ECON 439
Midterm: Normal Form Games

Kevin Hasker

This exam will start at about 12:20 and will end around 14:00

Points will only be given for work shown.

1. *(10 points)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I will also not offer assistance to others. Finally I will not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _
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   Signature:

   _

2. *(8 points)* Lil' Kevin does not like mixed strategies, so he wants to study Game Theory without them. Explain to him why this will not work, be sure to mention both the mathematical and intuitive problem with this approach.

3. *(44 points total)* In this game their are two states of the world, α and β. The prior probabilities of these states are equally likely ($\Pr(\alpha) = \Pr(\beta) = \frac{1}{2}$). What will differ in the different variations is what player's know about the state of the world, or a player's beliefs that the state of the world is $x \in \{\alpha, \beta\}$ given that the state $y \in \{\alpha, \beta\}$ is the true state. We will denote these $\Pr_i(x|y)$ for $i \in \{1, 2\}$. We will only consider two possibilities, that they are completely informed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = 1$) or they are completely uninformed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = \frac{1}{2}$). The games are:

α

		P2 L	C	R
	U	48; 24	96; 16	32; 12
P1	M	−48; −96	−48; 120	48; 48
	D	16; 12	12; 32	24; 24

β

		P2 L	C	R
	U	12; 36	24; 24	16; 24
P1	M	−48; 96	12; −96	48; 48
	D	96; 24	96; 16	24; 12

(a) *(16 points total)* $\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = 1$ for $i \in \{1, 2\}$ (Complete information)

i. *(12 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

ii. *(4 points)* Find the pure strategy Nash equilibrium in both games. Write down the strategies players use below.

(b) *(10 points total)* $\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = \frac{1}{2}$ for $i \in \{1, 2\}$ (No information)

i. *(2 points)* Write down the expected payoffs from the game in the table below:

		P2 L	C	R
	U	___;___	___;___	___;___
P1	M	___;___	___;___	___;___
	D	___;___	___;___	___;___

ii. *(6 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

iii. *(2 points)* Find the pure strategy Nash equilibrium. Write down the strategies players use below.

(c) *(10 points total)* $\Pr_1(\alpha|\alpha) = \Pr_1(\beta|\beta) = \frac{1}{2}$, $\Pr_2(\alpha|\alpha) = \Pr_2(\beta|\beta) = 1$ (Asymmetric Information, player 2 is informed and player 1 is not.)

i. *(3 points)* Write down player 2's best responses, be careful to denote in which state they play which action.

ii. *(3 points)* In the table below find player 1's expected payoff from each of player 2's best responses:

		P2's Best responses		
		_ _ _ _ _ _ _	_ _ _ _ _ _ _	_ _ _ _ _ _ _
P1's expected utilities	U	_ _ _ _ _ _ _	_ _ _ _ _ _ _	_ _ _ _ _ _ _
	M	_ _ _ _ _ _ _	_ _ _ _ _ _ _	_ _ _ _ _ _ _
	D	_ _ _ _ _ _ _	_ _ _ _ _ _ _	_ _ _ _ _ _ _

iii. *(4 points)* Find the pure strategy Nash equilibrium. Explain your answer.

(d) *(8 points)* Calculate player 2's expected utilities in each of the cases above (from an a-priori point of view, when both of the games are equally likely). Their is something odd about player 2's expected utilities, explain what is odd and why this can occur.

4. *(38 points total)* Consider a location model, consumers or voters are at one of five locations ($l \in \{1,2,3,4,5\}$). The total number at each location is given below, the total number at all location is $C = 48$.

Location (l)	1	2	3	4	5
Number at that location (c_l)	12	6	2	12	16

(a) *(20 points total)* Consider first the Hotelling location model. Consumers go the firm that is closest to them, and half go to each firm if the firms are equally close to their location. The two firms objectives are to maximize their demand. Let the demand of firm $j \in \{a,b\}$ given that they are at location l_j when their opponent is at location l_{-j} ($-j = \{a,b\} \setminus j$) be denoted $D_j(l_j, l_{-j})$.

i. *(10 points total)* Fill out the table below with $D_a(l_a, l_b)$ for all $(l_a, l_b) \in \{1,2,3,4,5\}^2$.

If $l_b =$	1	2	3	4	5
$D_a(1, l_b) =$					
$D_a(2, l_b) =$					
$D_a(3, l_b) =$					
$D_a(4, l_b) =$					
$D_a(5, l_b) =$	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _

ii. *(2 points)* Why do we not have to do the same exercise for firm b?

iii. *(8 points)* Find the unique pair of strategies to survive iterated deletion of dominated strategies. Show why it survives step by step.

(b) *(12 points total)* Now consider a model of political parties, consumers vote for the political party (a or b) that is closest to them, splitting their vote if both parties are equally close. Let $D_j(l_j, l_{-j})$ now be the number of votes party $j \in \{a, b\}$ receives. Then a political party's utility function is:

$$u_j(l_j, l_{-j}) = \begin{cases} 1 & \text{if } D_j(l_j, l_{-j}) > \frac{C}{2} \\ \frac{1}{2} & \text{if } D_j(l_j, l_{-j}) = \frac{C}{2} \\ 0 & \text{if } D_j(l_j, l_{-j}) < \frac{C}{2} \end{cases}.$$

i. *(2 points)* Using the you found for the Hotelling model above fill out the table below. (I.e. convert their demands into utilities.)

If $l_b =$	1	2	3	4	5
$u_a(1, l_b) =$					
$u_a(2, l_b) =$					
$u_a(3, l_b) =$					
$u_a(4, l_b) =$					
$u_a(5, l_b) =$	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _

ii. *(4 points)* Write down the best responses for political party a in the space below:

If $l_b =$	1	2	3	4
$BR_a(l_b) =$	________	________	________	________

iii. *(2 points)* Find the unique Nash equilibrium. Explain your reasoning.

iv. *(4 points)* Is one of the locations strictly dominated in this game? Why or why not? Do you think you would be able to solve the voting model using iterated deletion of dominated strategies? **NOTE**: I am not asking you to solve the model using this method, merely state whether you think you can.

(c) *(6 points)* Consider the welfare implications of both equilibria. Does this result seem welfare optimal in the Hotelling model? In the voting model? Why or why not? Note you can claim whatever you want, points will be given for backing your claim with a careful argument.

ECON 439
Midterm: Normal Form Games

Kevin Hasker

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1. *(10 points)* **Honor Statement:** Please read and sign the following statement:

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2. *(8 points)* Lil' Kevin does not like mixed strategies, so he wants to study Game Theory without them. Explain to him why this will not work, be sure to mention both the mathematical and intuitive problem with this approach.

3. *(44 points total)* In this game their are two states of the world, α and β. The prior probabilities of these states are equally likely ($\Pr(\alpha) = \Pr(\beta) = \frac{1}{2}$). What will differ in the different variations is what player's know about the state of the world, or a player's beliefs that the state of the world is $x \in \{\alpha, \beta\}$ given that the state $y \in \{\alpha, \beta\}$ is the true state. We will denote these $\Pr_i(x|y)$ for $i \in \{1, 2\}$. We will only consider two possibilities, that they are completely informed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = 1$) or they are completely uninformed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = \frac{1}{2}$). The games are:

α

		P2 L	P2 C	P2 R
P1	U	$-48; 96$	$48; 48$	$12; -96$
P1	M	$12; 36$	$16; 24$	$24; 24$
P1	D	$48; 24$	$24; 12$	$36; 16$

β

		P2 L	P2 C	P2 R
P1	U	$-48; -96$	$48; 48$	$-48; 120$
P1	M	$48; 24$	$32; 12$	$36; 16$
P1	D	$16; 12$	$24; 24$	$12; 32$

(a) *(16 points total)* $\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = 1$ for $i \in \{1, 2\}$ (Complete information)

i. *(12 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

ii. *(4 points)* Find the pure strategy Nash equilibrium in both games. Write down the strategies players use below.

(b) *(10 points total)* $\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = \frac{1}{2}$ for $i \in \{1, 2\}$ (No information)

i. *(2 points)* Write down the expected payoffs from the game in the table below:

		P2 L	P2 C	P2 R
P1	U	___;___	___;___	___;___
P1	M	___;___	___;___	___;___
P1	D	___;___	___;___	___;___

ii. *(6 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

iii. *(2 points)* Find the pure strategy Nash equilibrium. Write down the strategies players use below.

(c) *(10 points total)* $\Pr_1(\alpha|\alpha) = \Pr_1(\beta|\beta) = \frac{1}{2}$, $\Pr_2(\alpha|\alpha) = \Pr_2(\beta|\beta) = 1$ (Asymmetric Information, player 2 is informed and player 1 is not.)

i. *(3 points)* Write down player 2's best responses, be careful to denote in which state they play which action.

ii. *(3 points)* In the table below find player 1's expected payoff from each of player 2's best responses:

		P2's Best responses		
		_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _
	U	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _
P1's expected utilities	M	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _
	D	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _

iii. *(4 points)* Find the pure strategy Nash equilibrium. Explain your answer.

(d) *(8 points)* Calculate player 2's expected utilities in each of the cases above (from an a-priori point of view, when both of the games are equally likely). Their is something odd about player 2's expected utilities, explain what is odd and why this can occur.

4. *(38 points total)* Consider a location model, consumers or voters are at one of five locations ($l \in \{1,2,3,4,5\}$). The total number at each location is given below, the total number at all location is $C = 48$.

Location (l)	1	2	3	4	5
Number at that location (c_l)	16	12	2	6	12

(a) *(20 points total)* Consider first the Hotelling location model. Consumers go the firm that is closest to them, and half go to each firm if the firms are equally close to their location. The two firms objectives are to maximize their demand. Let the demand of firm $j \in \{a,b\}$ given that they are at location l_j when their opponent is at location l_{-j} ($-j = \{a,b\} \setminus j$) be denoted $D_j(l_j, l_{-j})$.

i. *(10 points total)* Fill out the table below with $D_a(l_a, l_b)$ for all $(l_a, l_b) \in \{1,2,3,4,5\}^2$.

If $l_b =$	1	2	3	4	5
$D_a(1, l_b) =$					
$D_a(2, l_b) =$					
$D_a(3, l_b) =$					
$D_a(4, l_b) =$					
$D_a(5, l_b) =$	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _

ii. *(2 points)* Why do we not have to do the same exercise for firm b?

iii. *(8 points)* Find the unique pair of strategies to survive iterated deletion of dominated strategies. Show why it survives step by step.

(b) *(12 points total)* Now consider a model of political parties, consumers vote for the political party (a or b) that is closest to them, splitting their vote if both parties are equally close. Let $D_j(l_j, l_{-j})$ now be the number of votes party $j \in \{a, b\}$ receives. Then a political party's utility function is:

$$u_j(l_j, l_{-j}) = \begin{cases} 1 & \text{if } D_j(l_j, l_{-j}) > \frac{C}{2} \\ \frac{1}{2} & \text{if } D_j(l_j, l_{-j}) = \frac{C}{2} \\ 0 & \text{if } D_j(l_j, l_{-j}) < \frac{C}{2} \end{cases}.$$

i. *(2 points)* Using the you found for the Hotelling model above fill out the table below. (I.e. convert their demands into utilities.)

If $l_b =$	1	2	3	4	5
$u_a(1, l_b) =$					
$u_a(2, l_b) =$					
$u_a(3, l_b) =$					
$u_a(4, l_b) =$					
$u_a(5, l_b) =$	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _

ii. *(4 points)* Write down the best responses for political party a in the space below:

If $l_b =$	1	2	3	4
$BR_a(l_b) =$	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _

iii. *(2 points)* Find the unique Nash equilibrium. Explain your reasoning.

iv. *(4 points)* Is one of the locations strictly dominated in this game? Why or why not? Do you think you would be able to solve the voting model using iterated deletion of dominated strategies? **NOTE**: I am not asking you to solve the model using this method, merely state whether you think you can.

(c) *(6 points)* Consider the welfare implications of both equilibria. Does this result seem welfare optimal in the Hotelling model? In the voting model? Why or why not? Note you can claim whatever you want, points will be given for backing your claim with a careful argument.

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2. *(8 points)* Lil' Kevin does not like mixed strategies, so he wants to study Game Theory without them. Explain to him why this will not work, be sure to mention both the mathematical and intuitive problem with this approach.

3. *(44 points total)* In this game their are two states of the world, α and β. The prior probabilities of these states are equally likely ($\Pr(\alpha) = \Pr(\beta) = \frac{1}{2}$). What will differ in the different variations is what player's know about the state of the world, or a player's beliefs that the state of the world is $x \in \{\alpha, \beta\}$ given that the state $y \in \{\alpha, \beta\}$ is the true state. We will denote these $\Pr_i(x|y)$ for $i \in \{1, 2\}$. We will only consider two possibilities, that they are completely informed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = 1$) or they are completely uninformed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = \frac{1}{2}$). The games are:

α

		P2		
		L	C	R
	U	$24; 24$	$-24; 60$	$-24; -48$
P1	M	$12; 12$	$6; 16$	$8; 6$
	D	$16; 6$	$18; 8$	$24; 12$

β

		P2		
		L	C	R
	U	$24; 24$	$6; -48$	$-24; 48$
P1	M	$12; 6$	$48; 8$	$60; 12$
	D	$8; 12$	$12; 12$	$6; 18$

(a) *(16 points total)* $\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = 1$ for $i \in \{1, 2\}$ (Complete information)

i. *(12 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

ii. *(4 points)* Find the pure strategy Nash equilibrium in both games. Write down the strategies players use below.

(b) *(10 points total)* $\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = \frac{1}{2}$ for $i \in \{1, 2\}$ (No information)

i. *(2 points)* Write down the expected payoffs from the game in the table below:

		P2		
		L	C	R
	U	___;___	___;___	___;___
P1	M	___;___	___;___	___;___
	D	___;___	___;___	___;___

ii. *(6 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

iii. *(2 points)* Find the pure strategy Nash equilibrium. Write down the strategies players use below.

(c) *(10 points total)* $\Pr_1(\alpha|\alpha) = \Pr_1(\beta|\beta) = \frac{1}{2}$, $\Pr_2(\alpha|\alpha) = \Pr_2(\beta|\beta) = 1$ (Asymmetric Information, player 2 is informed and player 1 is not.)

i. *(3 points)* Write down player 2's best responses, be careful to denote in which state they play which action.

ii. *(3 points)* In the table below find player 1's expected payoff from each of player 2's best responses:

P2's Best responses

		_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _
	U	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _
P1's expected utilities	M	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _
	D	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _

iii. *(4 points)* Find the pure strategy Nash equilibrium. Explain your answer.

(d) *(8 points)* Calculate player 2's expected utilities in each of the cases above (from an a-priori point of view, when both of the games are equally likely). Their is something odd about player 2's expected utilities, explain what is odd and why this can occur.

4. *(38 points total)* Consider a location model, consumers or voters are at one of five locations ($l \in \{1, 2, 3, 4, 5\}$). The total number at each location is given below, the total number at all location is $C = 96$.

Location (l)	1	2	3	4	5
Number at that location (c_l)	32	24	4	12	24

(a) *(20 points total)* Consider first the Hotelling location model. Consumers go the firm that is closest to them, and half go to each firm if the firms are equally close to their location. The two firms objectives are to maximize their demand. Let the demand of firm $j \in \{a, b\}$ given that they are at location l_j when their opponent is at location l_{-j} ($-j = \{a, b\} \setminus j$) be denoted $D_j(l_j, l_{-j})$.

i. *(10 points total)* Fill out the table below with $D_a(l_a, l_b)$ for all $(l_a, l_b) \in \{1, 2, 3, 4, 5\}^2$.

If $l_b =$	1	2	3	4	5
$D_a(1, l_b) =$					
$D_a(2, l_b) =$					
$D_a(3, l_b) =$					
$D_a(4, l_b) =$					
$D_a(5, l_b) =$	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _

ii. *(2 points)* Why do we not have to do the same exercise for firm b?

iii. *(8 points)* Find the unique pair of strategies to survive iterated deletion of dominated strategies. Show why it survives step by step.

(b) *(12 points total)* Now consider a model of political parties, consumers vote for the political party (a or b) that is closest to them, splitting their vote if both parties are equally close. Let $D_j(l_j, l_{-j})$ now be the number of votes party $j \in \{a, b\}$ receives. Then a political party's utility function is:

$$u_j(l_j, l_{-j}) = \begin{cases} 1 & \text{if} \quad D_j(l_j, l_{-j}) > \frac{C}{2} \\ \frac{1}{2} & \text{if} \quad D_j(l_j, l_{-j}) = \frac{C}{2} \\ 0 & \text{if} \quad D_j(l_j, l_{-j}) < \frac{C}{2} \end{cases} .$$

i. *(2 points)* Using the you found for the Hotelling model above fill out the table below. (I.e. convert their demands into utilities.)

If $l_b =$	1	2	3	4	5
$u_a(1, l_b) =$					
$u_a(2, l_b) =$					
$u_a(3, l_b) =$					
$u_a(4, l_b) =$					
$u_a(5, l_b) =$	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _

ii. *(4 points)* Write down the best responses for political party a in the space below:

If $l_b =$	1	2	3	4
$BR_a(l_b) =$	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _

iii. *(2 points)* Find the unique Nash equilibrium. Explain your reasoning.

iv. *(4 points)* Is one of the locations strictly dominated in this game? Why or why not? Do you think you would be able to solve the voting model using iterated deletion of dominated strategies? **NOTE**: I am not asking you to solve the model using this method, merely state whether you think you can.

(c) *(6 points)* Consider the welfare implications of both equilibria. Does this result seem welfare optimal in the Hotelling model? In the voting model? Why or why not? Note you can claim whatever you want, points will be given for backing your claim with a careful argument.

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2. *(8 points)* Lil' Kevin does not like mixed strategies, so he wants to study Game Theory without them. Explain to him why this will not work, be sure to mention both the mathematical and intuitive problem with this approach.

3. *(44 points total)* In this game their are two states of the world, α and β. The prior probabilities of these states are equally likely ($\Pr(\alpha) = \Pr(\beta) = \frac{1}{2}$). What will differ in the different variations is what player's know about the state of the world, or a player's beliefs that the state of the world is $x \in \{\alpha, \beta\}$ given that the state $y \in \{\alpha, \beta\}$ is the true state. We will denote these $\Pr_i(x|y)$ for $i \in \{1, 2\}$. We will only consider two possibilities, that they are completely informed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = 1$) or they are completely uninformed ($\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = \frac{1}{2}$). The games are:

α

		P2 L	P2 C	P2 R
P1	U	$48; 8$	$36; 12$	$12; 6$
P1	M	$12; 12$	$6; 18$	$8; 12$
P1	D	$6; -48$	$-24; 48$	$24; 24$

β

		P2 L	P2 C	P2 R
P1	U	$6; 16$	$8; 6$	$12; 12$
P1	M	$48; 8$	$24; 12$	$16; 6$
P1	D	$-24; 60$	$-24; -48$	$24; 24$

(a) *(16 points total)* $\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = 1$ for $i \in \{1, 2\}$ (Complete information)

i. *(12 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

ii. *(4 points)* Find the pure strategy Nash equilibrium in both games. Write down the strategies players use below.

(b) *(10 points total)* $\Pr_i(\alpha|\alpha) = \Pr_i(\beta|\beta) = \frac{1}{2}$ for $i \in \{1, 2\}$ (No information)

i. *(2 points)* Write down the expected payoffs from the game in the table below:

		P2 L	P2 C	P2 R
P1	U	_ _ _;_ _ _	_ _ _;_ _ _	_ _ _;_ _ _
P1	M	_ _ _;_ _ _	_ _ _;_ _ _	_ _ _;_ _ _
P1	D	_ _ _;_ _ _	_ _ _;_ _ _	_ _ _;_ _ _

ii. *(6 points)* Find the pure strategy best responses of both players in both games. You may mark them on the table above but you will loose three points if you do not explain your notation below.

iii. *(2 points)* Find the pure strategy Nash equilibrium. Write down the strategies players use below.

(c) *(10 points total)* $\Pr_1(\alpha|\alpha) = \Pr_1(\beta|\beta) = \frac{1}{2}$, $\Pr_2(\alpha|\alpha) = \Pr_2(\beta|\beta) = 1$ (Asymmetric Information, player 2 is informed and player 1 is not.)

i. *(3 points)* Write down player 2's best responses, be careful to denote in which state they play which action.

ii. *(3 points)* In the table below find player 1's expected payoff from each of player 2's best responses:

P2's Best responses

P1's expected utilities	U			
	M			
	D			

iii. *(4 points)* Find the pure strategy Nash equilibrium. Explain your answer.

(d) *(8 points)* Calculate player 2's expected utilities in each of the cases above (from an a-priori point of view, when both of the games are equally likely). Their is something odd about player 2's expected utilities, explain what is odd and why this can occur.

4. *(38 points total)* Consider a location model, consumers or voters are at one of five locations ($l \in \{1,2,3,4,5\}$). The total number at each location is given below, the total number at all location is $C = 96$.

Location (l)	1	2	3	4	5
Number at that location (c_l)	24	12	4	24	32

(a) *(20 points total)* Consider first the Hotelling location model. Consumers go the firm that is closest to them, and half go to each firm if the firms are equally close to their location. The two firms objectives are to maximize their demand. Let the demand of firm $j \in \{a,b\}$ given that they are at location l_j when their opponent is at location l_{-j} ($-j = \{a,b\} \setminus j$) be denoted $D_j(l_j, l_{-j})$.

i. *(10 points total)* Fill out the table below with $D_a(l_a, l_b)$ for all $(l_a, l_b) \in \{1,2,3,4,5\}^2$.

If $l_b =$	1	2	3	4	5
$D_a(1, l_b) =$					
$D_a(2, l_b) =$					
$D_a(3, l_b) =$					
$D_a(4, l_b) =$					
$D_a(5, l_b) =$	____	____	____	____	____

ii. *(2 points)* Why do we not have to do the same exercise for firm b?

iii. *(8 points)* Find the unique pair of strategies to survive iterated deletion of dominated strategies. Show why it survives step by step.

(b) *(12 points total)* Now consider a model of political parties, consumers vote for the political party (a or b) that is closest to them, splitting their vote if both parties are equally close. Let $D_j(l_j, l_{-j})$ now be the number of votes party $j \in \{a, b\}$ receives. Then a political party's utility function is:

$$u_j(l_j, l_{-j}) = \begin{cases} 1 & \text{if} \quad D_j(l_j, l_{-j}) > \frac{C}{2} \\ \frac{1}{2} & \text{if} \quad D_j(l_j, l_{-j}) = \frac{C}{2} \\ 0 & \text{if} \quad D_j(l_j, l_{-j}) < \frac{C}{2} \end{cases}.$$

i. *(2 points)* Using the you found for the Hotelling model above fill out the table below. (I.e. convert their demands into utilities.)

If $l_b =$	1	2	3	4	5
$u_a(1, l_b) =$					
$u_a(2, l_b) =$					
$u_a(3, l_b) =$					
$u_a(4, l_b) =$					
$u_a(5, l_b) =$	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _	_ _ _ _

ii. *(4 points)* Write down the best responses for political party a in the space below:

If $l_b =$	1	2	3	4
$BR_a(l_b) =$	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _	_ _ _ _ _ _ _ _

iii. *(2 points)* Find the unique Nash equilibrium. Explain your reasoning.

iv. *(4 points)* Is one of the locations strictly dominated in this game? Why or why not? Do you think you would be able to solve the voting model using iterated deletion of dominated strategies? **NOTE**: I am not asking you to solve the model using this method, merely state whether you think you can.

(c) *(6 points)* Consider the welfare implications of both equilibria. Does this result seem welfare optimal in the Hotelling model? In the voting model? Why or why not? Note you can claim whatever you want, points will be given for backing your claim with a careful argument.