ECON 439

Quiz 3—Mixed Strategies

Kevin Hasker

1. *(1 point)* **Honor Statement:** Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person. I will also neither help others nor use a calculator or other electronic aid for calculation.

   Name and Surname: _

   Student ID: _

   Signature:

   _

2. *(19 points total)* For the following game:

	α	β	γ
A	$5;9^{2}$	$6;5^{1}$	$4;2$
B	$2;3$	$3;4$	$5;7^{12}$
C	$6;4^{1}$	$3;8^{2}$	$1;1$

	α	β
A	$5;9^{2}$	$6;5^{1}$
C	$6;4^{1}$	$3;8^{2}$

$$q = \frac{3}{4}$$

	α	β	γ
A	$8;5^{1}$	$5;7^{2}$	$2;4$
B	$7;6^{2}$	$8;4^{1}$	$3;3$
C	$1;2$	$4;1$	$5;3^{12}$

	α	β
A	$8;5^{1}$	$5;7^{2}$
B	$7;6^{2}$	$8;4^{1}$

$$q = \frac{1}{4}$$

	α	β	γ
A	$9;7^{1}$	$1;3$	$7;8^{2}$
B	$4;1$	$3;5^{12}$	$3;4$
C	$5;7^{2}$	$2;2$	$9;6^{1}$

	α	γ
A	$9;7^{1}$	$7;8^{2}$
C	$5;7^{2}$	$9;6^{1}$

$$q = \frac{2}{3}$$

	α	β	γ
A	$2;3$	$1;2$	$7;5^{12}$
B	$5;9^{2}$	$7;6^{1}$	$4;4$
C	$7;3^{1}$	$3;6^{2}$	$3;1$

	α	β
B	$5;9^{2}$	$7;6^{1}$
C	$7;3^{1}$	$3;6^{2}$

$$q = \frac{1}{3}$$

(a) *(6 points)* Find the pure strategy best responses of both players. You may mark them on the table but you will loose two points if you do not explain your notation below.

Solution 1 *I mark a 1 in the upper right hand corner if it is a best response for player 1, a 2 if it is a best response for player 2*

(b) *(2 points)* Are there any pure strategy Nash equilibria? *(1 point)* Explain your answer. *(1 point)*

Solution 2 *Yes, in every case there is one strategy pair that is a pure strategy best response against the other.*

(c) *(2 points)* Is there a cycle in best responses? If so indicate it below.

Solution 3 *This is indicated by the smaller game above. For example in the game:*

	α	β	γ
A	$9;7^1$	$1;3$	$7;8^2$
B	$4;1$	$3;5^{12}$	$3;4$
C	$5;7^2$	$2;2$	$9;6^1$

The cycle is $(A,\alpha) \to (A,\gamma) \to (C,\gamma) \to (C,\alpha) \to (A,\alpha)$

(d) *(4 points)* Find the *candidate* for a mixed strategy equilibrium where positive probability is placed only on actions in that cycle.

Solution 4 *It is easiest for me to do the analysis game by game. Let* p *be the first strategy in the cylce for P1 and* q *be the first for player 2:*

	α	β
A	$5;9^2$	$6;5^1$
C	$6;4^1$	$3;8^2$

$$\begin{aligned} u_1(A,q) &= 5q + 6(1-q) \\ u_1(C,q) &= 6q + 3(1-q) \end{aligned}$$

$$\begin{aligned} u_1(A,q) &= u_1(C,q) \\ 5q + 6(1-q) &= 6q + 3(1-q) \\ q &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} u_2(p,\alpha) &= 9p + 4(1-p) \\ u_2(p,\beta) &= 5p + 8(1-p) \end{aligned}$$

$$\begin{aligned} 9p + 4(1-p) &= 5p + 8(1-p) \\ p &= \frac{1}{2} \end{aligned}$$

	α	β
A	$8;5^1$	$5;7^2$
B	$7;6^2$	$8;4^1$

$$\begin{aligned} u_1(A,q) &= 8q+5(1-q)=7q+8(1-q)=u_1(B,q) \\ q &= \frac{3}{4} \\ u_2(p,\alpha) &= 5p+6(1-p)=7p+4(1-p)=u_2(p,\beta) \\ p &= \frac{1}{2} \end{aligned}$$

	α	γ
A	$9;7^1$	$7;8^2$
C	$5;7^2$	$9;6^1$

$$\begin{aligned} 9q+7(1-q) &= 5q+9(1-q) \\ q &= \frac{1}{3} \\ 7p+7(1-p) &= 8p+6(1-p) \\ p &= \frac{1}{2} \end{aligned}$$

	α	β
B	$5;9^2$	$7;6^1$
C	$7;3^1$	$3;6^2$

$$\begin{aligned} 5q+7(1-q) &= 7q+3(1-q) \\ q &= \frac{2}{3} \\ 9p+3(1-p) &= 6p+6(1-p) \\ p &= \frac{1}{2} \end{aligned}$$

we have only written the p above.

(e) *(3 points)* Find the expected utility for both players and all strategies against the candidate you found in the last part of this question.

Solution 5 *For the strategies in the cycle you only need to check one of them.*

	α	β	γ
A	$5;9^2$	$6;5^1$	$4;2$
B	$2;3$	$3;4$	$5;7^{12}$
C	$6;4^1$	$3;8^2$	$1;1$

the two critical utilities for player 1 are:

$$\begin{aligned} U_1(A,q) &= 5\left(\frac{3}{4}\right)+6\left(1-\frac{3}{4}\right)=\frac{21}{4} \\ U_1(B,q) &= 2\left(\frac{3}{4}\right)+3\left(1-\frac{3}{4}\right)=\frac{9}{4} \end{aligned}$$

For player 2:

$$\begin{aligned} u_2(p,\alpha) &= 9\left(\frac{1}{2}\right)+4\left(1-\frac{1}{2}\right)=\frac{13}{2} \\ u_2(p,\gamma) &= 2\left(\frac{1}{2}\right)+1\left(1-\frac{1}{2}\right)=\frac{3}{2} \end{aligned}$$

	α	β	γ
A	$8;5^1$	$5;7^2$	$2;4$
B	$7;6^2$	$8;4^1$	$3;3$
C	$1;2$	$4;1$	$5;3^{12}$

the two critical utilities for player 1 are:

$$\begin{aligned} U_1(A,q) &= 5\left(\frac{3}{4}\right)+6\left(1-\frac{3}{4}\right)=\frac{21}{4} \\ U_1(B,q) &= 2\left(\frac{3}{4}\right)+3\left(1-\frac{3}{4}\right)=\frac{9}{4} \end{aligned}$$

For player 2:

$$\begin{aligned} u_2(p,\alpha) &= 9\left(\frac{1}{2}\right)+4\left(1-\frac{1}{2}\right)=\frac{13}{2} \\ u_2(p,\gamma) &= 2\left(\frac{1}{2}\right)+1\left(1-\frac{1}{2}\right)=\frac{3}{2} \end{aligned}$$

	α	β	γ
A	$9;7^1$	$1;3$	$7;8^2$
B	$4;1$	$3;5^{12}$	$3;4$
C	$5;7^2$	$2;2$	$9;6^1$

the two critical utilities for player 1 are:

$$\begin{aligned} U_1(A,q) &= 9\left(\frac{2}{3}\right)+7\left(1-\frac{2}{3}\right)=\frac{25}{3} \\ U_1(B,q) &= 4\left(\frac{2}{3}\right)+3\left(1-\frac{2}{3}\right)=\frac{11}{3} \end{aligned}$$

For player 2:

$$\begin{aligned} u_2(p,\alpha) &= 7\left(\frac{1}{2}\right)+7\left(1-\frac{1}{2}\right)=7 \\ u_2(p,\gamma) &= 3\left(\frac{1}{2}\right)+2\left(1-\frac{1}{2}\right)=\frac{5}{2} \end{aligned}$$

	α	β	γ
A	$2;3$	$1;2$	$7;5^{12}$
B	$5;9^{2}$	$7;6^{1}$	$4;4$
C	$7;3^{1}$	$3;6^{2}$	$3;1$

the two critical utilities for player 1 are:

$$
\begin{aligned}
U_1(A,q) &= 2\left(\frac{1}{3}\right)+1\left(1-\frac{1}{3}\right)=\frac{4}{3} \\
U_1(B,q) &= 5\left(\frac{1}{3}\right)+7\left(1-\frac{1}{3}\right)=\frac{19}{3}
\end{aligned}
$$

For player 2:

$$
\begin{aligned}
u_2(p,\alpha) &= 9\left(\frac{1}{2}\right)+3\left(1-\frac{1}{2}\right)=6 \\
u_2(p,\gamma) &= 4\left(\frac{1}{2}\right)+1\left(1-\frac{1}{2}\right)=\frac{5}{2}
\end{aligned}
$$

(f) *(2 points)* Is your candidate a mixed strategy equilibrium? If you did not find it you can answer in general.

Solution 6 *In general, for it to be a true mixed strategy equilibrium the expected payoff of strategies that have strictly positive probability must be higher than those that are assumed to have probability zero.*

In specific in each of these games the expected utility of the strategies with strictly positive payoff satsifies this, thus the candidate is a Nash equilibrium.