? Assert that rational people will go around and around the cycle not knowing what to do? Or, alternatively, are you going to accept that people might not want to be predictable and find a Nash equilibrium where they satisfy this desire? (This one I had thought of, but many of you presented it, and frankly this is what would have convinced Lil' Kevin because this was a practical problem he would have to solve.)*

   Intuitive:

(a) *Want to play Rock/Paper/Scissors? You are going to play a pure strategy, right?*

(b) *Are you always certain about what others are going to do? How would you represent this uncertainty? (Again an excellent point raised frequently in your answers, and again one that would have absolutely convinced Lil' Kevin.)*

3. *(44 points total)* In this game their are two states of the world, α and β. The prior probabilities of these states are equally likely ($\Pr(\alpha) = \Pr(\beta) = \frac{1}{2}$). What will differ in the different variations is what player's know about the state of the world, or a player's beliefs that the state of the world is $x \in \{\alpha, \beta\}$ given that the state $y \in \{\alpha, \beta\}$ is the true state. We will denote these $\Pr_i(x|y)$ for $i \in \{1, 2\}$. We will only consider two possibilities, that they are completely informed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = 1$) or they are completely uninformed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = \frac{1}{2}$). The games are:

Remark 2 *At this point I must apologize for my errors. Of the four variations one had two pure strategy Nash equilibria in the no information game, and one did not have a pure strategy equilibrium under asymmetric equilibrium. Since the former error was caught during the exam it was not much of a problem, the second one no one asked me about (or at least explained what they were asking) thus I was simply generous in that case. I am quite certain it did not cost anyone more than a point or two. But I apologize.*

How can I be certain my answers are correct now? Because I wrote a simple program in Excel to make sure they were. It is quite easy to find the best responses using Excel, and create the No Information game payoffs.

α (P2)

		L	C	R
P1	U	$48; 8^{1}$	$36; 12^{12}$	$12; 6$
	M	$12; 12$	$6; 18^{2}$	$8; 12$
	D	$6; -48$	$-24; 48^{2}$	$24; 24^{1}$

β (P2)

		L	C	R
P1	U	$6; 16^{2}$	$8; 6$	$12; 12$
	M	$48; 8^{1}$	$24; 12^{12}$	$16; 6$
	D	$-24; 60^{2}$	$-24; -48$	$24; 24^{1}$

$\frac{1}{2}\alpha + \frac{1}{2}\beta$ (P2)

		L	C	R
P1	U	$\frac{1}{2}(48) + \frac{1}{2}(6); \frac{1}{2}(8) + \frac{1}{2}(16)$	$\frac{1}{2}(36) + \frac{1}{2}(8); \frac{1}{2}(12) + \frac{1}{2}(6)$	$\frac{1}{2}(12) + \frac{1}{2}(12$
	M	$\frac{1}{2}(12) + \frac{1}{2}(48); \frac{1}{2}(12) + \frac{1}{2}(8)$	$\frac{1}{2}(6) + \frac{1}{2}(24); \frac{1}{2}(18) + \frac{1}{2}(12)$	$\frac{1}{2}(8) + \frac{1}{2}(16)$
	D	$\frac{1}{2}(6) + \frac{1}{2}(-24); \frac{1}{2}(-48) + \frac{1}{2}(60)$	$\frac{1}{2}(-24) + \frac{1}{2}(-24); \frac{1}{2}(48) + \frac{1}{2}(-48)$	$\frac{1}{2}(24) + \frac{1}{2}(24$

$\frac{1}{2}\alpha + \frac{1}{2}\beta$ (P2)

		L	C	R
P1	U	$27; 12^{2}$	$22; 9^{1}$	$12; 9$
	M	$30; 10^{1}$	$15; 15^{2}$	$12; 9$
	D	$-9; 6$	$-24; 0$	$24; 24^{12}$

		$BR_2(U) = (C, L)$	$BR_2(M) = (C, C)$	$BR_2(D) = (C, L)$	
	U	$\frac{1}{2}(36) + \frac{1}{2}(6)$	$\frac{1}{2}(36) + \frac{1}{2}(8)$	$\frac{1}{2}(36) + \frac{1}{2}(6)$	$=$
P1	M	$\frac{1}{2}(6) + \frac{1}{2}(48)$	$\frac{1}{2}(6) + \frac{1}{2}(24)$	$\frac{1}{2}(6) + \frac{1}{2}(48)$	
	D	$\frac{1}{2}(-24) + \frac{1}{2}(-24)$	$\frac{1}{2}(-24) + \frac{1}{2}(-24)$	$\frac{1}{2}(-24) + \frac{1}{2}(-24)$	

		$BR_2(U) = (C, L)$	$BR_2(M) = (C, C)$	$BR_2(D) = (C, L)$
	U	21	22^1	21
P1	M	27^1	15	27^1
	D	-24	-24	-24

Their is no pure strategy Nash equilibrium under asymmetric information.

α

	P2	L	C	R
	U	$48; 24^{12}$	$96; 16^1$	$32; 12$
P1	M	$-48; -96$	$-48; 120^2$	$48; 48^1$
	D	$16; 12$	$12; 32^2$	$24; 24$

β

	P2	L	C	R
	U	$12; 36^2$	$24; 24$	$16; 24$
P1	M	$-48; 96^2$	$12; -96$	$48; 48^1$
	D	$96; 24^{12}$	$96; 16^1$	$24; 12$

$\frac{1}{2}\alpha + \frac{1}{2}\beta$

	P2	L	C	R
	U	$\frac{1}{2}(48) + \frac{1}{2}(12); \frac{1}{2}(24) + \frac{1}{2}(36)$	$\frac{1}{2}(96) + \frac{1}{2}(24); \frac{1}{2}(16) + \frac{1}{2}(24)$	$\frac{1}{2}(32) + \frac{1}{2}($
P1	M	$\frac{1}{2}(-48) + \frac{1}{2}(-48); \frac{1}{2}(-96) + \frac{1}{2}(96)$	$\frac{1}{2}(-48) + \frac{1}{2}(12); \frac{1}{2}(120) + \frac{1}{2}(-96)$	$\frac{1}{2}(48) + \frac{1}{2}($
	D	$\frac{1}{2}(16) + \frac{1}{2}(96); \frac{1}{2}(12) + \frac{1}{2}(24)$	$\frac{1}{2}(12) + \frac{1}{2}(96); \frac{1}{2}(32) + \frac{1}{2}(16)$	$\frac{1}{2}(24) + \frac{1}{2}($

$\frac{1}{2}\alpha + \frac{1}{2}\beta$

	P2	L	C	R
	U	$30; 30^1$	$60; 20^1$	$22; 18$
P1	M	$-48; 0$	$-18; 12$	$48; 48^{12}$
	D	$56; 18^1$	$54; 24^2$	$24; 18$

P2

		$BR_2(U) = (L, L)$	$BR_2(M) = (C, L)$	$BR_2(D) = (C, L)$
	U	$\frac{1}{2}(48) + \frac{1}{2}(12) = 30$	$\frac{1}{2}(96) + \frac{1}{2}(12) = 54^1$	$\frac{1}{2}(96) + \frac{1}{2}(12) = 54^1$
P1	M	$\frac{1}{2}(-48) + \frac{1}{2}(-48) = -48$	$\frac{1}{2}(-48) + \frac{1}{2}(-48) = -48$	$\frac{1}{2}(-48) + \frac{1}{2}(-48) = -48$
	D	$\frac{1}{2}(16) + \frac{1}{2}(96) = 56$	$\frac{1}{2}(12) + \frac{1}{2}(96) = 54^1$	$\frac{1}{2}(12) + \frac{1}{2}(96) = 54^1$

Was it intended for their to be two best responses? No, but their is still only one pure strategy Nash equilibrium, $(D)(C, L)$

α

	P2	L	C	R
	U	$-48; 96^2$	$48; 48^1$	$12; -96$
P1	M	$12; 36^2$	$16; 24$	$24; 24$
	D	$48; 24^{12}$	$24; 12$	$36; 16^1$

β

	P2	L	C	R
	U	$-48; -96$	$48; 48^1$	$-48; 120^2$
P1	M	$48; 24^{12}$	$32; 12$	$36; 16^1$
	D	$16; 12$	$24; 24$	$12; 32^2$

$\frac{1}{2}\alpha + \frac{1}{2}\beta$

P2

		L	C	R
P1	U	$\frac{1}{2}(-48) + \frac{1}{2}(-48); \frac{1}{2}(96) + \frac{1}{2}(-96)$	$\frac{1}{2}(48) + \frac{1}{2}(48); \frac{1}{2}(48) + \frac{1}{2}(48)$	$\frac{1}{2}(12) + \frac{1}{2}(-48)$
	M	$\frac{1}{2}(12) + \frac{1}{2}(48); \frac{1}{2}(36) + \frac{1}{2}(24)$	$\frac{1}{2}(16) + \frac{1}{2}(32); \frac{1}{2}(24) + \frac{1}{2}(36)$	$\frac{1}{2}(24) + \frac{1}{2}(36);$
	D	$\frac{1}{2}(48) + \frac{1}{2}(16); \frac{1}{2}(24) + \frac{1}{2}(12)$	$\frac{1}{2}(24) + \frac{1}{2}(24); \frac{1}{2}(12) + \frac{1}{2}(24)$	$\frac{1}{2}(36) + \frac{1}{2}(12);$

$\frac{1}{2}\alpha + \frac{1}{2}\beta$

P2

		L	C	R
P1	U	$-48; 0$	$48; 48^{12}$	$-18; 12$
	M	$30; 30^{2}$	$24; 30^{2}$	$30; 20^{1}$
	D	$32; 18^{1}$	$24; 18$	$24; 24^{2}$

α

P2

		L	C	R
P1	U	$-48; 96^{2}$	$48; 48^{1}$	$12; -96$
	M	$12; 36^{2}$	$16; 24$	$24; 24$
	D	$48; 24^{12}$	$24; 12$	$36; 16^{1}$

β

P2

		L	C	R
P1	U	$-48; -96$	$48; 48^{1}$	$-48; 120^{2}$
	M	$48; 24^{12}$	$32; 12$	$36; 16^{1}$
	D	$16; 12$	$24; 24$	$12; 32^{2}$

P2

		$BR_2(U) = (L, R)$	$BR_2(M) = (L, L)$	$BR_2(D) = (L, R)$
P1	U	$\frac{1}{2}(-48) + \frac{1}{2}(-48) = -48$	$\frac{1}{2}(-48) + \frac{1}{2}(-48) = -48$	$\frac{1}{2}(-48) + \frac{1}{2}(-48) = -48$
	M	$\frac{1}{2}(12) + \frac{1}{2}(36) = 24$	$\frac{1}{2}(12) + \frac{1}{2}(32) = 22$	$\frac{1}{2}(12) + \frac{1}{2}(36) = 24$
	D	$\frac{1}{2}(48) + \frac{1}{2}(12) = 30^{1}$	$\frac{1}{2}(48) + \frac{1}{2}(24) = 36^{1}$	$\frac{1}{2}(48) + \frac{1}{2}(12) = 30^{1}$

$NE\ is\ (D)(L, R)$

α

P2

		L	C	R
P1	U	$24; 24^{1}$	$-24; 60^{2}$	$-24; -48$
	M	$12; 12$	$6; 16^{2}$	$8; 6$
	D	$16; 6$	$48; 8^{1}$	$24; 12^{12}$

β

P2

		L	C	R
P1	U	$24; 24^{1}$	$6; -48$	$-24; 48^{2}$
	M	$12; 6$	$48; 8^{1}$	$60; 12^{12}$
	D	$8; 12$	$12; 12$	$6; 18^{2}$

$\frac{1}{2}\alpha + \frac{1}{2}\beta$

P2

		L	C	R
P1	U	$\frac{1}{2}(24) + \frac{1}{2}(24); \frac{1}{2}(24) + \frac{1}{2}(24)$	$\frac{1}{2}(-24) + \frac{1}{2}(6); \frac{1}{2}(60) + \frac{1}{2}(-48)$	$\frac{1}{2}(-24) + \frac{1}{2}(-24);$
	M	$\frac{1}{2}(12) + \frac{1}{2}(12); \frac{1}{2}(12) + \frac{1}{2}(6)$	$\frac{1}{2}(6) + \frac{1}{2}(48); \frac{1}{2}(16) + \frac{1}{2}(8)$	$\frac{1}{2}(8) + \frac{1}{2}(60); \frac{1}{2}(6$
	D	$\frac{1}{2}(16) + \frac{1}{2}(8); \frac{1}{2}(6) + \frac{1}{2}(12)$	$\frac{1}{2}(48) + \frac{1}{2}(12); \frac{1}{2}(8) + \frac{1}{2}(12)$	$\frac{1}{2}(24) + \frac{1}{2}(6); \frac{1}{2}(1$

$\frac{1}{2}\alpha + \frac{1}{2}\beta$

P2

		L	C	R
P1	U	$24; 24^{12}$	$-9; 6$	$-24; 0$
	M	$12; 9$	$27; 12^{2}$	$34; 9^{1}$
	D	$12; 9$	$30; 10^{1}$	$15; 15^{2}$

		α P2 L	C	R
	U	$24;24^1$	$-24;60^2$	$-24;-48$
P1	M	$12;12$	$6;16^2$	$8;6$
	D	$16;6$	$48;8^1$	$24;12^{12}$

		β P2 L	C	R
	U	$24;24^1$	$6;-48$	$-24;48^2$
P1	M	$12;6$	$48;8^1$	$60;12^{12}$
	D	$8;12$	$12;12$	$6;18^2$

P2

		$BR_2(U)=(C,R)$	$BR_2(M)=(C,R)$	$BR_2(D)=(R,R)$
	U	$\frac{1}{2}(-24)+\frac{1}{2}(-24)=-24$	$\frac{1}{2}(-24)+\frac{1}{2}(-24)=-24$	$\frac{1}{2}(-24)+\frac{1}{2}(-24)=-24$
P1	M	$\frac{1}{2}(6)+\frac{1}{2}(60)=33^1$	$\frac{1}{2}(6)+\frac{1}{2}(60)=33^1$	$\frac{1}{2}(8)+\frac{1}{2}(60)=34^1$
	D	$\frac{1}{2}(48)+\frac{1}{2}(6)=27$	$\frac{1}{2}(48)+\frac{1}{2}(6)=27$	$\frac{1}{2}(24)+\frac{1}{2}(6)=15$

The NE is $(M)(C,R)$

(a) *(16 points total)* $\Pr_i(\alpha|\alpha)=\Pr_i(\beta|\beta)=1$ for $i\in\{1,2\}$ (Complete information)

i. *(12 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

Solution 3 *They are marked on the table with a 1 in the upper right hand corner for the BR of player 1, and a 2 for player 2. I am quite pleased that only one student did not know what a best response was. Most lost either 3 points for not explaining their notation or a point or two for getting a BR wrong.*

ii. *(4 points)* Find the pure strategy Nash equilibrium in both games. Write down the strategies players use below.

Solution 4 *In the first game above they are (U,C) and (M,C) for the rest they are the squares with both a 1 and a 2 in the upper right hand corner.*

(b) *(10 points total)* $\Pr_i(\alpha|\alpha)=\Pr_i(\beta|\beta)=\frac{1}{2}$ for $i\in\{1,2\}$ (No information)

i. *(2 points)* Write down the expected payoffs from the game in the table below:

Solution 5 *See above. I was shocked at how many of you could not do this simple task. Expected utility is required for this class and after that it is a simple mathematical exercise. I should have given this question more points so that those who could not do it lost more.*

ii. *(6 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

Solution 6 *Like before I put a 1 in the upper right hand corner of the square if it is a best response for player 1, and a 2 for player 2.*

iii. *(2 points)* Find the pure strategy Nash equilibrium. Write down the strategies players use below.

Solution 7 *The proper notation is show in part a.ii, these are the squares with both a 1 and 2 in them in the appropriate games above.*

(c) *(10 points total)* $\Pr_1(\alpha|\alpha) = \Pr_1(\beta|\beta) = \frac{1}{2}$, $\Pr_2(\alpha|\alpha) = \Pr_2(\beta|\beta) = 1$ (Asymmetric Information, player 2 is informed and player 1 is not.)

i. *(3 points)* Write down player 2's best responses, be careful to denote in which state they play which action.

Solution 8 *See the games above, in all cases when I write (X, Y) X is the best response in state α and Y is the best response in the state β.*
I might have marked you off a point here if your notation was so bad I couldn't figure out if you knew what I was really asking. As in what was the strategies for player 2.

ii. *(3 points)* In the table below find player 1's expected payoff from each of player 2's best responses:

Solution 9 *see above*

iii. *(4 points)* Find the pure strategy Nash equilibrium. Explain your answer.

Solution 10 *see above. In all cases in c.i we find $BR_2(X)$ for $X \in \{U, M, D\}$, in c.ii we find the expted utilities and find $BR_1(BR_2(X))$. It is a pure strategy Nash equilibrium if $X \in BR_1(BR_2(X))$.*

(d) *(8 points)* Calculate player 2's expected utilities in each of the cases above (from an a-priori point of view, when both of the games are equally likely, if their are multiple consider the best case for player 2) Their is something odd about player 2's expected utilities, explain what is odd and why this can occur.

Remark 11 *I notice that I did not explicitly say "in equilibrium" but since none of you asked about this during the exam I guess you must have all understood what I meant.*

Solution 12 *I will answer in two cases. First when:*

α — *P2*

		L	*C*	*R*
P1	*U*	$48; 24^{12}$	$96; 16^{1}$	$32; 12$
	M	$-48; -96$	$-48; 120^{2}$	$48; 48^{1}$
	D	$16; 12$	$12; 32^{2}$	$24; 24$

β — *P2*

		L	*C*	*R*
P1	*U*	$12; 36^{2}$	$24; 24$	$16; 24$
	M	$-48; 96^{2}$	$12; -96$	$48; 48^{1}$
	D	$96; 24^{12}$	$96; 16^{1}$	$24; 12$

$$
\begin{aligned}
U_2\left(\textit{Complete Information}\right) &= \frac{1}{2}(24)+\frac{1}{2}(24)=12 \\
U_2\left(\textit{No Information}\right) &= 48 \\
U_2\left(\textit{Asymmetric Information}\right) &= \frac{1}{2}U_2(D,C,\alpha)+\frac{1}{2}U_2(D,L,\beta)=\frac{1}{2}(32)+\frac{1}{2}(24)=28
\end{aligned}
$$

The peculiarity in this is that when player 2 has more information (Complete or Asymmetric) they have a lower payoff. The solution to this apparent paradox is that player 1 knows this, and reacts to player 2's plans, thus we can not be sure more information will make your Nash equilibrium payoffs higher.

I will also answer the same question for the game:

α, P2

		L	C	R
P1	U	$48;8^1$	$36;12^{12}$	$12;6$
	M	$12;12$	$6;18^2$	$8;12$
	D	$6;-48$	$-24;48^2$	$24;24^1$

β, P2

		L	C	R
P1	U	$6;16^2$	$8;6$	$12;12$
	M	$48;8^1$	$24;12^{12}$	$16;6$
	D	$-24;60^2$	$-24;-48$	$24;24^1$

where their is no pure strategy Nash equlibrium under complete information. This is inspired by one of you, who answered this part of the question correctly even though you did not find the Nash equilibrium under asymmetric information.

$$
\begin{aligned}
U_2\left(\textit{Complete Information}\right) &= \frac{1}{2}(12)+\frac{1}{2}(12)=12 \\
U_2\left(\textit{No Information}\right) &= 24 \\
U_2\left(\textit{Asymmetric Information}\right) &= \frac{1}{2}X+\frac{1}{2}Y
\end{aligned}
$$

In order for $\frac{1}{2}X+\frac{1}{2}Y$ to be higher than 24 it must include $U_2(D,C,\alpha)$ and $U_2(D,L,\beta)$ however in the payoffs:

		$BR_2(U)=(C,L)$	$BR_2(M)=(C,C)$	$BR_2(D)=(C,L)$
P1	U	21	22^1	21
	M	27^1	15	27^1
	D	-24	-24	-24

we see that D is never a best response to any of these strategies, thus it is almost certain that $\frac{1}{2}X+\frac{1}{2}Y<24$, thus we can reach the same conclusion.

4. *(38 points total)* Consider a location model, consumers or voters are at one of five locations ($l \in \{1,2,3,4,5\}$). The total number at each location is given below, the total number at all location is C.

Location (l)	1	2	3	4	5
Number at that location (c_l)	$\frac{1}{3}C$	$\frac{1}{4}C$	$\frac{1}{24}C$	$\frac{1}{8}C$	$\frac{1}{4}C$

Remark 13 *I will answer the question for*

Location (l)	*1*	*2*	*3*	*4*	*5*
Number at that location (c_l)	24	12	4	24	32

since this is such a simple exercise. The variations on different exam were found by either doubling each cell ($C = 48$ *instead of* $C = 96$) *or flipping the number in each cell. (For example in one version* $l_5 \in \{16, 32\}$ *and* $l_1 \in \{12, 24\}$ *while in the other* $l_5 \in \{12, 24\}$ *and* $l_1 \in \{16, 32\}$*).*

(a) *(20 points total)* Consider first the Hotelling location model. Consumers go the firm that is closest to them, and half go to each firm if the firms are equally close to their location. The two firms objectives are to maximize their demand. Let the demand of firm $j \in \{a, b\}$ given that they are at location l_j when their opponent is at location l_{-j} ($-j = \{a, b\} \setminus j$) be denoted $D_j (l_j, l_{-j})$.

i. *(10 points total)* Fill out the table below with $D_a (l_a, l_b)$ for all $(l_a, l_b) \in \{1, 2, 3, 4, 5\}^2$.

Solution 14 *Let me emphasize how all you had to do to get these 10 points was follow the instructions. You merely had to do what was written on the page, prior knowledge was unnecessary. Say, for example that firm a is at location* 1 *while firm b is at location* 5*, then we create the following table:*

location	*1*	*2*	*3*	*4*	*5*
distance to a	$\lvert 1-1 \rvert = 0$	$\lvert 2-1 \rvert = 1$	$\lvert 3-1 \rvert = 2$	$\lvert 4-1 \rvert = 3$	$\lvert 5-1 \rvert = 4$
distance to b	$\lvert 1-5 \rvert = 4$	$\lvert 2-5 \rvert = 3$	$\lvert 3-5 \rvert = 2$	$\lvert 4-5 \rvert = 1$	$\lvert 5-5 \rvert = 0$
which is closer	*a*	*a*	*tie*	*b*	*b*
D_a	24	12	$\frac{4}{2}$	0	0

so $D_a (1, 5) = 38$*, do this 25 times and you have an easy 10 points. All you had to do was follow instructions.*
Much fewer steps were actually required, for example obviously $D_a (x, x) = 48$*, and if* $D_a (x, y) = m$ *then* $D_a (y, x) = 96 - m$*, so at most 11 calculations were needed. In fact only 8 is needed, but that requires some deep logic.*[1]

If $l_b =$	1	2	3	4	5
$D_a (1, l_b) =$	48	24	30	36	38
$D_a (2, l_b) =$	72	48	36	38	40
$D_a (3, l_b) =$	66	60	48	40	52
$D_a (4, l_b) =$	60	58	56	48	64
$D_a (5, l_b) =$	58	56	44	32	48

[1] This deep logic is recognizing that the only important question is the location of the marginal consumer. Thus $D_a (x, y) = D_a (x + 1, y - 1)$ as long as $x < y + 2$.

ii. *(2 points)* Why do we not have to do the same exercise for firm b?

Solution 15 *Since this is a symmetric game* $D_a(x,y) = D_b(y,x)$*, of course it is also true (but a bit less useful) to point out that* $D_a(x,y) = 96 - D_b(x,y)$ *so one matrix defines the other.*

iii. *(8 points)* Find the unique pair of strategies to survive iterated deletion of dominated strategies. Show why it survives step by step.

Solution 16 *I said unique, why did so many of you stop? Did you run out of time? I hope so. Anyway here is my overly detailed answer. In the game:*

If $l_b =$	1	2	3	4	5
$D_a(1,l_b) =$	48	24	30	36	38
$D_a(2,l_b) =$	72	48	36	38	40
$D_a(3,l_b) =$	66	60	48	40	52
$D_a(4,l_b) =$	60	58	56	48	64
$D_a(5,l_b) =$	58	56	44	32	48

We can see that $\{2,3,4\}$ *have a higher demand for every* l_b *than* 1*, thus* 1 *is dominated and can be removed, by symmetry we can do that for player 2 as well, resulting in the game:*

If $l_b =$	2	3	4	5
$D_a(2,l_b) =$	48	36	38	40
$D_a(3,l_b) =$	60	48	40	52
$D_a(4,l_b) =$	58	56	48	64
$D_a(5,l_b) =$	56	44	32	48

As we could have noted before, $\{3,4\}$ *have a higher demand than* 5 *at every state, so we can remove that one, like before we can also conclude that player 2 will as well.*

If $l_b =$	2	3	4
$D_a(2,l_b) =$	48	36	38
$D_a(3,l_b) =$	60	48	40
$D_a(4,l_b) =$	58	56	48

now both 3 *and* 4 *dominate* 2*.*

If $l_b =$	3	4
$D_a(3,l_b) =$	48	40
$D_a(4,l_b) =$	56	48

and of course 4 *dominated* 3*, thus* $(4,4)$ *is the unique pair of locations to survive iterated deletion of dominated strategies.*
You know, I have been always referring to an equilibrium by its strategies, and consistently ask for you to do that. Some of you

said in this game the equilibrium was $(48, 48)$*. That was of course wrong, because their are 5 pairs of strategies that result in this payoff. It was so much fun to mark you down for doing that.*

(b) *(12 points total)* Now consider a model of political parties, consumers vote for the political party (a or b) that is closest to them, splitting their vote if both parties are equally close. Let $D_j(l_j, l_{-j})$ now be the number of votes party $j \in \{a, b\}$ receives. Then a political party's utility function is:

$$u_j(l_j, l_{-j}) = \begin{cases} 1 & \text{if} \quad D_j(l_j, l_{-j}) > \frac{C}{2} \\ \frac{1}{2} & \text{if} \quad D_j(l_j, l_{-j}) = \frac{C}{2} \\ 0 & \text{if} \quad D_j(l_j, l_{-j}) < \frac{C}{2} \end{cases}.$$

i. *(2 points)* Using the demands you found for the Hotelling model above fill out the table below. (I.e. convert their demands into utilities.)

Remark 17 *All you have to do, again, is follow the instructions. If* $D_a(l_a, l_b) > 48$ *party a wins, if* $D_a(l_a, l_b) = 48$ *then party a ties. This results in the matrix:*

If $l_b =$	1	2	3	4	5
$u_a(1, l_b) =$	$\frac{1}{2}$	0	0	0	0
$u_a(2, l_b) =$	1	$\frac{1}{2}$	0	0	0
$u_a(3, l_b) =$	1	1	$\frac{1}{2}$	0	1
$u_a(4, l_b) =$	1	1	1	$\frac{1}{2}$	1
$u_a(5, l_b) =$	1	1	0	0	$\frac{1}{2}$

ii. *(4 points)* Write down the best responses for political party a in the space below:

Solution 18 *and the best responses:*

If $l_b =$	1	2	3	4	5
$BR_a(l_b) =$	$\{2, 3, 4, 5\}$	$\{3, 4, 5\}$	$\{4\}$	$\{4\}$	$\{3, 4\}$

The key point here was that the best responses are multi valued.

iii. *(2 points)* Find the unique Nash equilibrium. Explain your reasoning.

Solution 19 *I did not ask you to prove it was unique, notice that*

$$BR_a(4) = 4$$

and by symmetry this is true for political party b, thus

$$BR_b(BR_a(4)) = 4$$

and this is a Nash equilibrium. It is, of course, unique. Notice that if $l_b < 4$ *then* $BR_a > l_b$ *and if* $l_b = 5$ *then* $BR_a(5) < 5$*. Like in the Hotelling model this means that the unique Nash equilibrium is* $(4, 4)$ *(not* $\left(\frac{1}{2}, \frac{1}{2}\right)$*, did you not realize that referred to 5 pairs of locations?)*

iv. *(4 points)* Is one of the locations strictly dominated in this game? Why or why not? Do you think you would be able to solve the voting model using iterated deletion of dominated strategies? **NOTE**: I am not asking you to solve the model using this method, merely state whether you think you can.

Solution 20 *Yes, location 4 dominates location 1:*

If $l_b =$	1	2	3	4	5
$u_a(1, l_b) =$	$\frac{1}{2}$	0	0	0	0
$u_a(2, l_b) =$	1	$\frac{1}{2}$	0	0	0
$u_a(3, l_b) =$	1	1	$\frac{1}{2}$	0	1
$u_a(4, l_b) =$	1	1	1	$\frac{1}{2}$	1
$u_a(5, l_b) =$	1	1	0	0	$\frac{1}{2}$

removing this for both political parties:

If $l_b =$	2	3	4	5
$u_a(2, l_b) =$	$\frac{1}{2}$	0	0	0
$u_a(3, l_b) =$	1	$\frac{1}{2}$	0	1
$u_a(4, l_b) =$	1	1	$\frac{1}{2}$	1
$u_a(5, l_b) =$	1	0	0	$\frac{1}{2}$

we now see 4 *dominates* 2*,*

If $l_b =$	3	4	5
$u_a(3, l_b) =$	$\frac{1}{2}$	0	1
$u_a(4, l_b) =$	1	$\frac{1}{2}$	1
$u_a(5, l_b) =$	0	0	$\frac{1}{2}$

and now 4 dominates 5:

If $l_b =$	3	4
$u_a(3, l_b) =$	$\frac{1}{2}$	0
$u_a(4, l_b) =$	1	$\frac{1}{2}$

and now 4 *dominates* 3*. Can this be done in general? I suspect not but I have no idea. It requires that we have a location that always looses to every other location—and that this property iterates.*

(c) *(6 points)* Consider the welfare implications of both equilibria. Does this result seem welfare optimal in the Hotelling model? In the voting model? Why or why not? Note you can claim whatever you want, points will be given for backing your claim with a careful argument.

Solution 21 *Those of you who said "its the equilibrium so it must be welfare optimal" or (so much worse) "Pareto efficient" really did not deserve any points. I gave you one because this was an intentionally vague question. Equilibrium is only Pareto Efficient in one* *case—the competitive market (with unreasonable restrictions on externalities and public goods). In the real world equilibrium is rarely Pareto efficient, and certainly is not here. Since the firms get the same total demand no matter where they locate that indicates that if customers have any preferences against travelling then this is not optimal. Indeed it is quite easy to Pareto improve on this equilibrium, take one of the firms and move them close to some of the other customers. Assuming customers do not like to travel that is a clear Pareto improvement.*

But of course, you can argue however you want–as long as you do it well.

My answer: In the case of the Hotelling location model see above. Indeed their is no reasonable model that includes a dislike for travelling by the consumer that would make this equilibrium optimal.

In the case of the voting model, I think it might be welfare optimal. The structure of the payoff function implies this is a "winner take all" election like becoming President of Turkey or the majority party in the English Parliment. In that case I think it is a perfectly reasonable welfare criterion to state that the best political party would be a moderate one—at the median of the population. However of course you might disagree—many did and got significant points.