ECON 439

Practice Questions—Extensive Form Games of Complete Information

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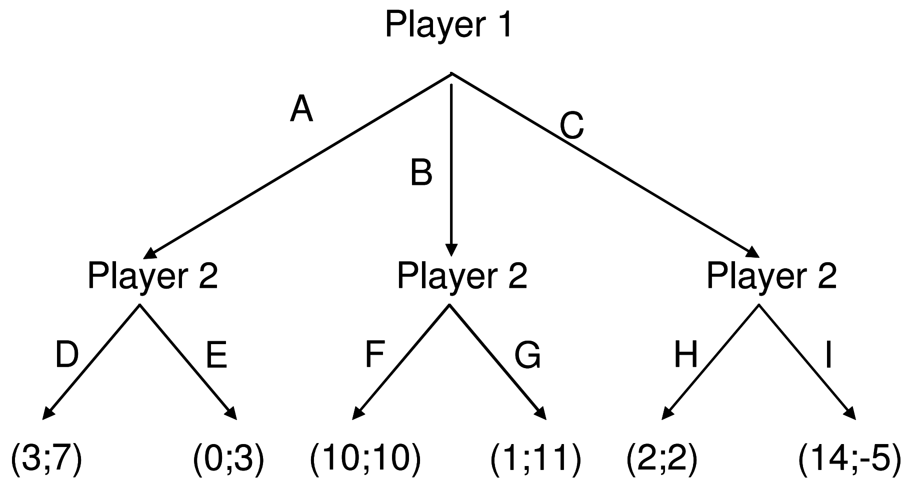
These questions are supposed to help you prepare for exams and quizzes, they are not to be turned in. Answers will be posted before the relevant exam.

1 Chapter 5—Sequential or Extensive form Games with Perfect Information.

1. Consider the following sequential game:



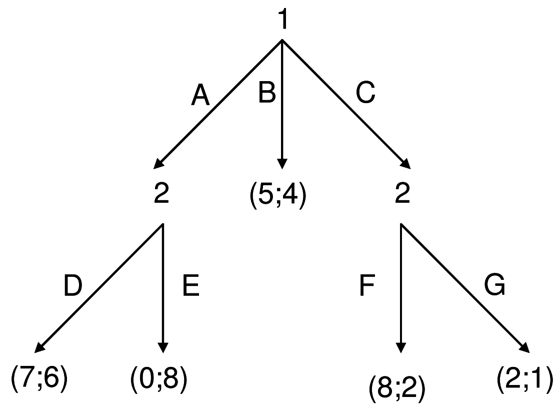
   (a) Write down all the strategies of both players.

   (b) Solve the game using backward induction, and write down the Subgame Perfect equilibrium strategies. You may mark your answers on the graph above, but you will loose two points if you do not explain your notation below.

   (c) Define the concept of an *empty threat*, and explain how this can result in a Nash equilibrium that is not the same as the Subgame Perfect equilibrium.

   (d) Write down the strategies of a Nash equilibrium of this game that is not the Subgame Perfect equilibrium.

2. Consider the following sequential game



(a) Find the set of strategies of both players.

(b) Find the Subgame Perfect equilibrium strategies. You may mark your answers on the game above but write the strategy below.

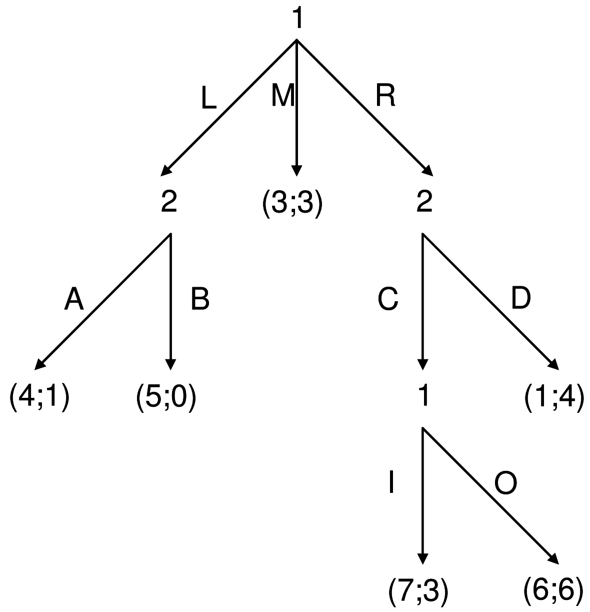
(c) Transform this game into a Strategic Form Game and draw the game table below.

(d) Find a Nash equilibrium that is not a Subgame Perfect equilibrium and yet gives the same outcome.

(e) Find a Nash equilibrium strategy that give a different outcome, and explain how there is an *empty threat* in this Nash equilibrium.

3. Consider the following sequential game or extensive form game of perfect

information.



(a) Find the best response at each decision node (or after every non-terminal history). *You may mark them above but explain your notation below or you will loose 2 points.*

(b) Write down all of each player's strategies.

(c) Find the subgame perfect equilibrium strategies. You will get no points for merely writing down the tactics used in equilibrium.

(d) Using this game explain why it is important to write down the equilibrium strategies instead of the tactics or the outcome.

4. Consider the following game:

		Player 2: α	Player 2: β
Player 1	A	$10; -1$	$2; 8$
	B	$8; 11$	$3; 4$
	C	$1; 7$	$6; 11$

(a) Find the best responses of both players. You may mark them in the table above but you will automatically loose one point if you do not explain your notation below.

(b) Find the Nash equilibrium.

(c) Now transform this into a sequential game where player one chooses between A, B, and C and then after observing what player one has done player two chooses between α and β. Draw this game.

(d) Find all the strategies of both players. Be clear in your notation.

(e) Solve the sequential game by backward induction. You may mark your answers on the game above but you will automatically loose one point if you do not explain your notation below.

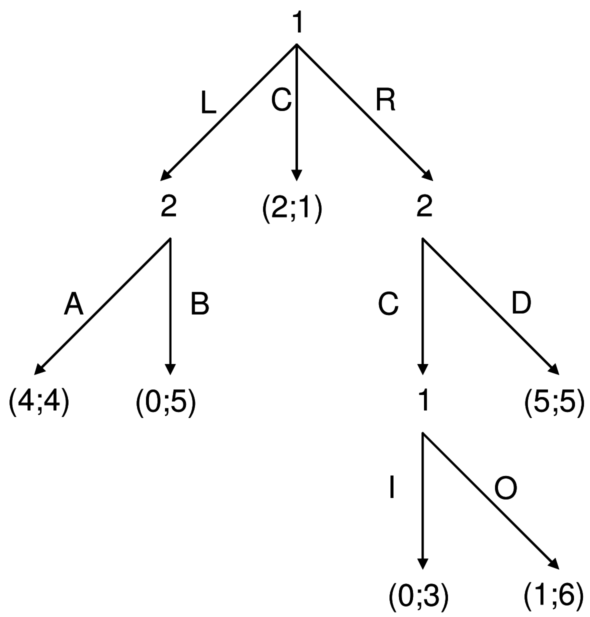
(f) Write down the Subgame Perfect equilibrium strategies of both players. *Hint: When I say strategies I mean strategies, zero points will be awarded for incomplete strategies.*

(g) Explain by example why it is important to write down the full strategies, not the tactics that will be used on the equilibrium path.

(h) What is an *empty threat*? Find a Nash equilibrium that is not a Subgame Perfect Equilibrium of the sequential game and explain how empty threats are used in this strategy.

5. Give a precise definition of *first mover's advantage* and prove that it is true.

6. Consider the following sequential game (or extensive form game of complete information.)



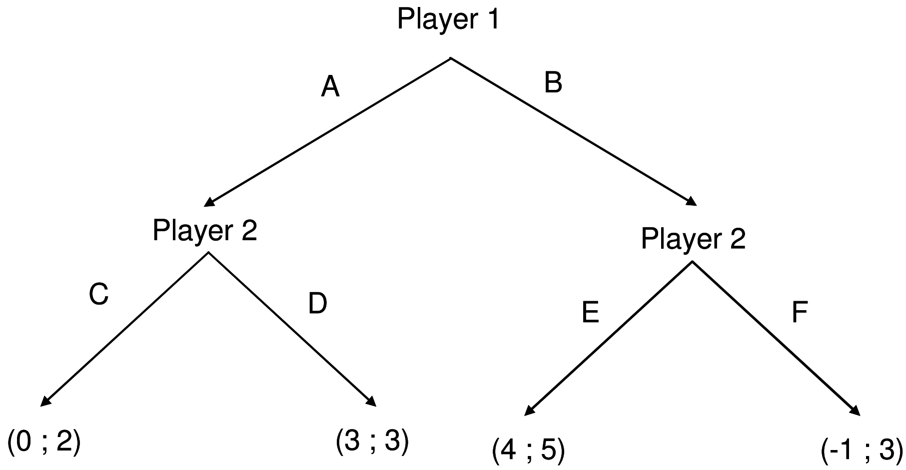
(a) Find the best response at each decision node (or after every non-terminal history). *You may mark them above but explain your notation below.*

(b) Find the subgame perfect equilibrium.

(c) Using this game explain why it is important to write down the equilibrium strategies instead of the tactics or the outcome.

7. What is the definition of a (pure strategy) subgame perfect equilibrium. Be sure to define any technical terms you use in the definition. Notice your answer only has to cover the games that we have analyzed so far.

8. Consider the following sequential game:



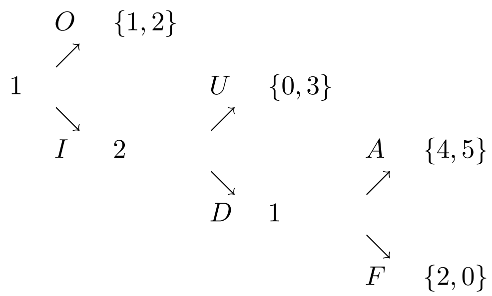
   (a) Solve the game using backward induction, and write down the equilibrium *strategies* below.
   (b) Write down all of the strategies of both players.
   (c) Draw a normal form game that is strategically equivalent to the game above.
   (d) Find the best responses and the Nash Equilibria of the normal form game.
   (e) Define an *empty threat*. Do empty threats always help the person who makes them?
   (f) For the Nash Equilibria that are not Subgame Perfect equilibria explain how they depend on empty threats.

9. A strategy is *Subgame Perfect Dominated* if there is a subgame where it is strictly dominated.

   Zermelo's Theorem tells us that as long as no two payoffs are the same for any person that in every Extensive Form Game of Perfect Information there is a unique Subgame Perfect Equilibrium.

   Prove that there is also a unique strategy that survives iterated removal of subgame perfect dominated strategies.

10. Consider the following sequential game (or extensive form game with perfect information.)



I will also describe this game using words. First 1 chooses between I and O, then if 1 chooses I then 2 chooses between U and D, then if 2 chooses D 1 chooses between A and F.

(a) Find the best responses of both players at every decision node and the Subgame perfect equilibrium.

(b) List all of the strategies of both players.

(c) Draw a strategic form game that is equivalent to this sequential game.

(d) Find the best responses of both players and all of the Nash equilibria of the game.

(e) Only one of these Nash equilibria is Subgame Perfect, explain what is wrong with the other Nash equilibria.

2 Chapter 6—Sequential Games, Illustrations.

1. An Extensive Form War of Attrition: Assume that there are two players, a and b, and let i be an arbitrary player, $i \in \{a, b\}$. In this game instead of deciding whether or not to fight for the next t periods in each period the two players decide whether or not to fight in the current period. Thus in period t a player's strategic choices are to fight (F_{it}) or Acquiesce (A_{it}). If a player choose to acquiesce in period t then the game ends and the other player wins the prize, otherwise you go to the next period. The payoffs are identical to the payoffs in the normal form game. Let t_i be the last period player i chooses to fight. Then:

$$u_i(t_i, t_j) = \begin{cases} 1 - c_i t_j & \text{if} \quad t_i > t_j \\ \frac{1}{2} - c_i t_i & \text{if} \quad t_i = t_j \\ -c_i t_i & \text{if} \quad t_i < t_j \end{cases}$$

where $c_i > 0$. We will only consider the game where $t \leq T$, where T is some fixed value.

(a) First consider $t = T$, and assume that both players have chosen to fight up to this point.

i. Draw the normal form game for this period.

ii. Show that there is a $\bar{c}_T$ such that if $c_i < \bar{c}_T$ player i has a dominant strategy to choose F_{iT}.

(b) Now consider $t = T - 1$, and assume that $c_a < \bar{c}_T$ and $c_b < \bar{c}_T$.

i. Draw the normal form game for this period—be sure to consider what will happen next period if they choose (F_{aT-1}, F_{bT-1}).

ii. Show that there is a $\bar{c}_{T-1}$ such that if $c_i < \bar{c}_{T-1}$ player i has a dominant strategy to choose F_{iT-1}.

(c) For general t find a $\bar{c}_t$ such that if both players will fight from $t + 1$ on player i has a dominant strategy to choose F_{it}.

(d) Show that the Subgame Perfect equilibria of this game still can be characterized as $t_a = 0$ and $t_b = k + \tau$ for $\tau \geq 0$ and a critical value of $k > 0$. Further show when $t_a = 0$ $t_b = T$ is not a Subgame Perfect Equilibrium.

(e) Discuss how the equilibria of this model have the flavor of "the strong will win the prize." Or if $c_a < c_b$ then a will win. Be sure to discuss what you think a reasonable value for T is.

2. Someone wants to buy a used car which has value 40 with probability ρ_a, 12 with probability ρ_b and 6 with probability ρ_c ($\rho_a + \rho_b + \rho_c = 1$). The way the game proceeds is first the buyer makes an offer, p. Then nature (player 0) randomizes to determine which of the three types of cars the seller has, then the seller decides either to accept the offer or reject it. Let w be the value of the seller's car. The seller's payoff is p if he accepts the offer, and w if he rejects it. The buyers utility is $3w - p$ if his offer is accepted and zero if it is rejected. Assume that the seller will always accept the offer if he is indifferent between accepting it and rejecting it.

(a) Show that $p \in \{40, 12, 6\}$, or the buyer will never make an offer that is not the value of some type of car. *Even if you can not show this you may assume it for the rest of the question.*

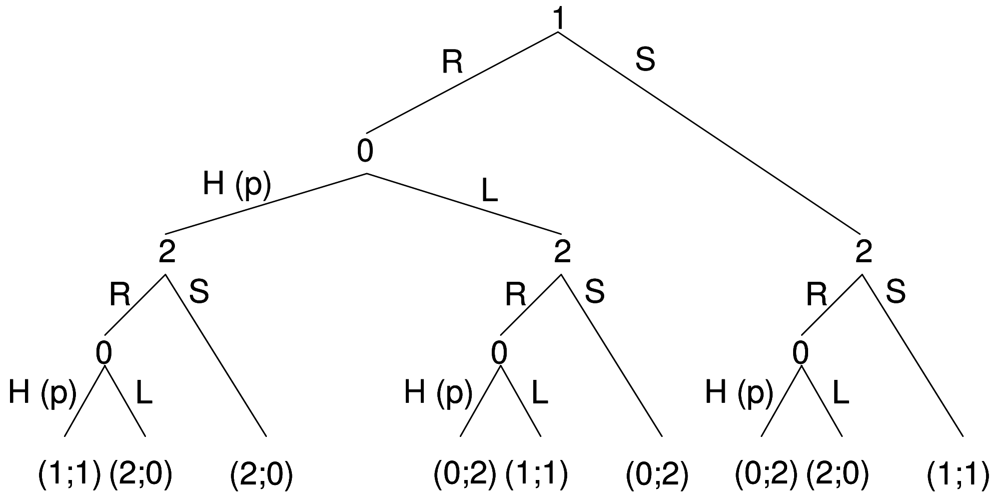
(b) Assuming that $p \in \{40, 12, 6\}$, draw an extensive form game that describes this interaction. You do not need to calculate all the payoffs.

(c) Given that $\rho_a = \frac{1}{4}$, $\rho_b = \frac{1}{4}$ and $\rho_c = \frac{1}{2}$ find the buyer's expected utility from each of his actions.

(d) Find the optimal value for p in this game.

3. Consider the following sequential game with chance moves—or moves by nature. Nature is player 0, every time nature randomizes H occurs with probability $p = \frac{2}{3}$ and L occurs with probability $1-p$. **Assume throughout this question that** $p = \rho$.

(Story behind the game: In College American Football if there is a tie at the end of regulation time both teams get one chance to score. They can take the safe action (S) of kicking a fieldgoal—which will give them 3 points for sure—or they can take the riskier action (R) of trying to score a touchdown—which will give them 7 points if they succeed and 0 if they fail. Their probability of success is p. First team one makes their choice and both teams see the outcome, then team two makes their choice. The winner is the one with more points after both tries. A winner in this game gets two points, if the teams tie each gets one point. In College American football this process repeats if there is a tie, but for our analysis we can ignore this.)



(a) Find all the strategies of both players. You will be given one half point per strategy (rounded up). Be clear in your notation.

(b) Solve this game using backward induction. You may mark your answers on the game above but you will automatically loose one point if you do not explain your notation below.

(c) Write down the Subgame Perfect equilibrium strategies of both players. *Hint: When I say strategies I mean strategies, zero points will be awarded for incomplete strategies.*

(d) Find the expected equilibrium payoffs of both players. Is player one's payoff higher than player two's or lower?

(e) Now transform this game into a simultaneous action game where both players have two strategies, R and S. If they choose the strategy R they get the outcome H with probability p and L with probability $1 - p$.

   i. Write down the payoff matrix for this game.

   ii. Find the best responses of both players. You may mark them in the table above but you will automatically loose one point if you do not explain your notation below.

iii. Find the unique Nash equilibrium in Pure Strategies.

(f) What does comparing the equilibrium payoffs in the simultaneous move game and the sequential game tell us about first mover's advantage? Explain your result. How does this explain the strategic decisions made in many sports?

4. Consider a committee model where the set of options is $X = \{a, b, c, d, e\}$ and the preferences of the three people are:

1	2	3
a	e	c
b	b	d
c	c	e
d	d	b
e	a	a

An *agenda* is an ordering over the options, for example (b, e, c, a, d). The way options in an agenda are voted over is:

1. In the first round the first two options are voted over. The option that gets the majority of votes becomes the *status quo.*

t. In every following round the committee votes over the status quo versus the next option in the agenda. The option that gets the majority of votes becomes the new *status quo.*

The game ends when every option in the agenda has been considered. Assume that committee members always use weakly undominated strategies.

(a) For the agenda (a, b, c, d, e) find the outcome if:

i. Committee members vote *naively* (vote for their favorite option among the two options in front of them.)

ii. Committee members vote strategically.

(b) Define what it means to be *Pareto Efficient.* Find the Pareto efficient options for this committee.

(c) Define what it means to be in the *Top Cycle.* Find the top cycle for this committee.

(d) Find the set of options such that there is an agenda with this option as the equilibrium outcome for this committee.

5. In the market for pocket calculators there are two firms. Firm 1 can either produce 7 or exit the industry (and never reenter). Firm 2 can either produce 3 or exit the industry (and never reenter). The costs of production are the same for both firms, $c_1(q) = 7q_1$, $c_2(q) = 7q_2$. This industry is in decline since cell phones, computers, PDA's and etcetera all do the same job as pocket calculators but do more interesting things

as well. Thus the inverse demand curve in this industry is $P(t, Q_t) = \max\{25 - t - Q_t, 0\}$.

Just to be clear, if firm 1 wants to produce in period 5 then they have to produce in every period before that date. If they choose to produce nothing in period 4 they have exited the industry and can never produce in this industry again (for example they must produce nothing in period 5). *Assume firms will produce if they are indifferent between producing and not producing.*

(a) Define t_1 as the highest value of t such that $P(t, 7) \geq 7$, t_2 as the highest value of t such that $P(t, 3) \geq 7$, and t_b as the highest value of t such that $P(t, 10) \geq 7$. Find all three values. For $i \in \{1, 2\}$ what is the relationship between t_i and the last period firm i will produce?

(b) Find the equilibrium strategy of both firms in $t > \max\{t_1, t_2, t_b\}$.

(c) Find the equilibrium strategy of both firms in $t \leq \max\{t_1, t_2, t_b\}$.

6. There are three committee members ($\{1, 2, 3\}$) and four alternatives ($\{a, b, c, d\}$). The preferences of the committee members are represented by the following table:

1	2	3
a	d	c
b	a	d
c	b	a
d	c	b

The alternatives are voted over in the following order. First committee members vote on b versus c, the winner being decided by majority voting. Then the winner is voted on against the outcome d, finally the winner is pitted against the outcome a.

(a) Prove that any outcome is a Subgame Perfect equilibrium of this game.

From this point on assume people use weakly undominated strategies.

(b) Find the outcome in the third round assuming each of the other alternatives $\{b, c, d\}$ wins the previous rounds.

(c) Find the outcome in the second round assuming each of the other alternatives $\{b, c\}$ wins the previous round.

(d) Find the outcome in the first round and the outcome of the game.

7. A committee consists of three members $\{1, 2, 3\}$. There are four options they are considering $\{A, B, C, D\}$. The preferences of the three members over the four outcomes are:

1	2	3
B	C	A
A	D	C
D	B	D
C	A	B

they have agreed that they should vote on options in the order of $\{A, B, C, D\}$ but are still arguing about the procedure to use. Assume that committee members always use weakly undominated strategies, or that when there vote will not matter they vote for the option that will lead to their preferred outcome.

(a) In this version they first vote on whether to accept option A or reject it and consider option B. They then vote on whether to accept option B or reject it and consider option C. Then they vote on whether to accept option C or accept option D. In each case the outcome is determined by the majority of voters.

i. Draw an extensive form game that represents this method. Please explain your notation.

ii. Find the outcome of the voting at each stage and the outcome of this procedure.

(b) In this version they first vote on option A versus option B. Then the winner is matched against option C and then the winner of that contest is matched against option D. In each contest the outcome is determined by the majority of voters.

i. Draw an extensive form game that represents this method. Please explain your notation.

ii. Find the outcome of the voting in each pairing and the outcome of this procedure.

3 Chapter 7

1. For an arbitrary extensive form game:

(a) Define *Common Knowledge.*

(b) Define a *Subgame*.

(c) Define a Subgame Perfect Equilibrium, you do not need to give a technical definition of a Nash equilibrium.

2. Consider a game of Bank Runs. There are I people who can either deposit money in the bank (B) or their mattress (M). If they deposit money in their mattress they always get a return of 1. If $2 \leq K \leq I$ deposit their money in the bank then those who deposit money in the bank get a return of $1 + r$, where $r > 0$, otherwise they get zero.

(a) Consider this as a simultaneous game, in other words all players choose between M and B at the same time.

i. Find the best response for each player.

ii. Find the pure strategy Nash equilibria of this game. Prove that they are equilibria.

(b) Now consider this as a sequential game, where first player 1 chooses between M and B, then player 2 chooses, and so on.

i. Find the best responses of the last player to make a decision.

ii. Given those best responses, find the best responses of the next to last player to make a decision.

iii. Now find the unique Subgame Perfect equilibrium of this game. Prove your answer.

iv. Find the other Nash equilibrium outcome of this game, specifying each player's strategy in this equilibrium.

v. Now find the minimal size group that must be making an empty threat for the Nash equilibrium outcome to occur. Assuming only the minimal size group is making an empty threat, which players can *not* be making an empty threat? Use this to identify who must be in this minimal size group.

3. Consider the following T period extensive form game. In period $t < T$, if t is odd,

t.a Player 1 makes an offer $s_1 \in [0, 1]$

t.b Player 2 accepts or declines. If player 2 accepts then the game terminates with the payoffs: $u_1(s_1, A) = \delta^{t-1} s_1$ and $u_2(s_1, A) = \delta^{t-1}(1 - s_1)$. If player 2 declines then the game goes to period $t+1$.

If t is even the only difference is that in $t.a$ player 2 makes the offer s_1 and player 1 accepts or declines in $t.b$.

If $t = T$ then the only difference is that in the case of a rejection $u_1(s_1, R) = u_2(s_1, R) = 0$.

Assume throughout that if a person is indifferent between accepting and rejecting that he will accept.

(a) Assume that $T = 1$.

i. Find the best response of player 2 in $1.b$.

ii. Find the subgame perfect equilibrium value of s_1.

(b) Assume that $T = 2$.

i. Find the subgame perfect best response of player 2 in $1.b$.

ii. Find the subgame perfect equilibrium value of s_1 in $1.a$.

iii. Why didn't I ask you any questions about period 2?

(c) Assume that $T = 3$

i. Find the subgame perfect best response of player 2 in $1.b$.

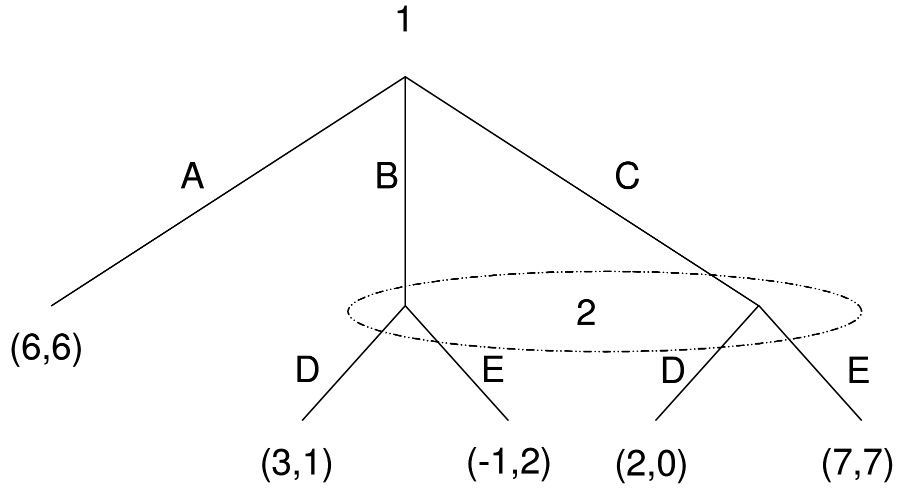
ii. Find the subgame perfect equilibrium value of s_1 in $1.a$.

iii. Why didn't I ask you any questions about periods 2 and 3?

(d) Assume that T is odd, show that the subgame perfect equilibrium value of s_1 in $1.a$ is $s_1 = \frac{1}{1+\delta}\left(1 + \delta^T\right)$.

4 Chapter 10

1. Consider the following general extensive form game.



   (a) Treating this as a game of complete information, find the Subgame Perfect equilibrium. You may solve this on the game above, but you will loose two points if you do not explain your notation below.

   (b) Rewrite these strategies as strategies of the game as given. Are they a weak sequential equilibrium? If so be sure to carefully specify the beliefs, if not explain why not.

   (c) Write down an equivalent normal form game for this extensive form game.

   (d) Find the Subgame Perfect equilibria of this extensive form game.

   (e) One of these Subgame Perfect equilibria is not a Weak Sequential Equilibrium. Explain the problem with this strategy.

2. Consider a model of bank runs. In this question I am going to change notation from before, and make it a game of incomplete information, but otherwise it will be the same game as before. First of all, their are N players who all begin the game with one unit of money in the bank. Each player has the choice between withdrawing their unit of money and using it for purchases—W, or saving their money for the next period—S. If they withdraw their unit of money they get a return of $1 + \mu_i$, where μ_i has a known distribution $F(\cdot)$ over $[-1, 3]$. To be clear only person i knows μ_i, for all other players it is only known to have the given distribution. If they choose to save their unit of money they get $1 + r$ if at least $2 \leq K \leq N$ people (including this person) choose S. **However we will only analyze the case where** $K = N \geq 2$. We assume that $r \leq 1$. Throughout

this analysis players will use a *cut-off* strategy and we will ignore the equilibrium where everyone chooses W unless it is the only equilibrium.

(a) For an arbitrary game, define a cutoff strategy and in this game explain what cutoff strategy players will use.

(b) First assume that $F(\mu) = \frac{1}{2}(\mu+1)^{\frac{1}{2}}$, and that $N = 2$.

   i. Consider the simultaneous game, where both players choose S or W at the same time. Find the equilibrium in symmetric strategies.

   ii. Consider the sequential game, where player 1 chooses S or W, then player 2 chooses S or W after seeing what player 1 does.

      A. Prove that player 2 uses the Pareto Efficient strategy. I.e. S if Player 1 chooses S and $\mu_2 \leq r$, W otherwise.

      B. Find the equilibrium strategy for player 1.

   iii. Show that the probability that either player saves their money in the simultaneous model is strictly lower than the probability that either player saves his money in the sequential model. What does this tell you about the probability of bank runs in the two models? Explain this result.

(c) Now consider the general sequential game, where player $i \in \{1, 2, 3, ..., N\}$ choose S_i or W_i observing the choices of players $j < i$.

   i. For this part of the question assume that $F(\cdot)$ is binary: with probability ρ $\mu_i = 3$ and with probability $1 - \rho$ $\mu_i = 0$.

      A. Prove that player N will use the Pareto Efficient strategy. I.e. S_N if $\mu_N = 0$ and all other players have played S, W_N otherwise.

      B. Prove that there is an K^* such that if $i \geq N - K^*$ i will use the Pareto Efficient strategy. I.e. S_i if $\mu_i = 0$ and all previous players have played S, W_i otherwise.

      C. Prove that if $i < K^*$ player i will never choose S_i.

      D. Find a explicit formula for K^*, and note that it is independent of N. Ignore the fact that it will usually not be an integer in this analysis. Comment on your result.

   ii. Now assume that $F(\cdot)$ has an arbitrary distribution.

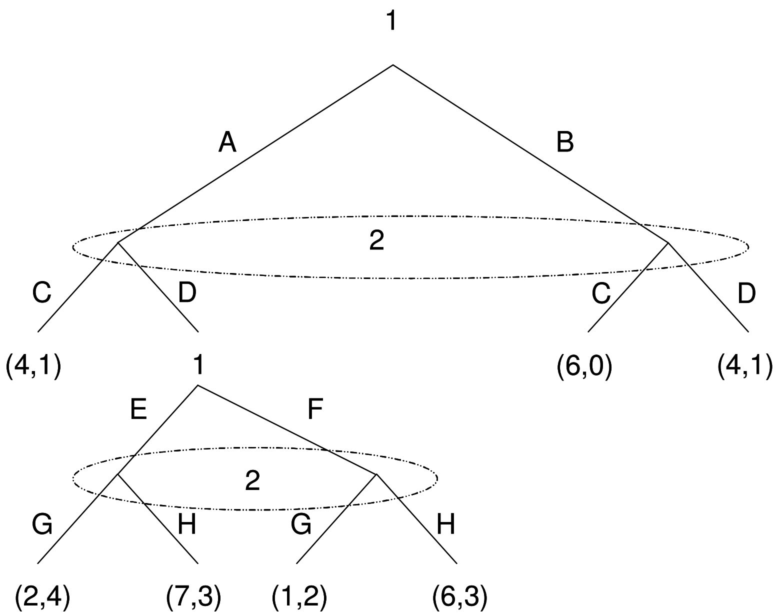
      A. Prove that player N will use a Pareto Efficient strategy.

      B. Show that player $N-1$ will not use a Pareto Efficient strategy. I.e. the probability he will choose W_{N-1} is strictly positive even if $\mu_{N-1} < r$.

      C. Prove by induction that if $j < i$ then the probability j chooses W_j is either strictly higher than the probability i chooses W_i or equal to one.

D. Conclude that if N is large enough the only equilibrium is everyone playing W. Do you think this would also be true if $K < N$? Why or why not?

3. Consider the following general extensive form game. Player 1 has the first decision, choosing between A and B.



   (a) Write down all of the strategies of both players.
   (b) Treating this as a game of complete information, find the Subgame Perfect equilibrium. You may solve this on the game above, but explain your notation below.
   (c) Rewrite these strategies as strategies of the game as given. Are they a weak sequential equilibrium? If so be sure to carefully specify the beliefs, if not explain why not.
   (d) Are there any other weak sequential equilibria? If so write them down and clearly specify the beliefs, if not prove this.
   (e) Are there any Nash equilibria **outcomes** that are not weak sequential equilibria? If so write them down and explain why they are Nash equilibria, if not prove this. *(Notice: I am explicitly asking about outcomes, not strategies.)*

4. About Weak Sequential Equilibrium:

   (a) Define an *assessment*.
   (b) Define the *sequential rationality* of an assessment.

(c) Define the *consistency* of an assessment. You do not need to define Bayes' Rule.

(d) Define a *Weak Sequential Equilibrium*.

(e) Why do we need to be so careful to specify beliefs properly in a weak sequential equilibrium?

5. A General Extensive Form War of Attrition: Assume that there are two players, a and b, and let i be an arbitrary player, $i \in \{a, b\}$. In this game instead of deciding whether or not to fight for the next t periods in each period the two players decide whether or not to fight in the current period. Thus in period t a player's strategic choices are to fight (F_{it}) or Acquiesce (A_{it}). If a player choose to acquiesce in period t then the game ends and the other player wins the prize, otherwise you go to the next period. The payoffs are identical to the payoffs in the normal form game. Let t_i be the last period player i chooses to fight. Then:

$$u_i(t_i, t_j) = \begin{cases} 1 - c_i t_j & \text{if} \quad t_i > t_j \\ \frac{1}{2} - c_i t_i & \text{if} \quad t_i = t_j \\ -c_i t_i & \text{if} \quad t_i < t_j \end{cases}$$

where $c_i \geq 0$. We will only consider the game where $t \leq T$, where T is some fixed value. Up to this point this is exactly the same game as you saw on the last exam. Let me remind you that we found on that exam that if $c_i < \tilde{c} = \frac{1}{2}\frac{1}{T}$ then i will fight to the end of the game.

Now we will transform this into a general extensive form game by assuming players have an unknown type. To be precise i knows $c_i \geq 0$ but all i knows about c_j is that it has a probability density function $g(c)$ and a cumulative density function $G(c)$ where $c \in [0, \infty)$. We assume that for any $0 < c < \infty$, $g(c) > 0$ and note that this implies $0 < G(c) < 1$ and that $\Pr(c_i < c) = \Pr(c_i \leq c) = G(c)$

(a) Prove that there is a $\bar{c}_1$ such that if $c_i > \bar{c}_1$then in any weak sequential equilibrium i chooses A_{i1}. *(Hint: Consider the best possible case for player i).*

(b) Given this fact, prove that there is a $\underline{c}_1$ such that if $c_i < \underline{c}_1$then in any weak sequential equilibrium i chooses F_{i1}.*(Hints: You may assume that in period 2 i choose* A_{i2}. *You should consider the worst possible case for player i, except that without loss of generality you can assume that if* $c_j = \bar{c}_1$ *player j chooses* A_{j1}.*)*

(c) We have now shown that in this generalized version of the war of attrition that the strong always win if they are strong enough. This matches empirical observation and is good.

Unfortunately in our current model there is frequently fighting, which does not match empirical observation.

Discuss how you could add a preliminary stage to the game that would remove the fighting in most cases. Specifically explain what the players have to do in this stage and what characteristics this interaction would have to take. Be careful to define all of the terms you use.

Would your method ever completely remove the fighting? Does your resulting model fit empirical observation? (To be concrete, consider the specific case where two bears come across a freshly killed deer at the same time.)

6. For a general extensive form game:

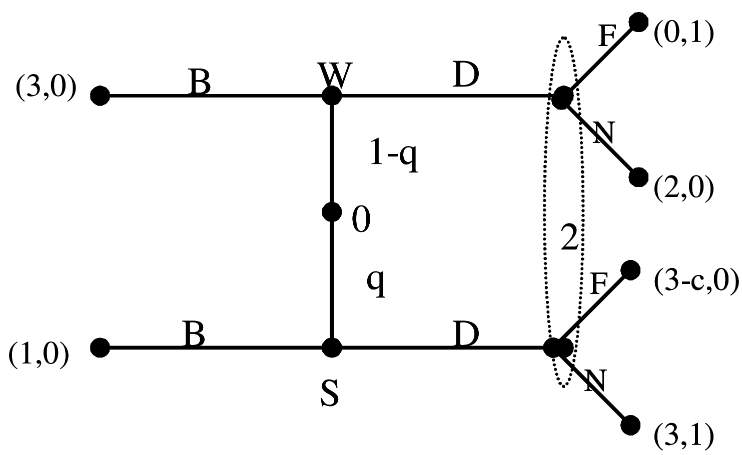
   (a) Define an *assessment.*

   (b) The *Sequential Rationality* of an assessment.

   (c) *Consistency* of an assessment. (You do not need to give a precise definition of Bayes Rule.)

   (d) Define *Weak Sequential Equilibrium:*

7. The Bully at the Door: You are eating in an otherwise empty restaurant when you hear someone yell at the top of their lungs: "I'm going to fight the next person who walks out this door." You look out the window and see a bully, not too strong looking, but a bully. Now you have two choices. You can sneak out the back (B) and leave the bully sitting there, or you can go out the front door (D) and face the bully. You may be one of two types: strong, S, with probability q or weak, W, with probability $1-q$. While you know your type you know the bully will not be able to figure it out, but you know the bully knows q and you think he will not want to fight you if you are strong. Notice that the bully does have a choice between fighting (F) and not fighting (N) once you walk out the door. The payoffs of this game are:



note that c is common knowledge.

(a) Write down all of the strategies of both players.

(b) Treating this as a game of complete information, solve the game using backward induction. You may mark your answers on the game above, but you will loose one point if you do not explain your notation below.

(c) Write the strategy you just found as a strategy of the game as given, and solve for when it is a weak sequential equilibrium. Be sure to find off the equilibrium path beliefs, if there are any.

(d) Which of the types of player one has a dominant strategy? Does the other one sometimes have dominant strategy? When?

(e) Find another pure strategy weak sequential equilibrium. Be sure to check when this strategy is an equilibrium and find off the equilibrium path beliefs, if there are any.

(f) Is there anything about the equilibria of this game that makes you doubt the bully's sanity?

8. Define the following terms, for simplicity I will give you the definition of an *assessment.*

Definition (σ, β) is an *assessment* if $\sigma = \{\sigma_i\}_{i=1}^{I}$ is a strategy and $\beta = \{\beta_i\}_{i=1}^{I}$ are beliefs about what other player will do in a game.

(a) *Sequential Rationality:*

(b) *Consistency* of an assessment:

(c) *Weak Sequential Equilibrium:*

9. About *signals:*

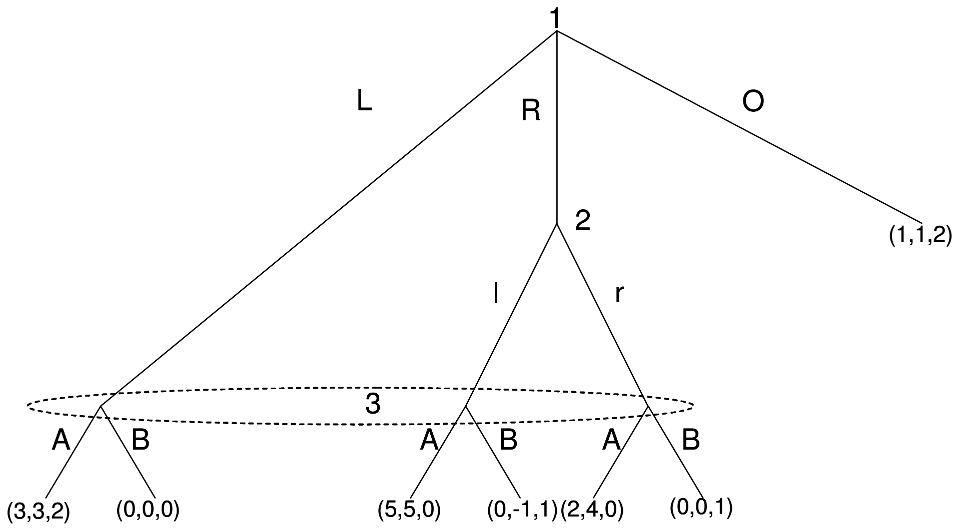
(a) Define a *signal*.

(b) We often say that education must be at least partially a signal. Explain why.

(c) Give two more examples of signals and explain why these things can be seen as signals. Two of the six points will be determined by the originality of your answers. I.e. if your signals were not things that we discussed in class.

10. Consider the following general extensive form game. Player 1 moves first and has three actions to choose between ($\{L, R, O\}$). Player 2 then chooses between l and r knowing what player 1 has chosen. Then player 3 chooses

between A and B knowing only whether player 1 choose O or not.



(a) Write down all the strategies of all players in this game.

(b) Treating this game as one of complete and perfect information, solve the game using backward induction. Then rewrite these strategies for the game as given, is this a weak sequential equilibrium of this game? A subgame perfect equilibrium? A Nash equilibrium?

(c) Find a pure strategy weak sequential equilibrium for this game that doe not use the strategy you just found. Be careful to specify the beliefs of all players that make this a weak sequential equilibrium.

11. Consider a Spence signalling model. There are high and low productivity types. Education has no value in the workplace, but it costs less for high productivity workers to get an education. Thus firms believe that workers who get a more education are more likely to be high productivity.

    To be precise the cost of e units of education for the high type is $c(E|H) = 2E$ and for the low type is $c(E|L) = 4e$. The productivity of the high types is 22 and the productivity of the low types is 10. The a-priori probability a worker is a high type is $\frac{2}{3}$, and after getting an education of length e a worker will earn their expected productivity. Let the firm's beliefs that a worker is a high type given e units of education be $\beta(e) = \Pr(H|e)$.

    (a) What two types of equilibria will we find in this model? Explain each type.

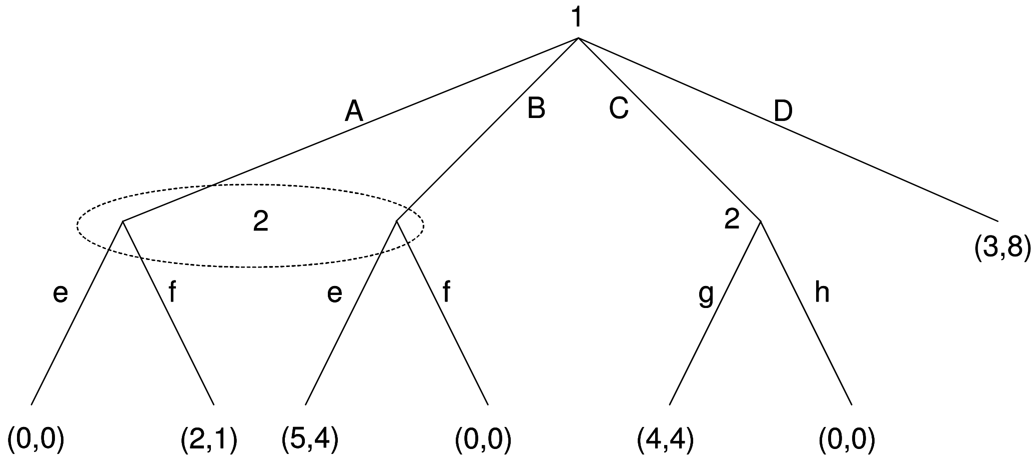
    (b) If low types get no education and high types get 4, can this be an equilibrium or not? If it is specify a firm's beliefs that makes this an equilibrium, if it is not prove that it is not.

(c) Find all equilibria where the two types of workers do not get the same level of education, be sure to prove your work and specify the beliefs.

(d) If everyone gets 1, can this be an equilibrium or not? If it is specify a firm's beliefs that makes this an equilibrium, if it is not prove that it is not.

(e) Find all equilibria where everyone gets the same level of education, be sure to prove your work and specify the beliefs.

12. Consider the following extensive form game.



(a) Write down the strategies of both players.

(b) Solve this game as if it was one of complete information using backward induction. You may solve it using the game above but explain your notation below. *Only for this question should you treat this as a game of complete information*

(c) Rewrite the strategies you just found as ones of the game as given and verify whether or not they are equilibrium strategies of this game.

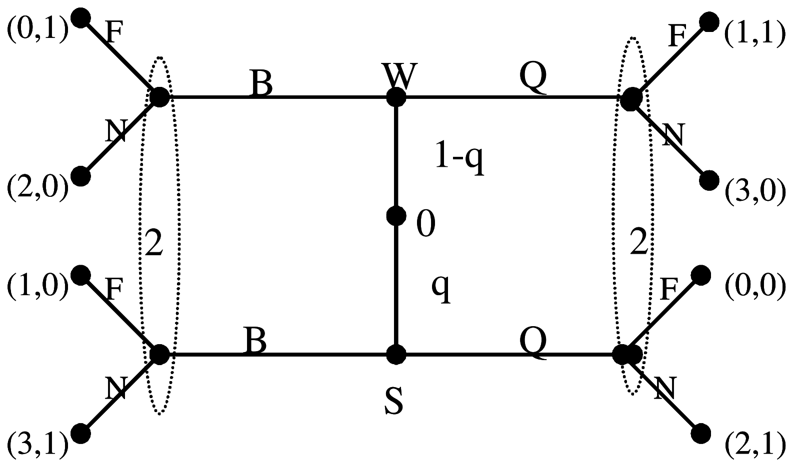
(d) Find all weak sequential equilibria of this game, be careful to write down the complete strategies and beliefs of both players.

(e) Find a Nash equilibrium of this game that is not a weak sequential equilibrium, and explain how this strategy relies on empty threats to be an equilibrium.

13. Give a precise definition of a weak sequential equilibrium, being sure to define any technical terms you use. *(You do not have to define "Bayes' rule.")*

14. Consider the following Beer/Quiche game. The way the action happens in this game is that first nature (player 0) determines whether player 1 is

weak (W) or strong (S). Player 1 is strong with probability $q \in \left[\frac{1}{2}, 1\right]$. Then player 1 decides whether to drink Beer (B) or eat Quiche (Q) for breakfast. (Notice that weak players prefer Quiche and strong players prefer Beer.) Then player 2 decides whether to Fight (F) player 1 or Not (N). Player 2 does not observe whether player 1 is strong or weak but does see what player 1 eats or drinks for breakfast. (Notice that player 2, the bully, only wants to fight a weak player 1, and that player 1 never likes getting in a fight.)



(a) Assuming player 2 can observe whether player 1 is weak or strong solve this game using backward induction. You can write your answer on the extensive form game but explain your notation below.

Note: From now on analyze the game using the information structure as given above. DO NOT treat it as a game of complete information.

(b) Rewrite the strategy you just found as a strategy in the game as given. Verify whether this is or is not an equilibrium strategy.

(c) Write down player 2's four strategies.

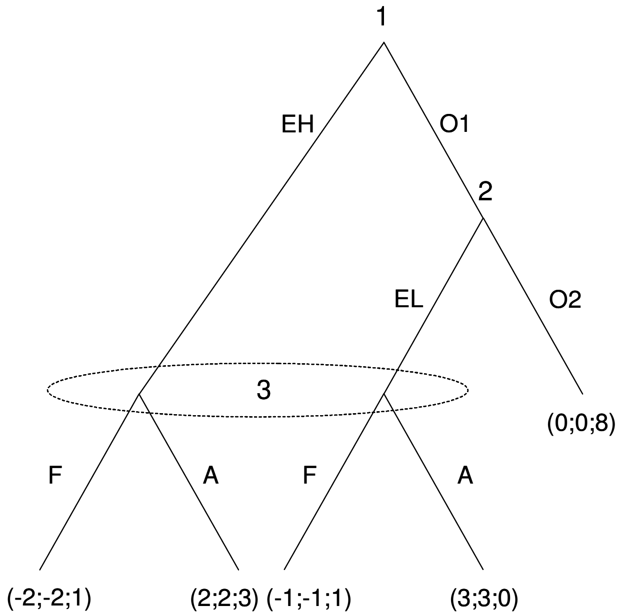
(d) For each of the four strategies of player 2 write down player 1's best response.

(e) For each of the best responses of player 1 (to the four strategies of player 2) write down player 2's best responses. If necessary give conditions when these are best responses.

(f) What are the equilibria of this game? Write down the beliefs of player 2 in each of these equilibria (both on and off the equilibrium path.)

(g) What is a *signal*? Is there signalling going on in the equilibria you found for this game?

15. Consider the following entry game. Players 1 and 2 are in a partnership where they split the profits from entering an industry. Player 1 can choose to enter with high costs (EH) or stay out of the market (O1). If player 1 chooses to stay out then player 2 can over rule this decision and enter as a low cost firm (EL) or they can agree and the partnership will not enter. Player 3 (the incumbent firm) can choose to Fight (F) or not (A,accommodate). Player 3 does not know if the entering partnership is high cost (EH) or low cost (EL).



(a) Solve this game as one of complete information. Then rewrite the strategy you found as an equilibrium strategy in the game as given. Is this strategy a Nash Equilibrium? Is it a Subgame Perfect Equilibrium? Is it a Weak Sequential equilibrium?

(b) For each action of player 3 solve the game using backward induction. Are any of the strategies you found Weak Sequential Equilibrium strategies?

(c) For each weak sequential equilibrium strategy you found in the last part of the question find the beliefs of all players that make these strategies equilibria.

16. A monopolist has just created a new product. The value of the product is unknown to the consumers, it will be $v_h = 15$ with probability $q = \frac{2}{3}$ and $v_l = 6$ the rest of the time. In order to reassure consumer's of the quality of the good the can offer a warranty (w) of either $0, 1, 2, 3, 4, 5,$ or 6 years (the warranty must be in one year increments.) If the good has a

high value then a warranty costs $c_h = 2$ per year to the firm, if it is of low value then it costs $c_l = 4$ per year.

In equilibrium the price offered for the good must be equal to the expected value of the good to a consumer. Thus an equilibrium can be described as a p_h offered if the warranty is w_h, a p_l offered if the warranty is w_l and a price $\tilde{p}$ that is offered if the warranty is any other length. If the good has a high value the firm will offer w_h, and if the good has a low value the firm will offer w_l.

(a) Write down the constraints on the firm that the warranty and prices must satisfy in any equilibrium.

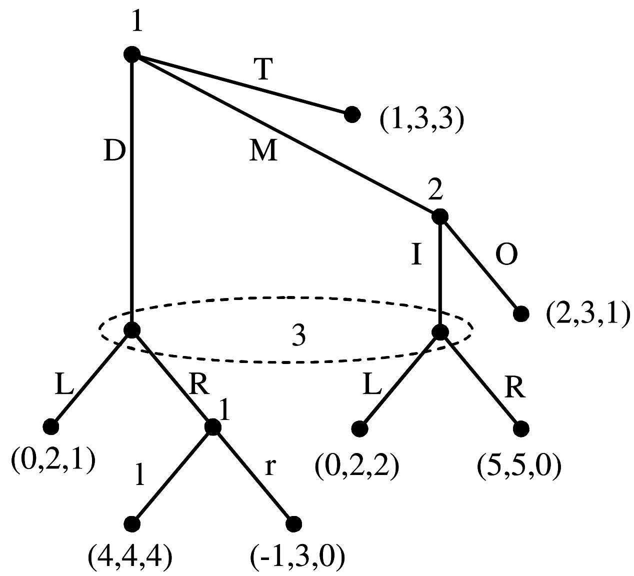
(b) Explain why if we want to find all of the equilibria then we can assume that $\tilde{p} = v_l$.

(c) Find all of the equilibria where $w_h = w_l$. What kind of equilibria are these?

(d) Find all of the equilibria where $w_h \neq w_l$. What kind of equilibria are these?

17. What is a Signal? Give two examples of signals in the real world.

18. Consider the following extensive form game:



(a) List all the strategies of the players.

(b) Find all of the proper subgames, and find all Nash equilibria of these subgames. (A *proper* subgame is a subgame that is not the game itself).

(c) Treating this game as one of complete information.

   i. Find the Backwards Induction equilibrium.

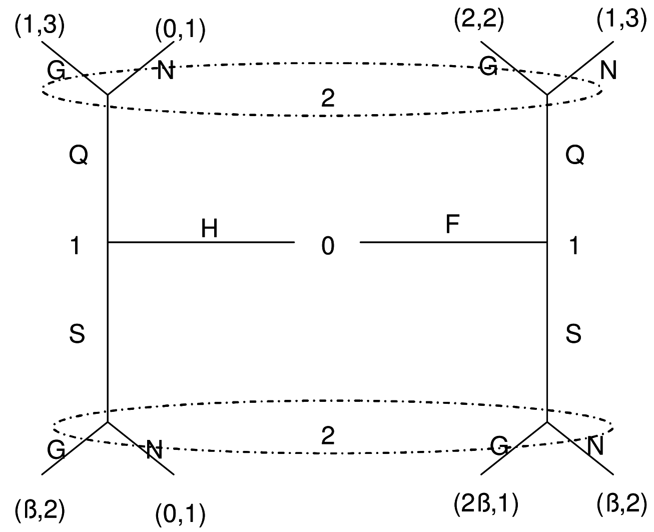
   ii. Write this as a strategy in the game above, I.e. when player 3 does not know if 2 played I or 1 played D.

   iii. Is the strategy you found in the last part a Nash equilibrium? A Subgame Perfect Equilibrium? A Weak Sequential Equilibrium? You must justify your answer for each part.

(d) Find all of the Weak Sequential Equilibria of this game. Be careful to specify the complete range of beliefs for player 3 that makes this an equilibrium.

(e) Find a Nash equilibrium that is not a Subgame Perfect or Weak Sequential Equilibrium of this game.

19. Consider the following signalling game. At the beginning of the game the baby bird (chick, C) is hungry (H) with probability $q > 0$ or full (F) with probability $1 - q > 0$. Then the chick can squawk (S, make a lot of noise) or keep quiet (Q). Finally the parent can decide whether to give the chick the food (G) or not (N). If the chick squawks its payoffs are discounted by β, $0 < \beta < 1$. The extensive form game can be represented as:



(a) Solve this game as if the parent knows whether the chick is hungry or not.

(b) Find weak sequential equilibria such that the strategy of the chicks is as you found in 19a. Note that the parent's strategy will change with q.

(c) Find the values of β such that there is a weak sequential equilibrium where parents only feed squawking chicks and only hungry chicks get fed.

(d) Find the values of β and q such that there is a weak sequential equilibrium where parents only feed squawking chicks but all chicks get fed. Is this equilibrium Pareto Efficient?

(e) Relate your finding about β above to when a signal can be effective.

20. A firm and a union are negotiating over the wage of the workers. The firm's revenue is R and the wage bill it pays its workers is $W \geq 0$. The union is strong with probability $q > 0$ and weak with probability $1-q > 0$ and the firm does not know whether the union is strong or weak. If it is strong it will accept offers of W_h or higher ($R > W_h$), if it is weak it will accept offers of $W_l = 0$ or higher. In order to signal its strength it can go on strike for a length of time S. If it is strong the cost of going on strike for S length of time is $C_s(S) = c_s S$, if it is weak the cost is $C_w(S) = c_w S$ where $c_w > c_s > 0$. If the union rejects the offer both parties get zero. You may assume that the firm never offers any wage above W_h or below W_l.

    (a) The structure of the game is that the union first goes on strike for a time S and then the firm makes a take it or leave it offer.

        i. Find the set of equilibria where neither type of union goes on strike.

        ii. Find the set of equilibria where both types of unions go on strike for the same length of time. (Hint: For some values of q the maximum length of the strike will be zero.)

        iii. Find the set of equilibria where the strong union goes on strike for longer than the weak union. How long will the weak union go on strike for?

    (b) Now assume that the uncertainty is symmetric, or that both the union and the firm do not know if the union is weak or strong. However there are now two rounds of negotiations. First the union goes on strike for some length of time S_1 then the firm makes a take it or leave it offer. If the union does not accept the offer it can go on strike for some length of time S_2, and then the firm makes a final take it or leave it offer. You should assume that S_1 and S_2 can only be positive integer quantities ($\{0, 1, 2, ...\infty\}$). After the first strike the union knows whether it is weak or strong. *Clarification: If $S_1 = 0$ then the union does not know what type it is. If $S_1 = 0$ and a strong union is offered a wage lower than W_h the union will not accept it.*

        i. Find the set of equilibria where neither type of union goes on strike.

        ii. Now find the set of equilibria where the total time both unions go on strike ($S_1 + S_2$) is the same and $S_1 > 0$. (Notice that these equilibria will only exist for some values of q). When will the unions prefer these equilibria to the equilibria where neither type goes on strike?

iii. Now find the set of equilibria where the strong union goes on strike for longer than the weak union. What is the minimum and maximum value for S_1 in such an equilibrium?

5 Chapters 14 and 15

1. This question will cover a large number of topics all based on the following game:

		Player 2		
		α	β	δ
	A	$2;8$	$13;13$	$4;6$
Player 1	B	$13;11$	$6;6$	$9;18$
	C	$15;8$	$17;3$	$2;1$

In this question you should **not** consider mixed or correlated strategies.

(a) Find the (pure strategy) best responses of both players. You may mark them on the table above but you will loose two points if you do not explain your notation below.

(b) Find the (pure strategy) Nash equilibrium strategies. Be sure to write down the strategies, and explain why these are Nash equilibria below.

(c) For the following questions we take the normal form game above and transform it into a sequential game by letting player 1 move first and then having player 2 move after observing what player 1 has done.

i. How many strategies does player 2 have in this new game? Explain.

ii. Find the Subgame Perfect equilibrium strategies of this game and write them down below. Explain your logic.

iii. Define *First Mover's Advantage* for an arbitrary game G with two players.

iv. Prove First Mover's Advantage.

v. Do we know anything about how the second mover's payoffs will change when we transform an arbitrary normal form game (G) into a sequential game? Why or why not?

(d) For the following questions we take the normal form game above as the stage game of a twice repeated game. We will focus on supporting the strategy pair X in the first period.

i. Prove that if player 1 does the correct thing in the first period then the strategy in the second period must be independent of what player 2 does.

ii. Write down an equilibrium strategy where players play X in the first period, and prove it is an equilibrium in all subgames.

(e) For the following questions we take the normal form game above as the stage game of an infinitely repeated game. We will focus on strategies supporting X on the equilibrium path.

i. Consider the following strategy:

$$s_t = \begin{cases} X & \text{if} \quad s_{t-1}^1 = x_1 \\ \mu & \text{else} \end{cases}$$

For some $\mu \in \{A, B, C\} \times \{\alpha, \beta, \delta\}$ show that this is **not** an equilibrium as $\delta \to 1$ for any μ, and for some μ find the $\{\underline{\delta}, \overline{\delta}\}$ such that if $\underline{\delta} \leq \delta \leq \overline{\delta}$ it is an equilibrium.

ii. Define the minmax payoff and find both these payoffs and the related strategies in this game.

iii. Write down the *best* equilibrium strategies in this game (as in this strategy gets cooperation for the lowest value of δ) and prove that it is an equilibrium in all subgames. Find the critical $\underline{\delta}$ such that if $\delta \geq \underline{\delta}$ it is an equilibrium.

2. Consider the following Normal form game as the stage game of an infinitely repeated game with a discount factor δ, where $0 \leq \delta < 1$.

		Player 2		
		α	β	γ
	A	$-1; 8$	$5; 12$	$6; 6$
Player 1	B	$0; -6$	$8; 0$	$8; -3$
	C	$2; 2$	$5; -1$	$9; 1$

(a) Find the best responses of both players in this stage game. You may mark them on the game but you will loose two points if you do not explain your notation below.

(b) Find the pure strategy Nash equilibria of this stage game.

(c) Define the *minimax* payoff in pure strategies for an arbitrary stage game G.

(d) Find the minimax payoff in pure strategies for both players in this stage game. Also write down the associated strategy pair.

(e) Find the unique Pareto Efficient and symmetric strategy pair in this game, and then find all other Pareto Efficient strategy pairs in this game. *(Note: Your answer should be in terms of strategies, not payoffs.)*

(f) Let $\vec{a}$ be the constant path where you play $a \in \{A, B, C\} \times \{\alpha, \beta, \gamma\}$ in every period. Prove that $V_i(\vec{a}) = \frac{1}{1-\delta} u_i(a)$ for $i \in \{1, 2\}$.

(g) Write down a strategy that will support the unique Pareto Efficient and symmetric strategy pair in this game, and find the minimal δ such that it is an equilibrium given your strategy. Be sure to prove that your strategy is an equilibrium in all subgames.

(h) Find the set of pure strategy pairs such that $a \in \{A, B, C\} \times \{\alpha, \beta, \gamma\}$ can be expected to be played in a subgame perfect equilibrium for high enough δ.

(i) Find a strategy that will support any of the strategy pairs you just said could be subgame perfect equilibria for high enough δ, and prove that for high enough δ any of these strategy pairs can be an equilibrium. Be sure to prove that your strategy is an equilibrium in all subgames. *Note: You do not have to solve the model in each and every case, just show that it can be an equilibrium.*

3. Consider the following stage game in an infinitely repeated game with a discount factor of $\delta \in [0, 1)$:

		Player 2		
		α	β	γ
	A	$2; 0$	$7; 6$	$2; 7$
Player 1	B	$7; 2$	$8; 1$	$-3; -3$
	C	$6; 6$	$11; 6$	$1; 11$

(a) Find the best responses for both players, you may mark them on the table above but you will loose two points if you do not explain your notation below.

(b) Find the pure strategy Nash equilibria.

(c) Prove that the value of getting x every period in the future is $V(x) = x/(1-\delta)$.

(d) Find a subgame perfect equilibrium strategy and the minimal δ (call it δ^*) such that playing (C, β) forever is the equilibrium path. *Be careful to prove it is an equilibrium in every subgame. Notice that 3 points will be given for correctly writing down the strategy.*

(e) Find a subgame perfect equilibrium strategy and the minimal δ (call it δ^*) such that playing (C, α) forever is the equilibrium path. *Be careful to prove it is an equilibrium in every subgame. Notice that 3 points will be given for correctly writing down the strategy.*

(f) For every pair of pure strategies state whether there can or can not be a subgame perfect equilibrium for high enough δ where this pair of strategies is played forever on the equilibrium path. You do not have to find the δ, simply state whether it is possible or not. **But be sure to explain your reasoning.**

4. Consider the repeated game with a Cournot Duopoly game as its stage game. Two firms each simultaneously choose to produce $q_i \geq 0$. The market price is $P = 24 - \frac{1}{2}(q_1 + q_2)$, and each firm has the same symmetric costs, $c(q_i) = 12q_i$. In this question I do not promise that all answers will be in integers, they should be simple fractions however.

(a) Find the unique Nash equilibrium of this stage game, including the profits.

(b) Find the amount each would produce in the symmetric collusive outcome—where firms maximize the sum of the profits. Find the profits.

(c) If firm 2 produces the symmetric collusive output what is the optimal amount for firm one to produce? What is it's profits?

(d) Find the minmax profits and the minmax strategies in this game. Is it a Nash equilibrium? Prove your answer.

(e) Now consider the finitely repeated game, where this stage game is repeated T times. Find the unique subgame perfect equilibrium for all $0 \leq \delta \leq 1$. Prove your answer.

(f) Now consider the infinitely repeated game with this game as a stage game. Find a trigger or Grimm strategy that will support firms producing the symmetric collusive outcome in every period, and find the minimal δ such that firms can produce the symmetric collusive outcome every period. Be certain to carefully prove your strategy is an equilibrium.

(g) Is it possible that the collusive outcome could be supported for a lower δ in a subgame perfect equilibrium? If so how would you do this? Precise mathematical answers are not needed, just a discussion.

5. Consider the following game:

		Player 2			
		α	β	ψ	ε
	A	$14; 14$	$15; 13$	$5; 15$	$4; 12$
Player 1	B	$14; 13$	$17; 17$	$-2; 10$	$0; 9$
	C	$14; 8$	$9; 12$	$8; 17$	$6; 14$
	D	$16; 10$	$10; 7$	$6; 9$	$11, 11$

(a) The Static Game:

i. Find all the of the best responses for both players. *You will automatically loose 2 points if you do not explain your notation below.*

ii. Find all of the pure strategy Nash equilibria. For at least one of them explain why it is a Nash equilibrium.

(b) The finite repeated game: This game is repeated twice, with the actions taken in the first period known before the second period actions are taken. The value of a sequence of action profiles is the sum of the stage game payoffs.

i. In the second period which strategy profiles can be played in a subgame perfect equilibrium? Explain why these are the only strategy profiles that can be played.

ii. Write down a subgame perfect equilibrium strategy where (A, α) is played in the first period. Prove that your strategy is a subgame perfect equilibrium.

iii. For all strategy pairs that are not either (A, α) or a Nash equilibrium of the static game show whether or not they can be equilibria of the twice repeated game.

(c) The infinitely repeated game: The game is repeated for an infinite number of periods, with the actions taken in period $t-1$ known before actions in period t are taken. The value of a sequence of action profiles is the discounted sum of the stage game payoffs, with the discount factor $\delta \in (0,1)$. (Thus a payoff of x t periods in the future is worth $\delta^{t-1}x$ today).

i. Show that for any x and $\delta \in (0,1)$ $\Sigma_{t=1}^{\infty}\delta^{t-1}x = \frac{1}{1-\delta}x$. *You may use this below even if you can not show this.*

ii. Write down a subgame perfect equilibrium strategy where people expect to play (A, α) forever (assuming no one deviates). Find the minimal δ such that any strategy is an equilibrium. **Be sure to check all subgames two points will be given for checking less obvious subgames.**

iii. What is a minimax strategy? Find the minimax strategies and payoffs of both players.

iv. For all strategy pairs that are not either (A, α) or a Nash equilibrium of the static game explain whether or not they can be equilibria of the infinitely repeated game as $\delta \to 1$. *Notice I am not requiring a rigorous proof, just an explanation will suffice.*

6. Consider the following Stage Game:

		Player 2		
		L	C	R
	U	$4;4$	$1;2$	$6;3$
Player 1	M	$7;0$	$0;3$	$3;2$
	D	$3;0$	$2;1$	$2;0$

(a) Find all the best responses of both players and the Nash equilibrium (or equilibria) strategies. *You may use the table above to mark your answers but explain your notation below.*

(b) Consider this now as the stage game of a standard T period repeated game (where $T < \infty$.) Find all the equilibria of this game as $T \to \infty$, prove your answer.

(c) Consider this now as the stage game of an infinitely repeated game, let the discount factor of both players be $\delta \in (0,1)$.

i. Find a strategy such that players play (U, L) in every period. This strategy must be a subgame perfect equilibrium for high enough δ.

ii. Prove that this is an equilibrium strategy for high enough δ, and find the critical δ^* such that if $\delta \geq \delta^*$ then this is a subgame perfect equilibrium.

iii. Consider the following strategy: "If U last period then (U, L) this period, otherwise (D, C)." Explain in intuitive terms why this can not be an equilibrium for high enough δ, and show precisely that it can never be an equilibrium.

iv. Which strategy pairs can be played every period in a subgame perfect equilibrium? Explain why.

7. You are explaining the restaurant quality problem to another student (restaurants have a short run incentive to produce low quality). This student says: "I can trust restaurants to produce high quality. If I don't like the food I am served I just won't go back for a while."

Explain in what situations this student would be correct, and when this student would not be correct. Further discuss what considerations should go into how long "a while" should be. ("a while" is the same as "for some period of time.")

8. Assume that the following stage game is repeated T times.

	α	β	ψ
A	$0; 4$	$0; -2$	$5; 6$
B	$0; 5$	$6; 7$	$3; 2$
C	$2; 3$	$8; 1$	$0; -2$

(a) Find all the best responses of both players and the Nash equilibria of this stage game. Write the Nash equilibrium strategies below.

(b) If $T = 2$.

i. Show that there is a Subgame Perfect equilibrium strategy where (B, β) can be the action pair that is played in the first period. Be careful to prove precisely that it is Subgame Perfect.

ii. Find all action pairs that can be played in a Subgame Perfect equilibrium in the first period. Explain your answer. *(Note, one explanation should be sufficient for all the action pairs.)*

(c) Find the minimal T such that (A, β) can be played in the first period of a Subgame Perfect Equilibrium.

9. Consider the infinitely repeated game with the following stage game.

	α	β	ψ
A	$2; 3$	$-4; 24$	$6; 6$
B	$3; 4$	$-3; 8$	$4; 1$
C	$6; -2$	$0; 0$	$8; -1$

For a sequence of action pairs $\mathcal{A} = \{a_t\}_{t=1}^{\infty}$ the value of player i of this sequence is $v_i(\mathcal{A}) = \sum_{t=1}^{\infty} \delta^{t-1} u_i(a_t)$ where $\delta \in (0,1)$ and $u_i(a_t)$ is the utility of person i from the stage game when the action pair a_t is played.

(a) Find all the best responses of both players and the Nash equilibria of this stage game. Write the Nash equilibria strategies below.

(b) Find a Subgame Perfect equilibrium strategy and the minimal δ such that (B, α) is the action pair that is played unless someone deviates from the strategy.

(c) Find a Subgame Perfect equilibrium strategy and the minimal δ such that (A, ψ) is the action pair that is played unless someone deviates from the strategy.

(d) Find all action pairs that can be the action pair played unless someone deviates from the strategy in a Subgame Perfect equilibrium. *(Note: one explanation should work for all action pairs)*

10. Consider the following stage game:

	α	β	ψ	δ	ε
A	$9;3$	$4;4$	$4;2$	$3;5$	$4;4$
B	$11;3$	$7;6$	$2;5$	$2;4$	$0;0$
C	$0;1$	$5;1$	$4;-1$	$1;0$	$5;2$
D	$10;9$	$6;10$	$3;-3$	$2;7$	$2;8$
E	$8;1$	$5;2$	$5;4$	$2;1$	$4;3$

(a) Find all of the best responses and Nash equilibria of this game.

(b) Define the *minimax* payoff, what is it's relationship with individual rationality? Find the minimax payoff for both players in this stage game.

(c) Now consider the finitely repeated game where this stage game is repeated T times. In all cases you want to show that a pure strategy pair of actions (like (D, α)) can or can not be supported as the pair of actions in the first period of a Subgame Perfect equilibrium. In every case be sure to check all of the Subgames.

i. If $T = 2$ show that (D, α) can be the first period action profile.

ii. If $T = 2$ show that (E, α) can be the first period action profile.

iii. If $T = 2$ find the three pairs of actions that *cannot* be first period action profile. Explain why they can not be.

iv. What is the minimal T such that *every* pair of actions can be the first period action profile?

(d) Now consider the infinitely repeated game with the game above as the stage game. Let the common discount factor be δ. In all cases you want to find when a particular pure strategy action pair (like

$(D,\alpha))$ can be the action pair that people expect to always play as part of a Subgame Perfect equilibrium. In other words the actions they expect to play in the first period and every period thereafter. In every case be sure to check all of the Subgames.

i. Find a strategy and the minimal δ (given your strategy) such that (D,α) can be supported as equilibrium.
ii. Find a strategy and the minimal δ (given your strategy) such that (A,ε) can be supported as equilibrium.
iii. Find all the pairs of actions that can be supported as equilibria and explain why they can be supported.
iv. Consider the strategy:

$$s_{1t} = \begin{cases} B & \text{if} \quad a_{2t-1} = \alpha \\ C & \text{else} \end{cases} \qquad s_{2t} = \begin{cases} \alpha & \text{if} \quad a_{2t-1} = \alpha \\ \varepsilon & \text{else} \end{cases}$$

where a_{2t-1} is the action player 2 chose last period. Show that this is not an equilibrium for any δ, and then explain how it could be altered to be an equilibrium.

11. Consider the following interaction. A customer can buy food from a restaurant but does not know if it is low or high quality. The restaurant can choose to produce either high (H) or low quality (L) food, and the customer can choose whether to buy (B) or not (N). If the consumer chooses not to buy then both parties get nothing. Assume that the cost of high quality is c_h, the cost of low quality is c_l, and that the value of high quality to the consumer is $v_h > c_h$, and the value of low quality food to the consumer is v_l. Assume that $v_h - c_h > v_l - c_l > 0$ and $c_h > v_l$. Let the price that the food is sold at be p.

(a) Analyzing this as a normal or strategic form game.

i. Letting the customer be the row player, draw a matrix that describes this game.
ii. Assuming $p > v_h$ find the best responses of both players and the Nash equilibrium.
iii. Assuming $v_h > p > c_h$ find the best responses of both players and the Nash equilibrium.
iv. Assuming $v_l > p > c_l$ find the best responses of both players and the Nash equilibrium.
v. Is there any common element to the equilibria in all cases? Given the equilibria of these games, what is the profit maximizing choice for p?

(b) Analyzing this as an extensive form game.

i. Draw an extensive form game with the customer choosing first that describes this interaction.

ii. Argue that the Weak Sequential equilibria of this game are the same as the Nash equilibria of the Normal form game.

iii. Does it matter whether the restaurant knows what the customer has done or not in the extensive form game? Why or why not?

(c) Analyzing this as a repeated game, where the discount factor is δ, $0 < \delta < 1$.

i. Show that

$$\Sigma_{t=0}^{T} \delta^t a = \frac{1 - \delta^{T+1}}{1 - \delta} a \ .$$

You may assume this throughout the rest of the question even if you think your answer is not correct.

ii. One of your friends claims that he can trust the restaurant to provide high quality because if the restaurant does not then he would never go back.

A. Formalize this intuition as a repeated game strategy.

B. Show that if δ is high enough then the restaurant will produce high quality if consumers use this strategy. Find a formula for how high δ has to be in terms of the other fundamentals of the model. Is it increasing or decreasing in p?

C. Find the profit maximizing value for p when consumers use this strategy (and δ is high enough that the firm will produce high quality). Are their profits higher or lower than in part 11a? Relate this finding to the success of chain restaurants like McDonald's and Burger King.

iii. Another friend claims that he doesn't have to never go back to that restaurant, if he only doesn't go back for one period it is enough.

A. Formalize this intuition as a repeated game strategy.

B. Let $c_l = 0$, $c_h = 2$, $p = 6$, and $\delta = \frac{3}{4}$, show that the restaurant will produce high quality if consumers use this strategy.