ECON 439
Quiz 5
Dr. Kevin Hasker

1. *(2 Points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test. I also promise not to provide this assistance and that I will not use a calculator or other electronic aid for calculation during this test.

   Name and Surname: _
   Student ID: _
   Signature:

   _

2. *(10 points total)* About *subgame perfect* equilibria.

   (a) *(2 points)* Define a *subgame.*

   Definition 1 *A* subgame *starts with a history, h, such that h is common knowledge and the game after that can be treated as a separate game.*

   (b) *(4 points)* Define a *subgame perfect equilibrium.*

   Definition 2 *A subgame perfect equilibrium is a Nash equilibrium in every subgame.*

   (c) *(4 points)* Prove that a subgame perfect equilibrium always exists, you may assume that a Nash equilibrium always exists.

   Proof. We will prove this using backward induction over the subgames. A *terminal* subgame is a subgame that does not contain any subgames. By definition every such subgame will have a Nash equilibrium, for each subgame choose one of them. Now go to the next to terminal subgames, we will assume that in the terminal subgames the outcome defined by the chosen Nash equilibrium will occur.

   Given these terminal outcomes, there must be a Nash equilibrium of this subgame, and we can now iterate until we are analyzing the entire game. ■

3. *(8 points)* Consider a symmetric duopoly where firms compete by choosing price. If π_s is the profit they both achieve in the game where they both choose price at the same time, and π_1 is the profit firm one achieves by choosing their price first, π_2 the profit firm two achieves by choosing price after observing the price firm one has chosen, explain why $\pi_s < \pi_1 < \pi_2$. To be clear I need you to explain why $\pi_s < \pi_1$ and why $\pi_1 < \pi_2$.

Solution 3 $\pi_s < \pi_1$ *is due to* first mover's advantage. *In the sequential game the second firm will do the same thing as it did in the simultaneous game, so this means that the first firm can push them around to their advantage, and thus* $\pi_s < \pi_1$

$\pi_1 < \pi_2$ *is because of* strategic compliments. *Since the price of the second firm is a strategic compliment to the first, when the first raises their price so will the second.*

This leads to the first firm pumping their price too high, and then the second firm just takes advantage of that to get even more profit.

Remark 4 *In their answers I told them I wanted both an intuitive and technical explanation, so don't give full credit for just the technical terms or just an intuitive argument.*